Field integrals and precession angles for GeN

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Abstract

This note tabulates the field integrals and precession angles for the three Q^2 settings used for the GeN experiment. All TOSCA calculations referenced in this document were done by Steve Lassiter.

1 Introduction

Charybdis is a dipole magnet placed between the target and the neutron polarimeter with an 8.25" gap and is equipped with 2" field clamps. It is used to precess the spin of the neutron. Following the treatment in Hagedorn[1], in the laboratory system at time t, we define the unit vectors $\hat{\beta}$ along the direction of motion of the particle and $\hat{n}$ normal to this vector such that $\hat{n}$ and $\hat{\beta}$ span the plane containing the polarization vector $\mathbf{P}$, as shown in Figure 1. The angle χ is the angle between the direction of motion of the particle and the polarization vector in the rest frame of the particle. In our specific case the magnetic field $\vec{B}$ is essentially perpendicular to this plane. The magnetic moment of the neutron is given by $g\mu_N\sigma$, where $g/2 = -1.913$ and $\mu_N = e/2M_p$ is the nuclear magneton in "natural" units ($\hbar = c = 1$).

The rate change of the precession angle χ (defined with respect to the direction of the particle's motion in the rest frame of the particle) in terms of laboratory quantities is

$$\frac{d\chi}{dt} = g\mu_N\hat{\beta}\cdot\vec{B}\times\hat{n} = g\mu_N B. \tag{1}$$

Integrating over time, we obtain

$$\chi = \int g\mu_N B dt = \frac{ge}{2M_pc}\frac{\int Bdl}{\beta_n}. \tag{2}$$

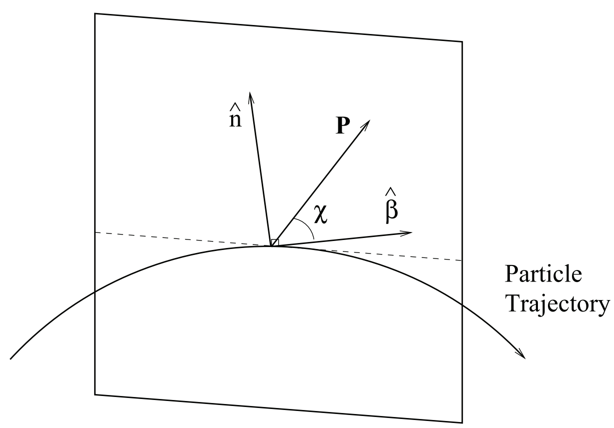

Figure 1: Unit vectors used in the discussion of the spin precession angle.

2 Precession by 90°

During the first part of the experiment we set the current in Charybdis such that the neutron spin was precessed by 90°. For the case where one of the precession angles is zero and the other (χ) is nonzero, $g = G_N^E/G_N^M$ is given by

$$g = K\left(\frac{\eta\sin\chi}{1-\eta\cos\chi}\right), \tag{3}$$

where K is a kinematic factor and η is the ratio of asymmetries for Charybdis off and on. The relative uncertainty in g is

$$\left|\frac{\delta g}{g}\right| = \left|\left(\frac{\cos\chi-\eta}{1-\eta\cos\chi}\right)\frac{\delta\chi}{\sin\chi}\right| = |\eta\delta\chi| \text{ when } \chi = 90°. \tag{4}$$

I extrapolated the values of the field integrals for the currents we used from a fourth-order polynomial fit to the field integrals derived from the set of TOSCA calculations available for the 8.25" gap/2" field clamp configuration. Most of the TOSCA calculations were performed for the T_20 experiment. Three of the results are specifically for the GeN experiment: namely, the field integrals for power supply currents of 373.9 A, 542.1 A, and 593.3 A. The TOSCA calculations were not tuned for these settings. An uncertainty of 0.1% was assigned to each field integral, based on the level of agreement between the measured field integrals[1] and the TOSCA calculations.

The result of the fit is shown in Figure 2 and the fit parameters are tabulated in Table 1. The field integrals and corresponding precession angles for the central path through the magnet are tabulated in Table 2. The uncertainty in the precession angle is from the uncertainty in the field integral alone. The field integral uncertainties are propagated from the uncertainties in the fit parameters (see Table 1).

3 Precession by $\pm\chi$

The relative uncertainty in g due to $\delta\chi$ is small when $\chi = 90°$, as shown in the last column of Table 2. Most of the time, however, we run with the magnet

[1]We measured the field *in situ* along the central axis of the magnet for the three currents (373.9 A, 542.1 A, and 593.3 A) using a Hall probe. We ramped the current up to about 680 A and down to the desired setting.

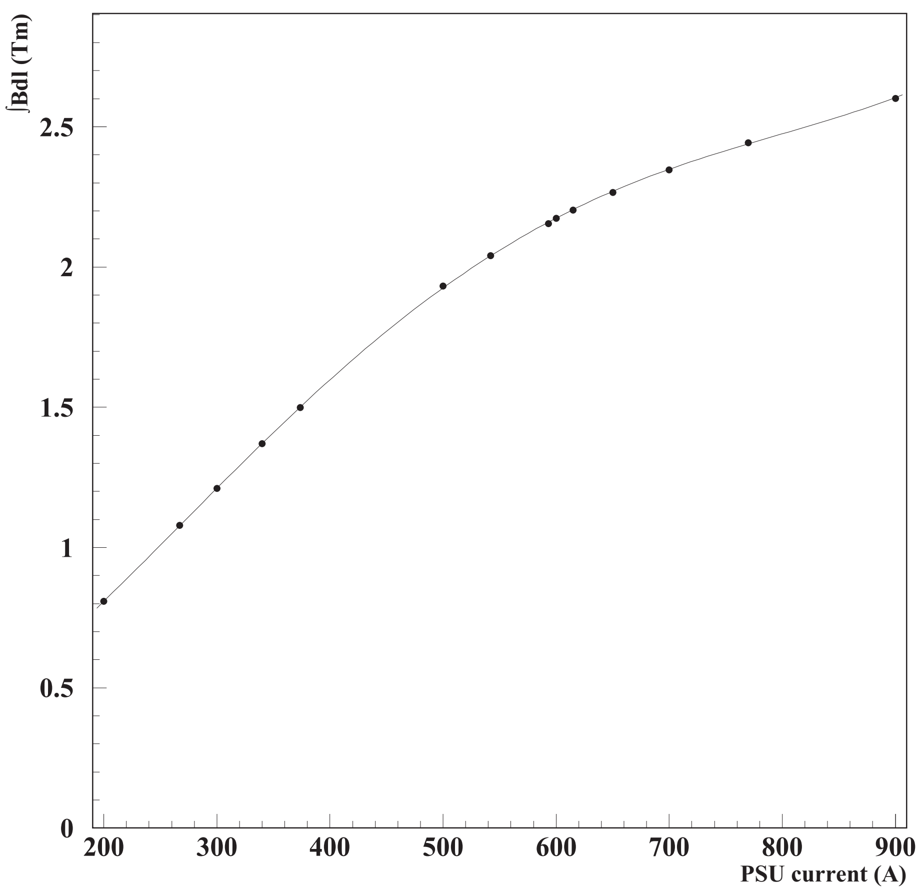

Figure 2: Field integral as a function of PSU current from the TOSCA calculations. Line is polynomial fit.

turned off. The largest contribution to the systematic error is expected to be the false asymmetry due to p-n conversion reactions in the Pb shielding. If we can keep Charybdis turned on all the time set to some intermediate current, we can eliminate the false asymmetry contribution by sweeping the charged particles out of the acceptance of the polarimeter at the possible expense of increased sensitivity to $\delta\chi$.

The proposed technique calls for splitting the running time between the two polarities of Charybdis at some fixed current (to be determined). The value of g is determined by

$$g = \frac{K_L}{K_S} \tan\chi \, \frac{\eta + 1}{\eta - 1}, \tag{5}$$

where χ is the precession angle, K_L and K_S are kinematic factors, and η is the ratio of asymmetries for precession by $-\chi$ and $+\chi$. The relative uncertainty in g due to uncertainty in χ is given by

$$\frac{\delta g}{g} = \sec\chi \csc\chi \, \delta\chi. \tag{6}$$

Table 3 tabulates the field integrals, power supply currents, and estimated uncertainties in χ for a set of possible values for χ for 3.395 GeV beam. To obtain the field integrals for $\chi < 35^\circ$ I used a linear extrapolation of the TOSCA results for currents in the range of 200-350 A down to lower currents for which we have no data. For $\chi \geq 35^\circ$ the extrapolation is no longer necessary since the required power supply currents are within the range of the TOSCA results, so the polynomial fit can be used (hence the smaller error estimates for $\delta\chi$). The uncertainty in χ is due to the uncertainty in the field integral alone. As before, these results are for the central path through the magnet. Table 4 shows the results for 884 MeV beam.

Parameter	Value	Error
p_0	0.2422	0.0019
p_1	1.085×10^{-3}	1.0×10^{-5}
p_2	1.249×10^{-5}	1.8×10^{-8}
p_3	-2.070×10^{-8}	2.8×10^{-11}
p_4	9.692×10^{-12}	2.2×10^{-14}

Table 1: Fit parameters and errors for the fit to the $\int Bdl$ vs. I plot. The functional form is $\int Bdl = p_0 + p_1 I + p_2 I^2 + p_3 I^3 + p_4 I^4$.

Q^2 $(GeV/c)^2$	I (A)	$\lvert\int Bdl\rvert$ (Tm)	$\chi(^\circ)$	$\delta g/g$ (%)
1.13±0.05	538.2	2.0302±0.0089	89.65±0.39	0.14
	540.3	2.0356±0.0089	89.70±0.39†	
1.474	592.3	2.1581±0.0108	90.14±0.45	0.15
(0.447)	(387.2)	(1.5511±0.0050)	(90±0.3)‡	(0.096)

Table 2: Field integrals and precession angles for the three Q^2 points. These results are for the central path through the magnet. The last column assumes the Galster value for g. †The first Q^2 point was split up into two running periods with slightly different beam energies, leading to slightly different central β_n values and field integrals. ‡Since the lowest Q^2 point has not been measured yet, the values in this row are projections.

$\chi(^\circ)$	I (A)	$\lvert\int Bdl\rvert$ (Tm)	$\delta\chi(^\circ)$	$\delta g/g$ (%)
15.8	92.9	0.3785±0.0078	0.33	2.2
20.0	118.0	0.4791±0.0081	0.34	1.8
25.0	147.9	0.5989±0.0085	0.35	1.6
30.0	177.8	0.7186±0.0090	0.38	1.5
35.0	207.7	0.8389±0.0022	0.092	0.34
40.0	237.6	0.9583±0.0026	0.11	0.39
45.0	267.5	1.0796±0.0030	0.13	0.45
50.0	297.0	1.1993±0.0035	0.15	0.53

Table 3: Field integrals, power supply current, and precession angle uncertainties for a set of values for χ for 3.395 GeV beam.

$\chi(^\circ)$	I (A)	$\lvert\int Bdl\rvert$ (Tm)	$\delta\chi(^\circ)$	$\delta g/g$ (%)
25.0	106.0	0.4310±0.0080	0.46	2.1
30.0	127.5	0.5172±0.0082	0.48	1.9
35.0	149.0	0.6034±0.0086	0.50	1.9
40.0	170.5	0.6896±0.0089	0.52	1.8

Table 4: Field integrals, power supply current, and precession angle uncertainties for a set of values for χ for 0.884 GeV beam.

References

[1] R. Hagedorn, *Relativistic Kinematics: A Guide to the Kinematic Problems of High-energy Physics* (Reading, Massachusetts: The Benjamin/Cummings Publishing Company, Inc., 1963), pp. 124-136.