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Calculation of the amount of light through the perpendicualr window using the prior Recombination Rate Calculation

1 Purpose

Calculate amount and wavelength of light through perpendicular window

2 QE of photocathode and power of light

By definition,

$$QE = \frac{N_{e^-}}{N_\gamma} \tag{1}$$

where N_{e^-} is the number of electrons emitted and N_γ is the number of photons incident on the photocathode. For a uniform laser (fixed wavelength λ and power P), the laser power in some time interval t is given by

$$P\,(W) = \frac{EN_\gamma}{t} = \frac{hc}{\lambda}\frac{N_\gamma}{t} \tag{2}$$

since $E_\gamma = \dfrac{hc}{\lambda}$ is the energy of a photon with wavelength λ. Solving for N_γ yields

$$N_\gamma = \frac{P\lambda t}{hc} \tag{3}$$

When the photons strike the photocathode in some area, some number N_{e^-} of electrons will be emitted. The SI unit of current I is the ampere, which is one coulomb per second, and can be related to electrons per second when noting that one coulomb is equivalent to the charge of 6.242×10^{18} electrons and so

$$N_{e^-} = I\,(\mathrm{A})\,t\,(\mathrm{s}) \times \left(\frac{6.242 \times 10^{18}\ \text{electrons}}{1\ \text{Coulomb}}\right) \tag{4}$$

Dividing (4)by (3) yields the equation for QE:

$$\begin{aligned} QE = \frac{N_{e^-}}{N_\gamma} &= I\,(\mathrm{A})\,t\,(\mathrm{s})\left(6.242 \times 10^{18}\right) \times \frac{hc}{P\lambda t} \\ &= I\,(\mathrm{A})\,t\,(\mathrm{s})\left(6.242 \times 10^{18}\right) \times \frac{1240\mathrm{eV\,nm}}{P\left(\frac{\mathrm{eV}}{\mathrm{s}}\right)\lambda\,(\mathrm{nm})\,t\,(\mathrm{s})} \\ QE &= \frac{I\,(\mathrm{A})\left(6.242 \times 10^{18}\right)}{P\left(\frac{\mathrm{eV}}{\mathrm{s}}\right)}\frac{1240\mathrm{eV\,nm}}{\lambda\,(\mathrm{nm})} \end{aligned} \tag{5}$$

We can solve this for the power of light P incident on the photocathode:

$$P\left(\frac{\mathrm{eV}}{\mathrm{s}}\right) = \frac{(1240\mathrm{eV\,nm})\,I\,(\mathrm{A})\left(6.242 \times 10^{18}\,\frac{electrons}{C}\right)}{QE \times \lambda\,(\mathrm{nm})} \tag{6}$$

Plugging in some typical numbers for the ghost beam: $I = 4\mathrm{nA} = 4 \times 10^{-9}\mathrm{A}$, $\lambda = 656\mathrm{nm}$ (most intense, visible spectral line of hydrogen gas), $QE = 0.01\,(= 1\%)$. Thus, the total power of all photons incident on the photocathode that produce the ghost beam is

$$\begin{aligned} P_{GB}\left(\frac{\mathrm{eV}}{\mathrm{s}}\right) &= \frac{\left(4 \times 10^{-9}\mathrm{A}\right)\left(6.242 \times 10^{18}\right)(1240\mathrm{eV\,nm})}{(0.01)\,(656\mathrm{nm})} \\ &\approx 4.72 \times 10^{12}\,\frac{\mathrm{eV}}{\mathrm{s}} \\ &= 0.7561\mu\mathrm{W} \end{aligned}$$

Based on this power, the number of incident photons per second is:

$$\begin{aligned} \frac{N_\gamma}{t} = \frac{P\lambda}{hc} &= \frac{\left(4.72 \times 10^{12}\,\frac{\mathrm{eV}}{\mathrm{s}}\right)(656\mathrm{nm})}{(1240\mathrm{eV\,nm})} \\ &= 2.50 \times 10^{12}\,\frac{photons}{s} \end{aligned}$$

3 Solid angle of light on photocathode active area from the center of the beamline at the perpendicular viewport

Although recombination photons are emitted in all directions, not all of the photons actually make it to the photocathode active area. Only photons emitted within a solid angle that encompasses the photocathode active area can produce the ghost beam. Assume that all of these recombinations take place at the center of the beamline 0.21m away from the photocathode. This corresponds to the middle of the magnetizing solenoid where the perpendicular viewport is. The photocathode active area has a radius of about 1mm. Thus, the angle θ subtended by the solid angle of the photocathode is

$$\begin{aligned}\theta_{PC} &= \tan^{-1}\left(\frac{1\times 10^{-3}\text{m}}{0.21\text{m}}\right)\\ &= 0.00476\text{radians}\end{aligned}$$

The solid angle Ω on a unit sphere corresponding to this θ is

$$\begin{aligned}\Omega &= 4\pi\sin\left(\frac{0.00476\text{radians}}{2}\right)\\ &= 0.0299\end{aligned}$$

(If $\theta = \pi$ radians, then $\Omega = 4\pi$). The fraction of surface area taken up by the solid angle compared to the total surface area of the unit sphere is

$$\frac{\Omega}{4\pi} = \frac{0.0299}{4\pi} \approx 0.00238 = 0.238\%$$

Note that the solid angle is *independent of radius*. Thus, if $2.50\times 10^{12}\frac{photons}{s}$ equates to 0.238% of all photons produced in the trap, then by proportionality, the total number of photons produced per second within the trap (independent of direction) is

$$\begin{aligned}\frac{2.50\times 10^{12}\frac{photons}{s}}{0.238\%} &= \frac{x}{100\%}\\ x &\approx 1.05\times 10^{15}\frac{photons}{s}\end{aligned}$$

Since in each radiative recombination, it is assumed that one photon is emitted, this is equal to the recombination rate. Working backwards, the total power of all recombination light within the trap is assuming $\lambda = 656$nm is

$$\begin{aligned}\frac{N_\gamma}{t} &= \frac{P\lambda}{hc}\\ P &= \frac{hc}{\lambda}\frac{N_\gamma}{t} = \frac{hc}{\lambda}x\\ P &= \frac{1240\text{eV nm}}{(656\text{nm})}\left(1.05\times 10^{15}\frac{photons}{s}\right)\\ P_{trap} &\approx 1.98\times 10^{15}\text{eV s}^{-1} \approx 0.318\text{mW}\end{aligned}$$

4 Solid angle of light through perpendicular viewport

We can now calculate the power of light incident on the perpendicular viewport based on its solid angle. The diameter of the window is 0.015m and the distance from the center of the beamline to the center of the window is 0.05715m (2.25 in.). Thus, the angle subtended by the solid angle of the window is

$$\begin{aligned}\theta_{window} &= \tan^{-1}\left(\frac{0.0075\text{m}}{0.05715\text{m}}\right)\\ &= 0.130\text{radians}\end{aligned}$$

The solid angle Ω on a unit sphere corresponding to this θ is

$$\begin{aligned}\Omega &= 4\pi\sin\left(\frac{0.130\text{radians}}{2}\right)\\ &\approx 0.816\end{aligned}$$

The fraction of surface area taken up by the solid angle compared to the total surface area of the unit sphere is

$$\frac{\Omega}{4\pi} \approx 0.0650 = 6.5\%$$

This means that approximately 6.5% of the photons produced in the trap are incident on the window. The amount of photons produced in one second (assuming $\lambda = 656\text{nm}$) and their corresponding power is

$$x = (0.0650)\left(1.05 \times 10^{15}\frac{photons}{s}\right) = 6.825 \times 10^{13}\frac{photons}{s}$$
$$P = \frac{hc}{\lambda}x = \frac{1240\text{eV nm}}{(656\text{nm})}\left(6.825 \times 10^{13}\frac{photons}{s}\right)$$
$$P_{window} \approx 1.29 \times 10^{14}\text{eV s}^{-1} = 0.02067\text{mW}$$

5 Transmittance of light through perpendicular viewport

Using data from https://www.thorlabs.com/NewGroupPage9.cfm?ObjectGroup_ID=3982, the transmittance of 656nm light is 86.25%. Thus, the power of light that makes it through the window is

$$P_{trans} = (0.8625)(0.02067\text{mW}) = 0.01783\text{mW}$$

Now, 656nm light is not the only spectral line of hydrogen. Below is a table of spectral lines of hydrogen with high relative intensities (from NIST) and their corresponding transmittances.

Wavelength (nm)	Color	Rel. Intensity (NIST)	Norm. Intensity	Energy (eV)	Transmittance (%)
388.9	Near UV	7.0×10^4	0.14	3.18	85.02
410.17	Violet	7.0×10^4	0.14	3.02	85.35
434.04	Blue	9.0×10^4	0.18	2.86	85.65
486.1	Green (Aqua)	1.8×10^5	0.36	2.55	85.93
656.3	Red	5.0×10^5	1	1.89	86.25
1875	IR	5.1×10^4	0.102	0.66	87.67

Table 1: Hydrogen spectral line data taken from NIST website

6 Relative power of light for each spectral line

The calculations made in sections 2-4 were made under the assumption that the 656nm line was the *only* wavelength of light produced in the trap. Using the normalized intensities above, we can calculate the partial power for each spectral line of hydrogen.

Assuming that the QE of the photocathode active area for 656nm light is 0.01, we can calculate the QE for other wavelengths using proportionality:

$$QE = \frac{I\,(\text{A})\left(6.242 \times 10^{18}\right)}{P\left(\frac{\text{eV}}{\text{s}}\right)}\frac{1240\text{eV nm}}{\lambda\,(\text{nm})}$$
$$\frac{QE_2}{QE_1} = \frac{\lambda_1}{\lambda_2} \rightarrow QE_2 = QE_1\frac{\lambda_1}{\lambda_2}$$

Below are QE values for each spectral line:

Wavelength (nm)	QE (%)
388.9	1.69
410.17	1.60
434.04	1.51
486.1	1.35
656.3	1.00
1875	0.350

Table 2: QE values for each spectral line of hydrogen assuming 1% QE at 656nm

The typical ghost beam current is $I_{GB} = 4\text{nA} = 4\times10^{-9}\text{A}$. From the relative intensities and QE values, we can determine each spectral line's contribution to this current in the following way:

$$
\begin{aligned}
I &= \frac{QE \times P\left(\frac{\text{eV}}{\text{s}}\right) \times \lambda\,(\text{nm})}{(6.242\times10^{18})\,(1240\text{eV nm})} \\
I_{GB} &= I_{388.9} + I_{410.17} + I_{434.04} + I_{486.1} + I_{656.3} + I_{1875} \\
&= (0.14 + 0.14 + 0.18 + 0.36 + 1 + 0.102)\, I_{656.3} \\
I_{656.3} &= \frac{I_{GB}}{1.922} = 2.08\times10^{-9}\text{A} \\
P_{656.3}\left(\frac{\text{eV}}{\text{s}}\right) &= \frac{I_{656.3}\times(6.242\times10^{18})\,(1240\text{eV nm})}{QE_{656.3}\times\lambda\,(\text{nm})} \\
&= 2.45\times10^{12}\frac{\text{eV}}{\text{s}} \\
&= 0.393\mu\text{W} \\
\frac{N_{656.3}}{t} &= \frac{P\lambda}{hc}
\end{aligned}
$$

Below are values for the current contribution, power, and photon production rate for all spectral lines of hydrogen

Wavelength (nm)	Current (nA)	Power (nW)	Photon production rate (photons/s)
388.9	0.291	55.0	1.08×10^{11}
410.17	0.291	55.1	1.14×10^{11}
434.04	0.375	71.0	1.55×10^{11}
486.1	0.750	142	3.46×10^{11}
656.3	2.08	393	1.30×10^{12}
1875	0.212	40.1	3.79×10^{11}

Table 3: Calculated values for current and power based on the relative intensities and QE values in the above two tables

Using these values and the solid angles of the photocathode active area and window with respect to the center of the beamline, we can calculate the total power of light, the total incident light, and the total transmitted light for each spectral line using similar calculations as those in sections 3-4:

Wavelength (nm)	Total power within trap (nW)	Total incident power (nW)	Total transmitted power (nW)
388.9	2311	150.2	127.7
410.17	2315	150.5	128.5
434.04	2983	193.9	166.1
486.1	5966	387.8	333.2
656.3	16510	1073	925.7
1875	1685	109.5	95.92

Table 4: Calculated values for the total power of light, total incident light, and total transmitted light for each spectral line

The sum of the total transmitted powers through the window for all wavelengths is $P_{trans} = 1.777\mu\text{W}$.

7 Photodiode specs

We need a photodiode that would be able to detect and resolve this relatively small amount of light