# 1 Intro/Purpose

The main goal of reading this book is to understand the theory behind how photocathodes are damaged by ion back-bombardment. The idea is that neutral residual gas in the accelerator vacuum can be ionized by a high-energy electron beam and accelerate back towards the photocathode, damaging it. As a result, the quantum efficiency (QE) of the photocathode decreases: it takes more and more laser power to eject the same number of electrons from the photocathode. In order to understand how to mitigate ion back-bombardment, knowledge of ion bombardment (collision) theory is required.

# 2 Chapter 2: Collisions between atoms

## 2.1 Introduction (pg. 6)

This chapter deals with mechanics of collisions between particles with forces between them. The interaction forces are due to interatomic potentials, the form of which determines the derived mechanics. Often this potential with be the Coulomb potential due to charged particles. Since methods relativistic corrections to collision parameters for high incident particle energies are exceedingly difficult, we often restrict ourselves by defining a cutoff distance on the order of the atomic diameter. Often this is a good approximation, since interaction forces decrease rapidly past this distance and so we can restrict our focus to an incident particle on a lattice atom itself as opposed to all atoms in the lattuce. (This is probably similar to the Coulomb Logarithm). Depending on the energy range of the particles in the two-body collisions, we may or may not need to consider quantum mechanical effects. Often, we can make good approximations using classical mechanics. For simplification purposes, it will be assumed that the collisions are elastic (no kinetic energy loss in the collision).

## 2.2 Collision Kinetics (pg. 6)

Figure 1 below diagrams the interaction between an incident particle with mass $M_1$ with a target atom with mass $M_2$..

The impact parameter $p$ denotes the minimum distance between the incident and target atom if there were no forces between them. If there is an interaction force $F(r)$, where $r$ is the distance between the atoms, then the incident atom scatters through an angle $\phi$ and the target atom moves at an angle $\psi$ with respect to the incident axis. Thus, energy is transfered from the incident atom to the target atom. If the collision is elastic, no kinetic energy is lost.

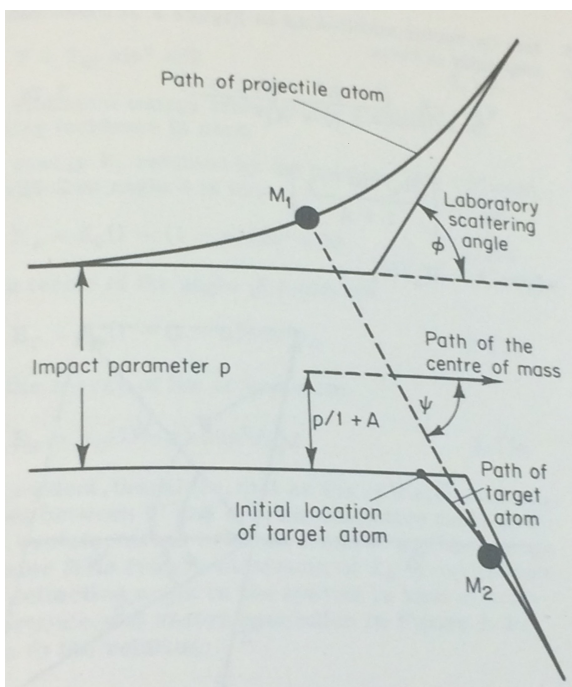


Figure 1: Diagram of an incident particle $M_1$ scattering off of a target atom $M_2$.

### 2.2.1 Collisions Between Elastic Spheres (pg. 7-9)

We can make a simplifying assumption that the two atoms can be represented by hard spheres of radius $R$. Figure 2 shows the collision between two of these spheres.

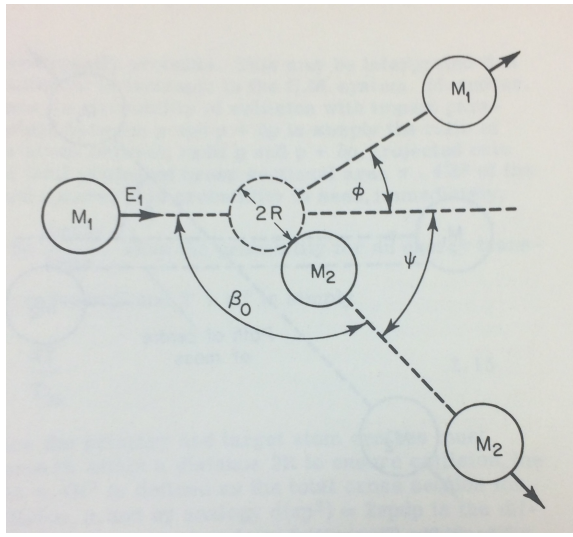


Figure 2: Diagram of the collision between two hard spheres of radius $R$.

Often the incident atom is moving much faster than the lattice atom. As a result, we can consider the lattice atom to be essentially at rest before the collision. When the incident atom makes contact with the target atom, if we draw a straight line through the centers of the two atoms, then the target atom would move straight along that line. The angle between the incident axis and this line is $\beta_0$ and so $\psi = 180° - \beta_0$. Note that this was not quite true before: it was only true *asymptotically*. The hard sphere approximation allows us to assume that the all effects of the collision happen instantaneously.

We can simplify calculations by switching to the center of mass frame, shown in Figure 3 below.

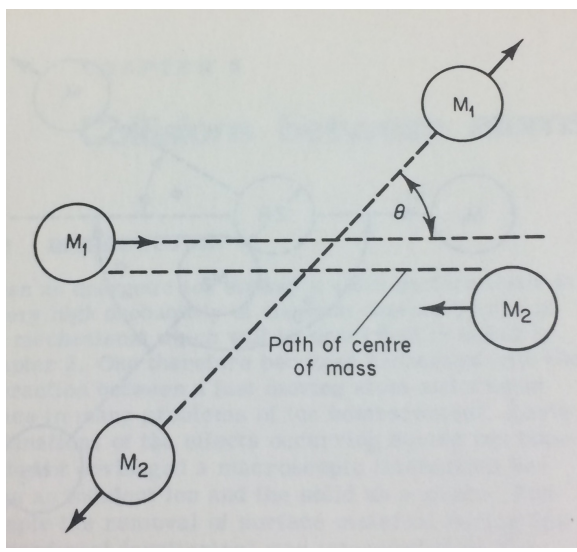


Figure 3: Diagram of a collision between two hard spheres in the center of mass frame.

In the center of mass frame, the center of mass passes within a distance $\dfrac{M_1 p}{M_1 + M_2}$ from the original center of the target atom and has an effective velocity $v_c = \dfrac{M_1 v_0}{M_1 + M_2}$. If $v_0$ is the incident atom's velocity before the collision, then with respect

to the center of mass frame, it has velocity $v_{0,c}$ before the collision, given by

$$v_{0,c} = v_0 - \frac{M_1 v_0}{M_1 + M_2} = v_0 - v_c = \frac{M_2 v_0}{M_1 + M_2} \tag{1}$$

The target atom has velocity $v_{tar,c}$ before the collision, given by

$$v_{tar,c} = -\frac{M_1 v_0}{M_1 + M_2} \tag{2}$$

Multiplying both sides of equations (1) and (2) by $M_1+M_2$ and subtracting the two equations, we see that total momentum in the center of mass frame is zero before and after the collision. Thus, if the incident and target atoms have velocities $V_1$ and $V_2$ after the collision, we must have that

$$M_1 V_1 = -M_2 V_2 \tag{3}$$

. From conservation of energy (in the absence of potential energy),

$$\begin{aligned} \frac{1}{2}M_1 V_1^2 + \frac{1}{2}M_1 V_2^2 &= \frac{1}{2}M_1 v_{0,c}^2 + \frac{1}{2}M_2 v_{tar,c}^2 \\ &= \frac{1}{2}M_1 \left(\frac{M_2 v_0}{M_1 + M_2}\right)^2 + \frac{1}{2}M_2 \left(\frac{M_1 v_0}{M_1 + M_2}\right)^2 \end{aligned} \tag{4}$$

Solving equations (3) and (4) for $V_1$ and $V_2$ yields

$$V_1 = \frac{M_2 v_0}{M_1 + M_2} = v_{0,c} \tag{5}$$

$$V_2 = -\frac{M_1 v_0}{M_1 + M_2} = v_{tar,c} \tag{6}$$

From here, we can solve for the final velocities after the collision in the lab frame. Figure (4) below shows the velocities $V_1$ and $V_2$ in the center of mass frame and $V_a$ and $V_b$ in the lab frame with $\theta$ being the deflection angle, as in Figure (3).

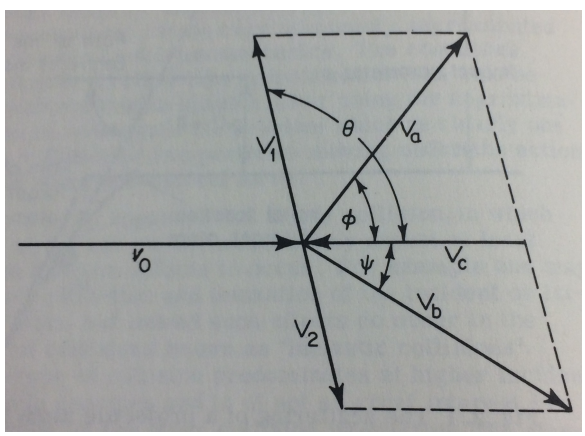


Figure 4: Diagram of relevant velocities in the collision.

From vector addition, we see that

$$V_a^2 = v_0^2 \frac{1 + A^2 + 2a\cos\theta}{(1+A)^2} \tag{7}$$

$$V_b^2 = \left[\frac{2v_0 \sin\frac{\theta}{2}}{1+A}\right]^2 \tag{8}$$

$$A = \frac{M_2}{M_1}$$

Using trigonometry, $\phi$ and $\psi$ is related to $\theta$ by

$$\tan\phi = \frac{V_1 \sin\theta}{V_c + V_1 \cos\theta} = \frac{A \sin\theta}{1 + A\cos\theta} \tag{9}$$

$$\psi = \frac{\pi}{2} - \frac{\theta}{2} \tag{10}$$

Using equation (8) and the fact that the energy gained by the target atom is equal to that given by the incident atom (by energy conservation), we see that the energy transfer $T$ is

$$\begin{aligned} T &= \frac{1}{2} M_2 V_b^2 \\ &= \frac{1}{2} M_2 \left( \frac{2v_0 \sin\frac{\theta}{2}}{1+A} \right)^2 \\ &= \frac{2M_2 v_0^2 \sin^2\frac{\theta}{2}}{(1+A)^2} \end{aligned} \tag{11}$$

If we define the quantity $\alpha = \left(\frac{A-1}{A+1}\right)^2$, we can write $T$ succinctly in terms of the incident particle's energy $E_0$

$$T = (1-\alpha) E_0 \sin^2\frac{\theta}{2} \tag{12}$$

For head on collisions, $\beta_0 = 0$, $\psi = 180°$ and $\theta = -180°$. The maximum energy transfer depends only on the masses of the two particles and the incident atom's initial energy:

$$T_m = (1-\alpha) E_0 = \frac{4M_1 M_2}{(M_1 + M_2)^2} E_0 \tag{13}$$

Thus, $T = T_m \sin^2\frac{\theta}{2}$.

### 2.2.2 Collision Probabilities (pg. 9-10)

From Figure 2, we see that $\sin\beta_0 = \cos\frac{\theta}{2} = \frac{p}{2R}$. Differentiating with respect to $\theta$ yields $-\sin\theta d\theta \sim pdp$. Thus, $p^2 = 4R^2\left[1 - \frac{T}{T_m}\right]$ and so

$$2\pi p \frac{dp}{dT} = \pi \frac{4R^2}{T_m} \tag{14}$$

Since all incident angles $\beta_0$ are equally probable, we have that the probability for an energy transfer between $T$ and $T + dT$ is

$$\frac{dT}{T_m} = \frac{2\pi p dp}{4\pi R^2} \tag{15}$$

The total cross section for collision is just the surface area of the sphere

$$\sigma = 4\pi R^2 \tag{16}$$

since for a collision between the spheres, the center of the incident atom must approach within a distance $2R$ of the target atom. The differential cross section is

$$d\sigma(p) = d\left(\pi p^2\right) = 2\pi p dp \tag{17}$$

Due to azimuthal symmetry of the possible scattering angles $\theta$, the probability that the incident atom will scatter into a solid angle between $\theta$ and $\theta + \delta\theta$ is $\frac{d\sigma(\theta)}{d(2\pi\cos\theta)}$. Note that since $-\sin\theta d\theta \sim pdp$, this probability is *independent* of $\theta$ and

so the scattering is isotropic (invariant in direction) in the center of mass frame. This is not true in the lab frame unless $M_1 \ll M_2$, in which case we can approximate equation (9) for large $A$

$$\tan\phi \approx \frac{A\sin\theta}{A\cos\theta} = \tan\theta$$

and so the scattering in the lab frame is approximately isotropic, as we would expect. If $M_1 \gg M_2$ then $A$ is very small, implying $\alpha \approx 1$ and so from equation (12), very little energy is transfered to the target atom. The probability that the incident particle would be reflected ($\phi = 180°$) in this case would be zero, since for an incident angle $\beta_0 = 0$, $\sin\beta_0 = \cos\frac{\theta}{2}$, implying $\theta = \pi$ and since $\cos\pi = 0$, $\frac{d\sigma(\theta)}{d(2\pi\cos\theta)} = 0$. In other words, it is not possible for a heavy atom cannot backscatter off of a light atom. The average energy transfer $T_{avg}$ is defined as

$$T_{avg} = \frac{\int_0^{T_m} T d\sigma(T)}{\int_0^{T_m} d\sigma(T)} = \frac{1}{2}T_m \tag{18}$$

as we would expect. The probability of an energy transfer $dT$ in the course of the collision is

$$g(E_0, E_2)\,\delta E_2 = \frac{\sin\theta}{2}\frac{d\theta}{dE_2}\delta E_2 \tag{19}$$

Here, $E_0$ and $E_2$ are the incident atom's energy before and after the collision. From equation (13), $E_2$ is given by

$$E_2 = E_0\left(1 - (1-\alpha)\sin^2\frac{\theta}{2}\right) \tag{20}$$

Thus, we can rewrite equation (19) as

$$g(E_0, E_2)\,\delta E_2 = -\frac{\delta E_2}{E_0(1-\alpha)} \tag{21}$$

Note that for $M_1 = M_2$, $A = 1$ and $\alpha = 0$ and equation (21) tells us that the probability of an energy transfer between $E_2$ and $E_2 + \delta E_2$ is the ratio of the differential cross section to the total cross section.