An Introduction to Optimization Using Genetic/Evolutionary Algorithms

Part I

Single and Multi-Objective Problems

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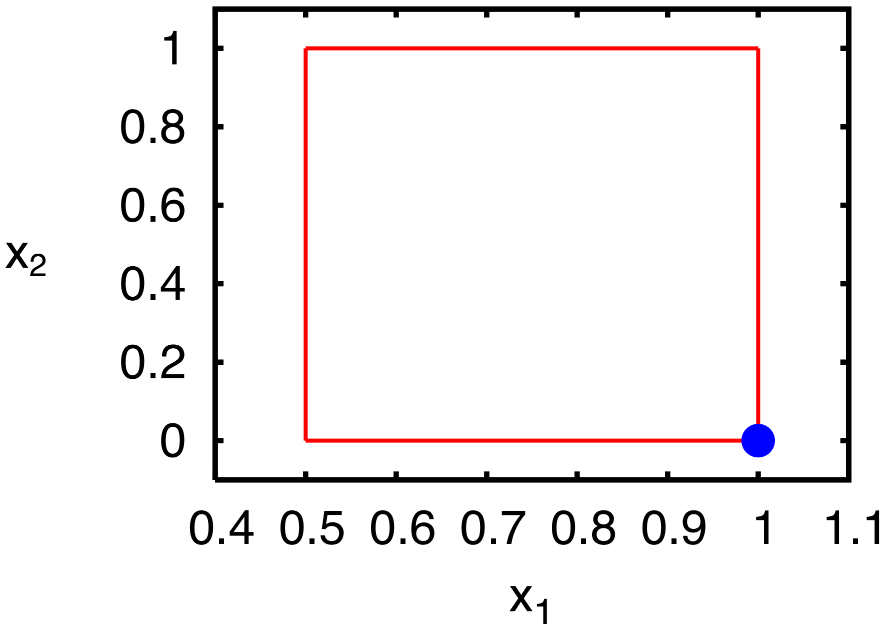
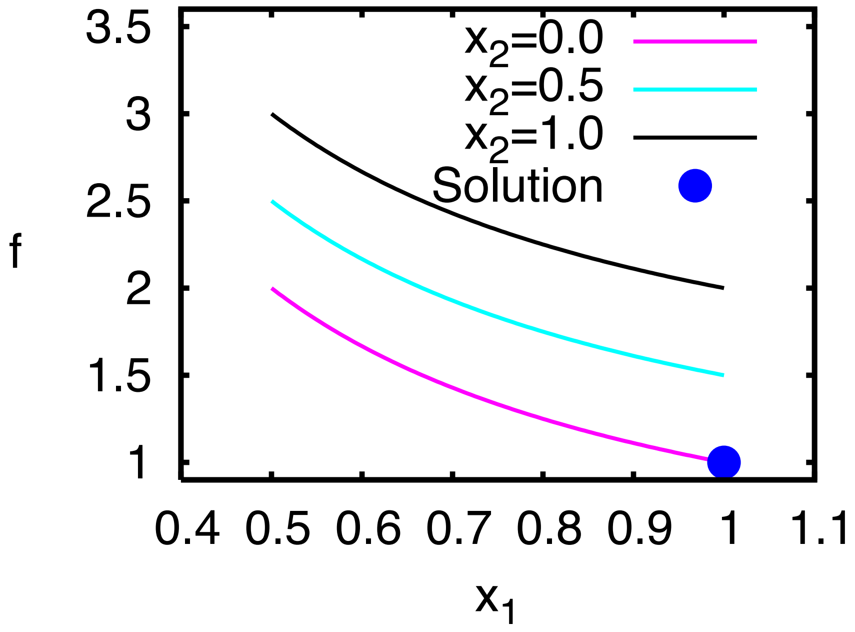
Single-Objective Example

Decision space: x_2 vs. x_1
(red box and interior)



f vs. x_1 for fixed x_2

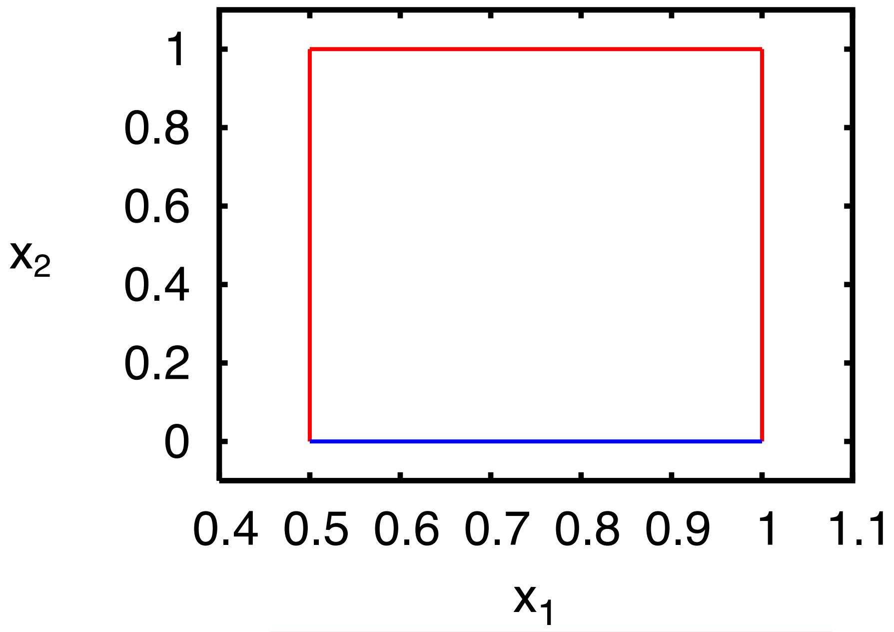


Minimize $f(x_1, x_2) = \frac{1}{x_1} + x_2$ — One multi-dimensional objective

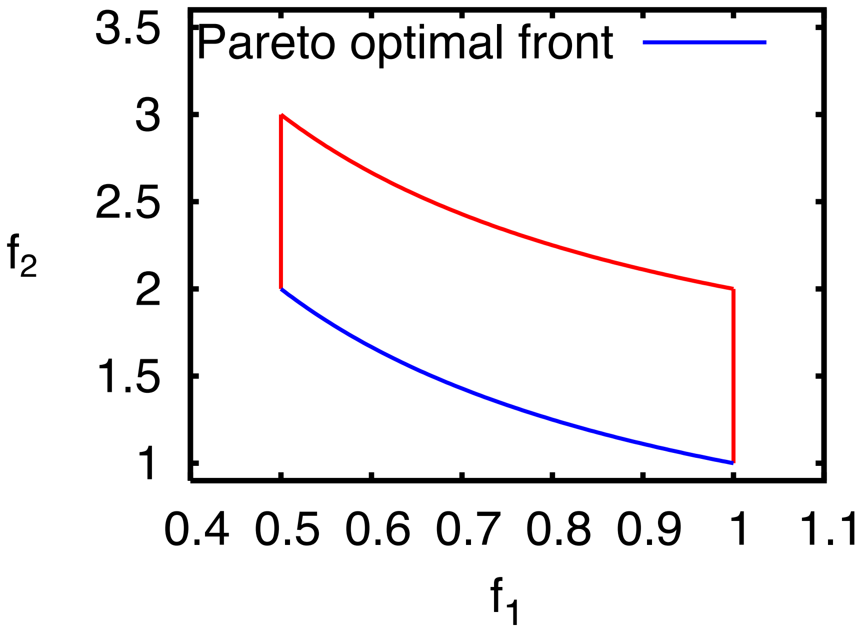
subject to $0.5 \leq x_1 \leq 1$, $0 \leq x_2 \leq 1$ — **Decision variable** bounds constraints

Multi-Objective Example 1

Decision space: x_2 vs. x_1
(red and blue box and interior)



Search space: f_2 vs. f_1
(red and blue structure and interior)



Minimize $f_1(x_1, x_2) = x_1$

Minimize $f_2(x_1, x_2) = \frac{1}{x_1} + x_2$

Two conflicting objectives

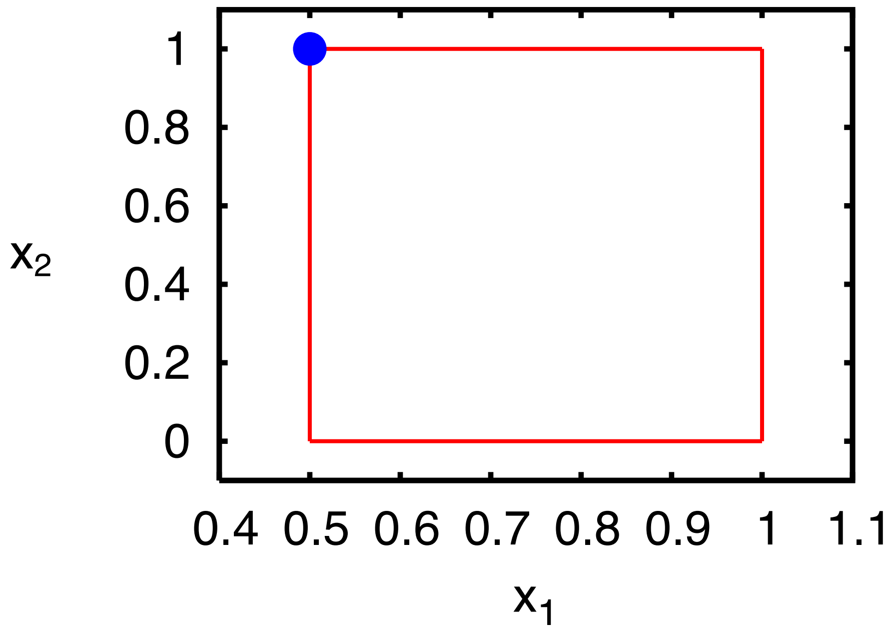
subject to $0.5 \le x_1 \le 1$

$0 \le x_2 \le 1$

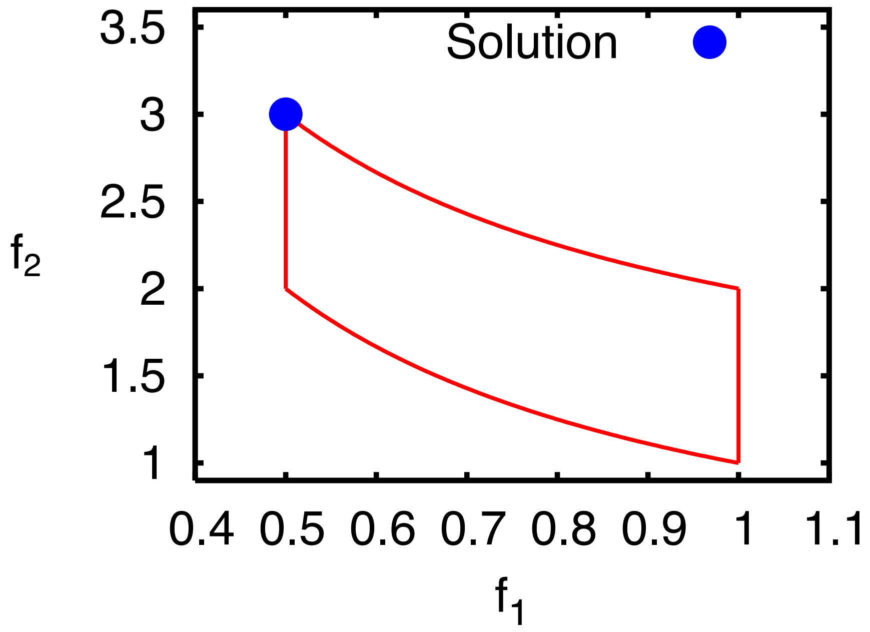
Decision variable bounds constraints

Multi-Objective Example 2

Decision space: x_2 vs. x_1
(red box and interior)



Search space: f_2 vs. f_1
(red structure and interior)



Minimize $f_1(x_1, x_2) = x_1$

Maximize $f_2(x_1, x_2) = \frac{1}{x_1} + x_2$

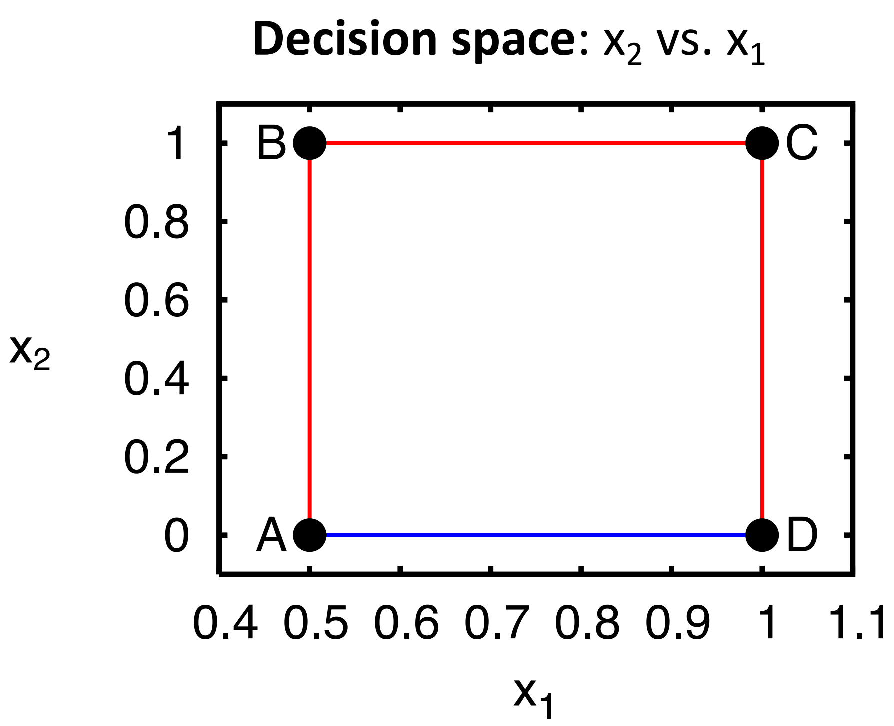
} Objectives do not conflict

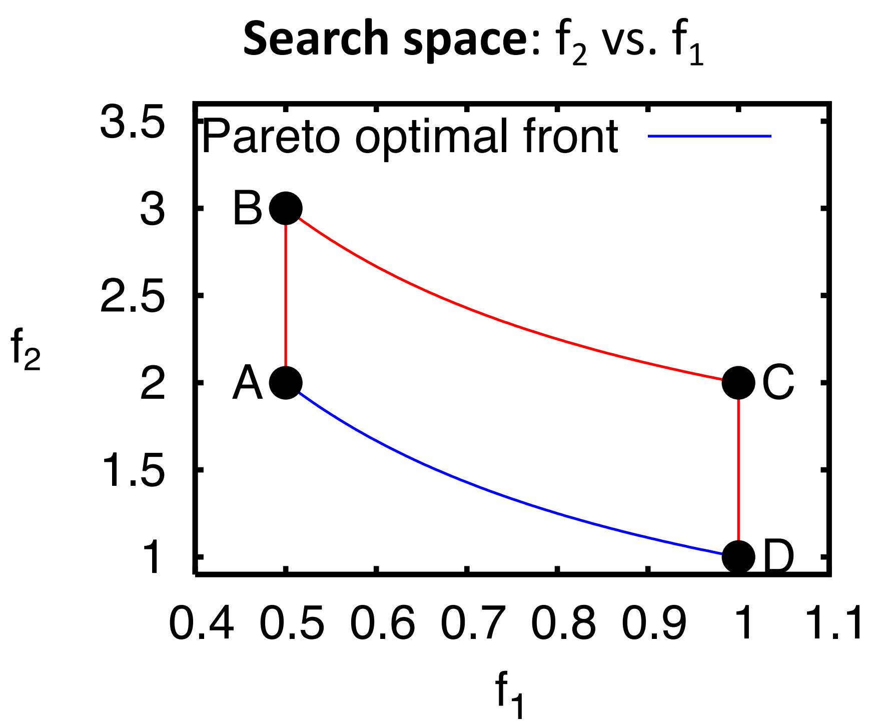
subject to $0.5 \le x_1 \le 1$

$0 \le x_2 \le 1$

} Decision variable bounds constraints

Multi-Objective Example 1





- **Dominance**
  - An individual dominates another if it is **better** in at least one objective and **no worse** in the remainder
  - "Better" = "<" and "no worse" = "≤" for minimization
- **Pareto optimality**
  - Trade-offs between objectives
  - **Non-dominated individuals** that dominate at least one other individual

- For A, B, C, and D in f_2 vs. f_1
  - A dominates B and C but not D
  - D dominates C but not A and B
  - B and C do not dominate
  - A and D are non-dominated
  - Blue curve is Pareto optimal front
  - A and D are on the Pareto optimal front

Terminology for Multi-Objective Optimizations and Genetic/Evolutionary Algorithms

- Decision variable = independent variable
- Decision space is the domain of the optimization problem and is the volume of all possible combinations of decision variable values
- Search space is the range of the optimization problem and is the volume of all possible combinations of objective values
- Conflicting objectives are objectives for which given decision variable values have the opposite effects (work against each other) and lead to sets of solutions
- Dominance: concept from natural selection used to categorize solution ”performance” with respect to other solutions and the optimization goals
- Pareto Optimality: concept from economics indicating that solutions are equally good and have trade-offs