

## Generation of angular-momentum-dominated electron beams from a photoinjector

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Various projects under study require an angular-momentum-dominated electron beam generated by a photoinjector. Some of the proposals directly use the angular-momentum-dominated beams (e.g., electron cooling of heavy ions), while others require the beam to be transformed into a flat beam (e.g., possible electron injectors for light sources and linear colliders). In this paper we report our experimental study of an angular-momentum-dominated beam produced in a photoinjector, addressing the dependencies of angular momentum on initial conditions. We also briefly discuss the removal of angular momentum. The results of the experiment, carried out at the Fermilab/NICADD Photoinjector Laboratory, are found to be in good agreement with theoretical and numerical models.

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### I. INTRODUCTION

Angular-momentum-dominated electron beams generated by photoinjectors have direct applications in several accelerator proposals presently under consideration, either in the field of high-energy colliders or accelerator-based light sources. In Ref. [1], an angular-momentum-dominated, or “magnetized,” beam is proposed to be accelerated to  $\sim 50$  MeV and used for electron beam cooling [2,3] of ion beams in the relativistic heavy ion collider (RHIC). In such a scheme, an electron beam is copropagated with the ion beam with a matched velocity. Collisions of ions with electrons lead to a transfer of thermal motion from the ion to the electron beam. As the two beams copropagate, the electron-ion effective interaction length is increased due to the helical trajectory of the electron in the magnetic field, thereby improving the cooling efficiency. The cooling rate is then mainly determined by the longitudinal momentum spread of the electron beam, which can be made much smaller than the transverse one. Reference [4] concerns the photoinjector production of flat beams, i.e., a beam with high transverse emittance ratio. The technique consists of manipulating an angular-momentum-dominated beam produced by a photoinjector using the linear transformation described in Ref. [5]. The latter linear transformation removes the angular momentum and results in a flat beam. In the context of linear

collider proposals, where a flat beam at the interaction point is needed to reduce beamstrahlung [6], the development of a flat-beam electron source is an attractive idea since it could simplify or eliminate the need for an electron damping ring. The flat beam technique is also proposed for generation of ultrashort x-ray pulses by making use of the smaller dimension of the flat beam [7], and also in enhancing beam-surface interaction in a Smith-Purcell radiator [8] or in an image charge undulator [9]. A proof-of-principle experiment conducted at the Fermilab/NICADD Photoinjector Laboratory (FNPL) [10] has demonstrated the flat beam production [11,12], where an emittance ratio of 50 was reported.

In this paper we report on recent results pertaining to the experimental investigation of some properties of an angular-momentum-dominated beam. We also briefly address the removal of angular momentum and the subsequent generation of a flat beam. Producing flat beams is our primary motivation for the present studies.

In Sec. II we briefly summarize theoretical aspects of the photoinjector production of angular-momentum-dominated beams. In Sec. III we describe the experimental setup of FNPL. Sections IV and V are dedicated to experimental results and their comparisons to theory and numerical simulations. Our conclusions appear in Sec. VI.

### II. THEORETICAL BACKGROUND

In this section we assume the beam and external focusing forces to be cylindrically symmetric. The cylindrical symmetry implies the conservation of the canonical angular momentum of each electron. In an axial magnetostatic field  $B_z(z)$ , the canonical angular momentum of an electron  $L$  in circular cylindrical coordinates  $(r, \phi, z)$  is [13]

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$$L = \gamma m r^2 \dot{\phi} + \frac{1}{2} e B_z(z) r^2, \quad (1)$$

where  $\gamma$  is the Lorentz factor,  $\dot{\phi}$  the time derivative of  $\phi$ , and  $m$  and  $e$  are, respectively, the electron rest mass and charge.

The average canonical angular momentum of the electrons  $\langle L \rangle$  is obtained by averaging Eq. (1) over the beam distribution. At the photocathode location, we have  $\langle \dot{\phi} \rangle = 0$  and

$$\langle L \rangle = \frac{1}{2} e B_0 \langle r^2 \rangle = e B_0 \sigma_c^2, \quad (2)$$

where  $B_0 = B_z(z=0)$  is the axial magnetic field on the photocathode, and  $\sigma_c$  is the transverse root-mean-square (rms) beam size on the photocathode in Cartesian coordinates. For a cylindrically symmetric beam,  $\sigma_c^2 = \langle r^2 \rangle / 2$ .

Outside the solenoidal field region, where  $B_z$  vanishes, an electron acquires axial mechanical angular momentum due to the torque exerted on it in the transition region. Since  $B_z(z) = 0$ , the second term of Eq. (1) vanishes and the canonical angular momentum is given by the first term of Eq. (1), which is the axial mechanical angular momentum. It is convenient to normalize  $\langle L \rangle$  with the axial momentum  $p_z$ , and introduce the quantity  $\mathcal{L}$  given by

$$\mathcal{L} = \frac{\langle L \rangle}{2 p_z} = \kappa \sigma_c^2, \quad (3)$$

where  $\kappa = e B_0 / (2 p_z)$ .

The beam angular momentum can be removed by means of a properly designed skew quadrupole section [14–16] and the beam is transformed into a flat beam (see Sec. V). The flat beam transverse emittances after the skew quadrupole section  $\epsilon_{\pm}$  are given by [16,17]

$$\epsilon_{\pm} = \sqrt{\epsilon_u^2 + \mathcal{L}^2} \pm \mathcal{L}. \quad (4)$$

Here  $\epsilon_u$  is the uncorrelated transverse emittance prior to the skew quadrupole section. Note that the four-dimensional emittance is conserved since  $\epsilon_u^2 = \epsilon_+ \epsilon_-$ .

The evolution of the transverse rms beam size of a relativistic electron bunch in a drift is given by the envelope equation [18]

$$\sigma'' - \frac{K}{4\sigma} - \frac{\epsilon_u^2}{\sigma^3} - \frac{\mathcal{L}^2}{\sigma^3} = 0, \quad (5)$$

where  $\sigma$  is the transverse rms size,  $K = 2I/I_0\gamma^3$  is the generalized perveance,  $I$  is the absolute value of the instantaneous beam current, and  $I_0$  is the Alfvén current for electrons ( $\sim 17$  kA). The second, third, and fourth terms, respectively, represent the effects due to space charge, emittance, and the angular momentum. For low energy beam, the space charge term is important. However, for the typical operating conditions considered in this paper, e.g.,  $\gamma \approx 30$ , bunch charge  $\approx 0.5$  nC, rms beam duration  $\sigma_t \approx 4$  ps,  $\sigma \approx 1.25$  mm [19],  $\gamma\epsilon_u \approx 4$  mm mrad [20],

$\gamma\mathcal{L} \approx 20$  mm mrad, the fourth term in Eq. (5) is much greater than the second and the third term. Such a beam is said to be angular momentum dominated.

### III. EXPERIMENTAL SETUP

The experimental production and characterization of angular-momentum-dominated electron beams were carried out at FNPL.

The photoinjector incorporates a photoemission source consisting of a  $1 + \frac{1}{2}$  cell cavity operating at 1.3 GHz, the so-called radio frequency (rf) gun. An ultraviolet (UV) laser beam impinges a cesium telluride photocathode located on the back plate of the rf gun half cell. The thereby photoemitted electron bunch exits from the rf gun at 4 MeV/c and is immediately injected into a TESLA-type superconducting cavity [21] (henceforth referred to as the booster cavity). The bunch momentum downstream of the booster cavity is approximately 16 MeV/c when the cavity is operated to yield the maximum energy gain. The typical operating conditions of the main subsystems of the photoinjector are gathered in Table I, and a block diagram of the facility is depicted in Fig. 1.

The transverse size of the UV drive-laser beam at the photocathode is set by a remotely controllable iris. The laser beam temporal profile is a Gaussian distribution with rms duration of  $\sim 3.5$  ps.

The rf gun is surrounded by three solenoidal magnetic lenses independently powered. This allows proper focusing of the electron bunch while maintaining the desired magnetic field on the photocathode.

Downstream of the booster cavity, the beam line includes a round-to-flat-beam (RTFB) transformer, consisting of four skew quadrupoles, that can be used to remove the mechanical angular momentum.

Several optical transition radiation (OTR) or fluorescent (YAG-based) screens serve as diagnostics to measure the beam's transverse density at various locations in the beam line. Transverse emittances can also be measured based on the multislit [22,23], or quadrupole scan techniques [24]. The multislit mask used for emittance measurements con-

TABLE I. Typical settings for the photocathode drive-laser beam, rf gun, and accelerating section. Values in square brackets correspond to the range used in the measurements.

Parameter	Value	Units
Laser beam injection phase	$25 \pm 5$	degree
Laser beam radius on cathode	$[0.6, 1.6] \pm 0.05$	mm
Laser beam pulse duration	$3.5 \pm 0.5$	ps
Bunch charge	$[0.2, 1.6]$	nC
$E_z$ on cathode	$35 \pm 0.2$	MV/m
$B_0$ on cathode	$[200, 1000]$	G
Booster cavity gradient	$\sim 12$	MV/m

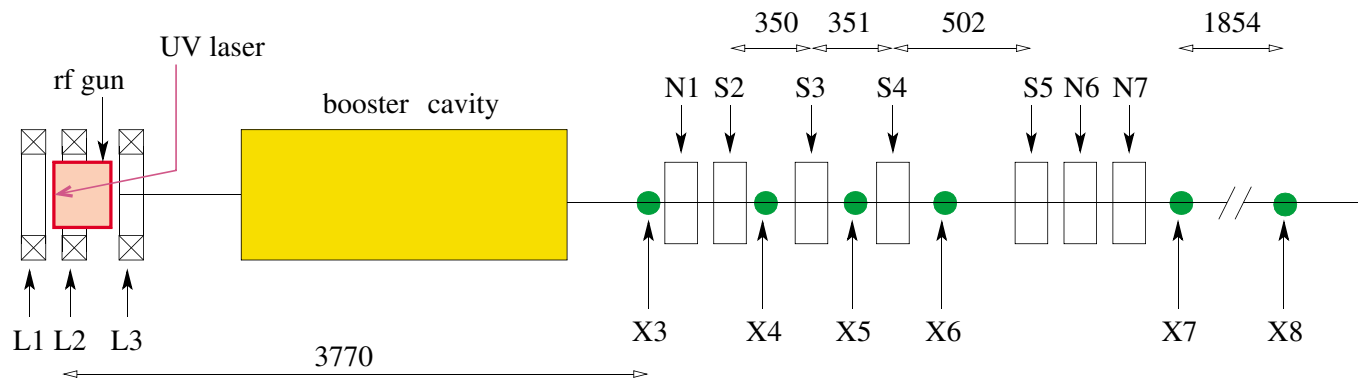


FIG. 1. (Color) Overview of the FNPL beam line. Here only the elements pertaining to the flat beam experiment are shown. The letters represent solenoidal magnetic lenses (L), normal (N) and skew (S) quadrupoles, and diagnostic stations (X). Dimensions are in mm.

sists of a 6-mm-thick tungsten mask with 48  $\mu\text{m}$ -wide slits spaced 1 mm apart.

#### IV. MEASUREMENTS OF CANONICAL ANGULAR-MOMENTUM

We now turn to the basic properties of the canonical angular momentum. We especially investigate the conversion of the canonical angular momentum of the photoemitted electron bunch into mechanical angular momentum downstream of the booster cavity.

The canonical angular momentum at the photocathode surface is obtained from Eq. (2). Given the experimental settings of the solenoidal lens currents, the magnetic field  $B_0$  is inferred via simulations using the POISSON [25] program, which is benchmarked against calibration of the solenoidal lenses [26]. The value of  $\sigma_c$  used in Eq. (2) is directly measured from an image of the UV laser beam on a “virtual photocathode.” The virtual photocathode is a calibrated UV-sensitive screen, located outside of the vacuum chamber, being a one-to-one optical image of the photocathode.

To elaborate the method used to measure the longitudinal mechanical angular momentum downstream of the booster cavity, we consider an electron in a magnetic-field-free region at longitudinal location  $z_1$  with transverse radial vector  $\mathbf{r}_1 = r_1 \hat{\mathbf{e}}_x$  ( $\hat{\mathbf{e}}_x$  stands for the  $x$ -axis unit vector). After propagating through a drift space, the electron reaches  $\mathbf{r}_2$  at location  $z_2$ . Let  $\theta = \angle(\mathbf{r}_1, \mathbf{r}_2)$  be the angle between the two aforementioned radial vectors ( $\theta$  is henceforth referred to as “shearing angle”; see Fig. 2). The longitudinal mechanical angular momentum of the electron at location  $z_1$ ,  $L$  is given by

$$\mathbf{L} = \mathbf{r}_1 \times \mathbf{p} = r_1 p_y \hat{\mathbf{e}}_x \times \hat{\mathbf{e}}_y, \quad (6)$$

where  $\mathbf{p}$  is the transverse momentum of the electron, and  $p_y$  its vertical component. By introducing  $y' = \frac{dy}{dz} = \frac{p_y}{p_z}$ , and noting that  $y'$  is a constant in a drift space for an angular-momentum-dominated beam, we see that the

change in vertical coordinate is  $\Delta y = y'D = r_2 \sin\theta$ , where  $D = z_2 - z_1$ . Hence Eq. (6) can be rewritten in the convenient form

$$L = r_1 p_z y' \hat{\mathbf{e}}_z = p_z \frac{r_1 r_2 \sin\theta}{D} \hat{\mathbf{e}}_z. \quad (7)$$

For a cylindrically symmetric laminar beam with rms transverse beam sizes  $\sigma_1$  and  $\sigma_2$  at respective locations  $z_1$  and  $z_2$  along the beam line, the averaged mechanical angular momentum can then be calculated via

$$\langle L \rangle = 2p_z \frac{\sigma_1 \sigma_2 \sin\theta}{D}. \quad (8)$$

Thus the measurements of rms beam sizes at locations  $z_1$  and  $z_2$  along with the corresponding shearing angle provide all the necessary information for calculating the mechanical angular momentum. Experimentally, the shearing angle is obtained by inserting at location  $z_1$  a multislit mask and measuring the corresponding shearing angle of the beamlets at the location  $z_2$ ; see Fig. 3. For the mechanical angular momentum measurement reported here we use the diagnostic stations X3 and X6 (see Fig. 1). The X3 diagnostic station includes an OTR screen and two insert-

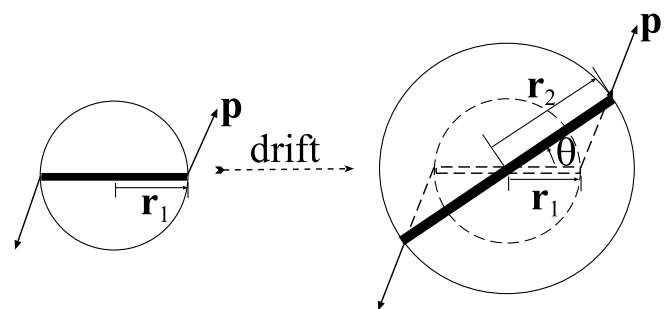


FIG. 2. Beam with angular-momentum-induced shearing while drifting. The dark narrow rectangle represents a slit inserted into the beam line to measure the shearing angle (see text for more details).

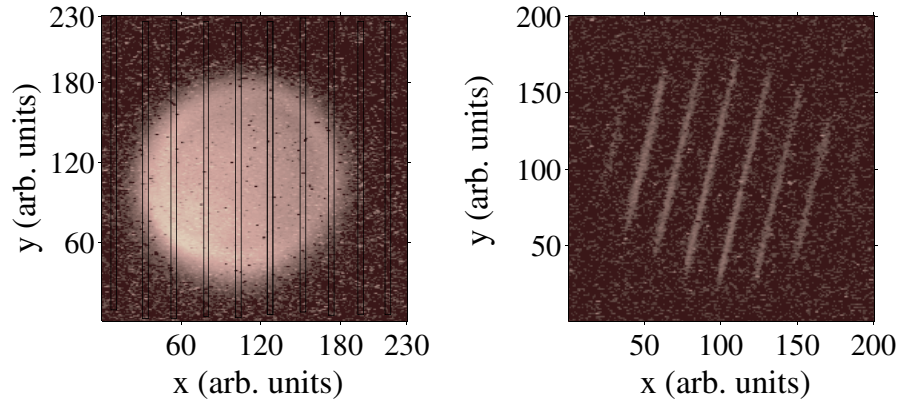


FIG. 3. (Color) Example of data set used for mechanical angular momentum measurement. Beam transverse density on X3 (left) and observed beamlets on X6 when the vertical multislit mask is inserted at X3 (right). The vertical lines superimposed on the X3 image are an illustration of vertical slits when the multislit mask is inserted.

able multislit masks (with vertical and horizontal slits). The station X6 is only equipped with an OTR screen.

A set of measurements of mechanical angular momentum versus  $B_0$  was reported in Ref. [27]. In the present paper, such measurements are performed by varying  $B_0$  over a wider range ( $B_0 \in [200\ 1000]$  G for a bunch charge of  $0.41 \pm 0.05$  nC; see details in Ref. [19]). The measurement technique discussed in the previous paragraph was also numerically tested for each experimental data point. In Fig. 4 we compare the measured mechanical angular momentum from Eq. (8) with the canonical angular momen-

tum calculated from Eq. (2), given the  $B_0$ . The measured values include both experimental data and simulated values, i.e., values that have been retrieved after simulating the measurement technique numerically with the particle tracking program ASTRA [28]. The uncertainties in the measurement of angular momentum are obtained via error propagation from the direct measurements of rms beam sizes and the shearing angle.

Conservation of canonical angular momentum is demonstrated in Fig. 5, where the angular momentum was measured at different locations along the beam line. In

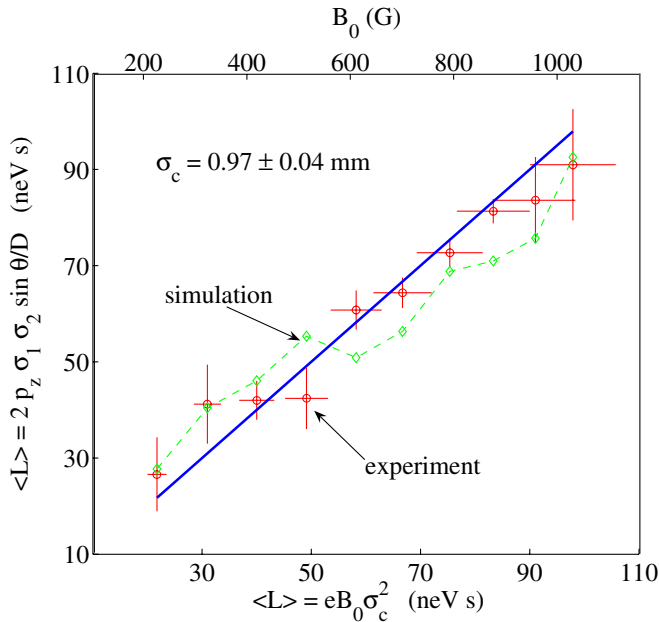


FIG. 4. (Color) Mechanical angular momentum from Eq. (8) versus the canonical angular momentum calculated from Eq. (2). The labels “experiment” and “simulation” correspond, respectively, to experimentally measured data points and simulated values found by modeling of the measurement technique. The solid diagonal line is drawn simply to aid the eye.

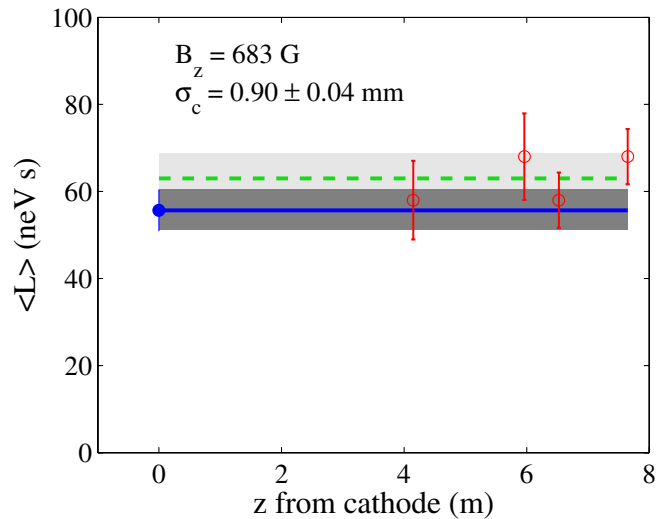


FIG. 5. (Color) Evolution of canonical angular momentum along the beam line. At photocathode location (dot), canonical angular momentum is calculated from Eq. (2) and solid line is this value extended along  $z$ . At other locations (circles), mechanical angular momentum is obtained from Eq. (8) and the dashed line is the average. The shaded area covers the uncertainties in the measurements either from Eq. (2) (darker strip) or from Eq. (8) (lighter strip).

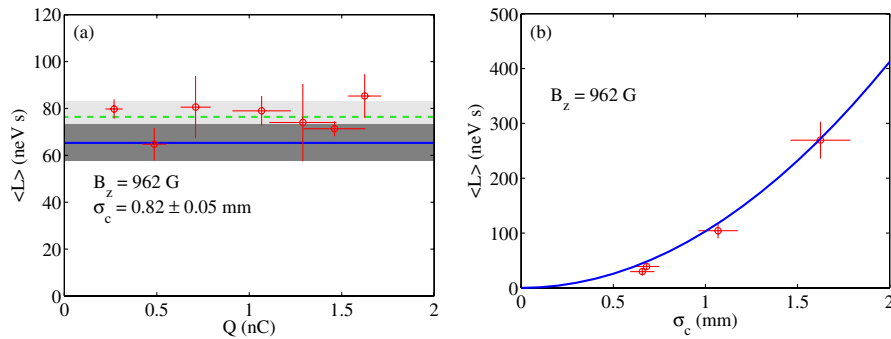


FIG. 6. (Color) Canonical angular momentum versus charge (a) and photocathode drive-laser beam spot size (b). The measured mechanical angular momenta (circles) are compared with the theoretical value of the canonical angular momentum calculated from the axial magnetic field (solid line). In (a) the dashed line represents the average value of all the data points, and the shaded area has the same meaning as in Fig. 5.

these measurements all quadrupoles are turned off so that the beam propagated in a drift space.

The dependence of mechanical angular momentum on the charge was also explored. In this experiment, the laser beam spot size was set to  $\sigma_c = 0.82$  mm, and the laser beam intensity was varied via a wave-plate attenuator located in the UV laser beam path. The results, shown in Fig. 6(a), indicate that the mechanical angular momentum, for our set of operating parameters, is charge-independent, confirming our assumption that the beam dynamics is angular-momentum-dominated in the range explored here.

Finally, the dependence of canonical angular momentum versus  $\sigma_c$  was investigated. The laser beam intensity was held constant and  $B_0$  was identical to the previous experiment ( $B_0 = 962$  G). The charge density in the bunch is therefore kept constant. The measurements [see Fig. 6(b)] support the expected quadratic dependence of the angular momentum on  $\sigma_c$  indicated in Eq. (2).

The measured dependencies of canonical angular momentum on the different parameters are all in good agreement with theoretical expectations. Such an agreement gives us some confidence in our ability to control the angular momentum of the incoming beam upstream of the RTFB section.

## V. REMOVAL OF ANGULAR-MOMENTUM AND FLAT-BEAM GENERATION

To remove angular momentum, it is necessary to apply a torque on the beam. A quadrupole can exert a net torque only on an incoming asymmetric beam. Thus more than one quadrupole is needed to remove the angular momentum of a cylindrically symmetric incoming beam. A first quadrupole followed by a drift space will introduce asymmetry in the  $x$ - $y$  space, while the other quadrupoles downstream are properly tuned to apply a total net torque such that the angular momentum is removed at the exit of the quadrupole section. For the series of measurements and simulations presented in this section, a set of three skew quadrupoles (S2, S3, S5 in Fig. 1) are used to remove the angular momentum and generate a flat beam.

Given the photoinjector parameters, numerical simulations of the beam line (from the photocathode up to the entrance of the RTFB transformer) are performed using ASTRA. The four-dimensional phase-space coordinates are propagated downstream of the transformer using a linear

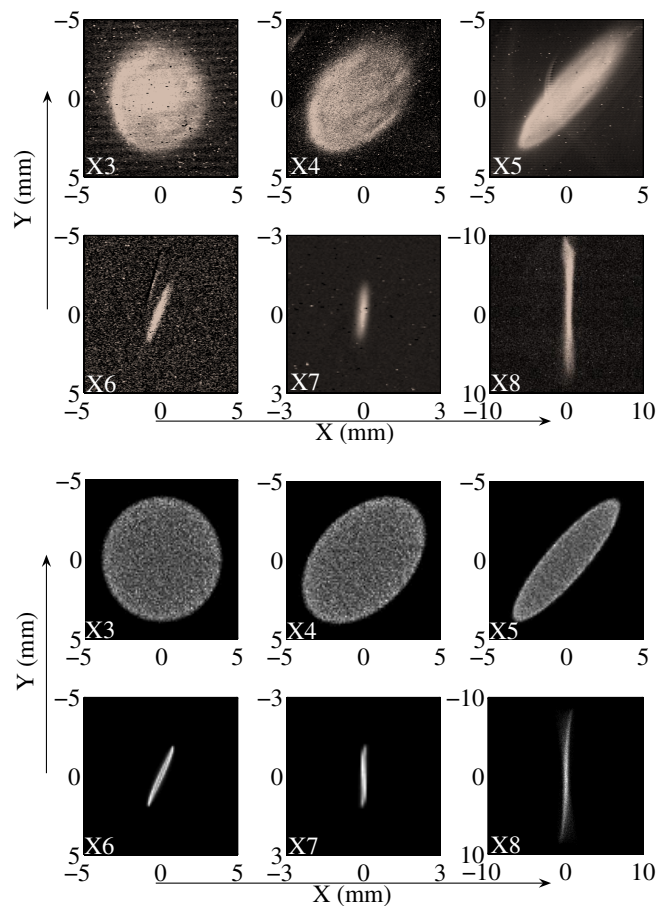


FIG. 7. (Color) Measured (top six photos) and simulated (bottom six plots) beam transverse density evolution in the RTFB section. The consecutive plots correspond to locations X3, X4, X5, X6, X7, and X8 shown in Fig. 1.

transfer matrix. The initial values of the skew quadrupole strengths are those derived, under the thin-lens approximation, in Ref. [29]. They are then optimized, using a least-square technique, to minimize the  $x$ - $y$  coupling terms of the beam matrix at the exit of the transformer. The final optimized quadrupole strengths are used for subsequent ASTRA simulation of the beam dynamics through the RTFB transformer.

Further empirical optimization around the predicted values is generally needed to ensure the angular momentum is totally removed, as inferred by observation of the  $x$ - $y$  coupling at several locations downstream of the RTFB section. Evolution of transverse density throughout the RTFB section is in good agreement with expectations from simulations, as shown in Fig. 7. Each of the top six photos is a superposition of five bunches with charge of  $0.55 \pm 0.10$  nC. In the sequence of measurements and simulations presented there, the incoming round beam (X3) is transformed into a flat beam characterized by the large asymmetry (X7 and X8). The mechanical angular momentum is removed: there is no noticeable shearing as the beam propagates from X7 to X8.

## VI. CONCLUSION

We have experimentally explored some parametric dependencies of angular momentum for an angular-momentum-dominated electron beam produced in a photo-injector. The results obtained are in good agreement with theoretical expectations, giving us some confidence in our understanding of the angular-momentum-dominated beam.

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