Error propagation at the microMott

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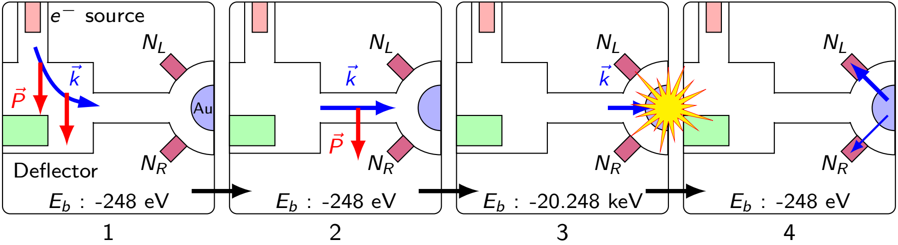
The microMott



- Counting experiment to measure an asymmetry between scattering angles
- Retarding field grids isolate elastic scatterings
- Asymmetry used to extract beam polarization

$$A = P_{\mathrm{b}} S(\theta) \Longrightarrow P_{\mathrm{b}} = \frac{A}{S(\theta)}$$

The data

- L/R are left and right detector, $+/-$ are for plus and minus helicity (HWP reversal)

Dark, d_1
3x Light, l_i
Dark, d_2

L+	L-	R+	R-	V
20.000	20.000	15.000	15.000	152.806
759853.000	647536.000	264401.000	375516.000	152.781
745859.000	626904.000	259332.000	372740.000	152.810
709136.000	610520.000	257920.000	370600.000	152.828
31.000	31.000	19.000	19.000	152.715
33.000	33.000	16.000	16.000	163.041
657430.000	563196.000	220662.000	315151.000	163.030
...				

Retarding Field at 150 V

- Retarding Field scanned from 150 to 320 V to include threshold voltage (248 V)

The data reduction

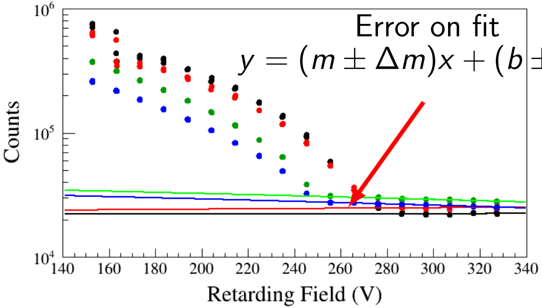
- Work now in cells of one retarding field voltage and only L+
- Remove dark count average from each I_i

$$I_i^* = I_i^* \pm \Delta d = I_i - \bar{d} \pm \Delta d$$

- Remaining counts above threshold are x-rays, remove them

Error on fit

$$y = (m \pm \Delta m)x + (b \pm \Delta b)$$

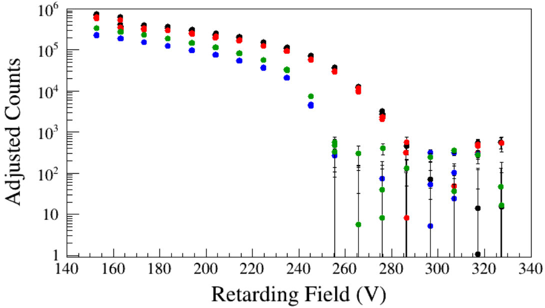


The data reduction

- Extrapolate to voltages below threshold
- Define the background $I_{bg}^{(i)}$ for each voltage and subtract to produce adjusted spectra

$$I_{bg}^{(i)} \pm \Delta I_{bg}^{(i)} = (mv_i + b) \pm \sqrt{(v_i \Delta m)^2 + (\Delta b)^2 + 2v_i \Delta(mb)^2}$$

$$c_i \pm \Delta c_i = I_i^* - I_{bg}^{(i)} \pm \Delta I_{bg}^{(i)}$$



The asymmetry

- Calculate asymmetry for each v_i in the cell — our data file now looks like below where $l_{pi} = c_i$ for each column

$$A_i = \frac{1-\sqrt{r_i}}{1+\sqrt{r_i}},\ r = \frac{N_i^-}{N_i^+},\ N_i^- = l_{mi} r_{pi},\ N_i^+ = l_{pi} r_{mi}$$

```
L+   L-   R+   R-   V
lp1  lm1  rp1  rm1  v1
lp2  lm2  rp2  rm2  v2
lp3  lm3  rp3  rm3  v3
lp1  lm1  rp1  rm1  v1
...
```

$$\Longrightarrow \Delta N_i^- = (l_{mi} r_{pi}) \sqrt{\left(\frac{\Delta l_{mi}}{l_{mi}}\right)^2 + \left(\frac{\Delta r_{pi}}{r_{pi}}\right)^2}$$

$$\Longrightarrow \Delta N_i^+ = (l_{pi} r_{mi}) \sqrt{\left(\frac{\Delta l_{pi}}{l_{pi}}\right)^2 + \left(\frac{\Delta r_{mi}}{r_{mi}}\right)^2}$$

$$\Longrightarrow \Delta r_i = \frac{N_i^-}{N_i^+} \sqrt{\left(\frac{\Delta N_i^-}{N_i^-}\right)^2 + \left(\frac{\Delta N_i^+}{N_i^+}\right)^2} \Longrightarrow \Delta A_i = \frac{A_i \Delta r_i}{\sqrt{2}} \sqrt{\frac{r_i + 1}{r_i (r_i - 1)^2}}$$

The asymmetry

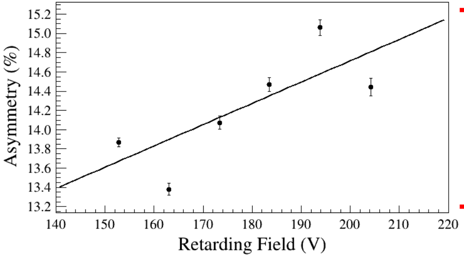
- Each Voltage cell now has the form

```
A    dA    V
A1   dA1   v1
A2   dA2   v2
A3   dA3   v3
A1   dA1   v1
...
```

- Calculate the average Asymmetry (and Voltage) for threshold extrapolation

$$A \pm \Delta A = \bar{A} \pm \frac{\sqrt{\sum_i (\Delta A_i)^2}}{3}, \;\; V = \bar{V}$$

- Cells are condensed to one asymmetry per cell, use 6 cells



Weighted fit applied
Error grows statistically
$y = (s \pm \Delta s)x + (i \pm \Delta i)$

The polarization

- Take value of fit at 248 V and divide by $S(\theta) = 0.201$

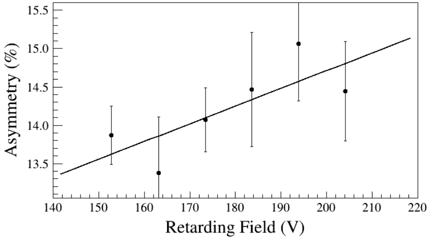
$$A_{248} \pm \Delta A_{248} = (s(248) + i) \pm \sqrt{(\Delta s(248))^2 + (\Delta i)^2 + 2(248)\Delta(si)^2}$$

$$\Longrightarrow P + \Delta P = \frac{A_{248}}{S(\theta)} \pm \frac{\Delta A_{248}}{S(\theta)}$$

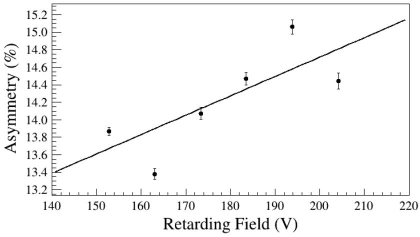
- Example polarization result is **78.50** $\pm$ **0.62** %
- Does not include error on the Sherman function
- Correlated error is included
- How does this stack up against old method? What changed?

The comparison

Old results



New results



- Old polarization result is **78.71** $\pm$ **3.37** %
- Error bars are **not** statistical, were calculated by the standard deviation of the asymmetries
- **No** error propagation from the counting statistics
- Fit was **unweighted** — doesn't account for random errors from σ

The takeaways

- Not accounting for error propagation from the counts discards information — cannot ensure statistical behavior
- Using the standard deviation for the error w/ an unweighted fit can underestimate the error
- Weighted fits are important to capture the statistical behavior of a counting experiment
- Essential to include correlated error in error propagation
- Good statistics are **IMPORTANT**, otherwise error can be large
- On another run, polarization is 83.02 ± 1.23 %, and error can get worse
- **NEED** to have > 20000 events for acceptable statistics