Chicane optimization

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Plan

1. Analytics optimization
2. ELEGANT's Simulations
3. Conclusion & Questions

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Rectengular dipole matrix

- The rectengular dipole matrix is defined as :

$$M_{dipole}(\rho\theta) = \begin{bmatrix} \cos\theta & \rho\sin\theta & 0 & \rho(1-\cos\theta) \\ -\frac{1}{\rho} & \cos\theta & 0 & \sin\theta \\ -\sin\theta & -\rho(1-\cos\theta) & 1 & (\frac{\rho\theta}{\gamma^2}) - \rho(\theta - \sin\theta) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- $L_{dipole} = \rho\theta$
- ρ is the bend radius.
- θ is the bend angle.
- $[y, y']$ and [x,x',z,δ] elements are decoupled.

Sector dipole matrix

- The sector dipole matrix is defined as :

$$M_{dipole}(\rho\theta) = \begin{bmatrix} \cos\theta & \rho\sin\theta & 0 & 0 \\ -\frac{1}{\rho} & \cos\theta & 0 & 0 \\ ? & ? & 1 & (\frac{\rho\theta}{\gamma^2}) - \rho(\theta - \sin\theta) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- $L_{dipole} = \rho\theta$
- ρ is the bend radius.
- θ is the bend angle.
- $[y, y']$ and [x,x',z,δ] elements are decoupled.

Achromaticity condition

To simplify the mathematics we apply:

Achromaticity criterion : $D = \begin{bmatrix} \eta_{x\,exit} \\ \eta'_{x\,exit} \end{bmatrix} = \begin{bmatrix} \eta_{x\,entrance} \\ \eta'^{x\,entrance} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$D_{exit} = \begin{bmatrix} M_x & 0 \end{bmatrix} \times D_{entrance} + \begin{bmatrix} R_{16} \\ R_{26} \end{bmatrix}$$

$$R_{16} = R_{26} = 0$$

Achromaticity condition

$$M_{chicane} = \begin{bmatrix} 1 & R_{12} & R_{13} & R_{14} & R_{15} & \color{red}{0} \\ R_{21} & 1 & R_{23} & R_{24} & R_{25} & \color{red}{0} \\ R_{31} & R_{32} & 1 & R_{24} & R_{25} & \color{red}{0} \\ R_{41} & R_{42} & R_{43} & 1 & R_{25} & \color{red}{0} \\ \color{red}{0} & \color{red}{0} & \color{red}{0} & \color{red}{0} & \color{blue}{1} & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z_{exit\,chicane} = \color{blue}{R_{55}} z_0 + R_{56}\delta_0$$

$$\Delta z = R_{56}\delta_0$$

Longitudinal beam chirp

- **Using z & $\frac{\Delta P}{P}$ space, we have:**

$$\kappa = \frac{d\delta_p}{dz} = \frac{-keV_0}{E0 + eV0\cos\phi}\sin\phi$$

  - $k = 2\pi\frac{f}{c}$ $[m^{-1}]$
  - f is the cavity frequency
  - eV_0 Cavity acceleration [MeV]
  - E_0 Central energy [MeV]
  - ϕ Cavity phase advance.

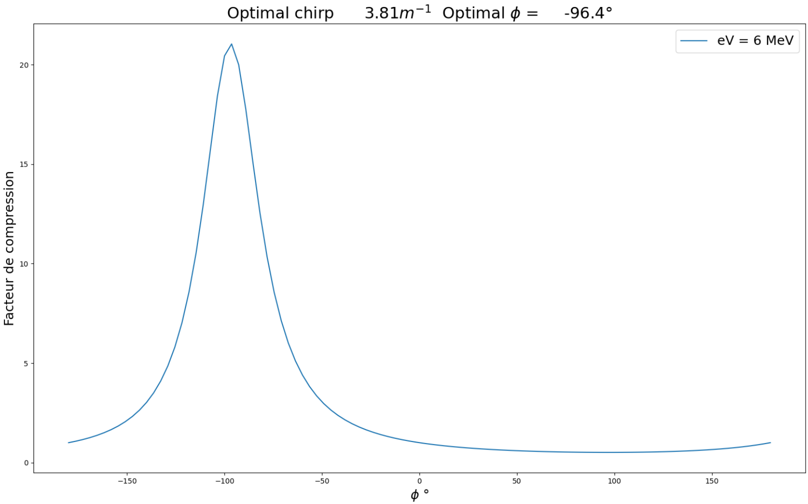
- **Compression factor**

$$C = \frac{1}{1 + [R_{56} \times \kappa]}$$

$$C = \frac{1}{1 + \left[R_{56} \times \frac{-keV_0}{E0+eV0\cos\phi}\sin\phi\right]}$$

Compression factor

- $R_{56} = -0.25$ m



Beam size along the chicane

- How to reduce the beam size along the chicane?
- **Answer : FODO**
- **Motivation:** $\frac{\Delta P}{P_0} = \pm 10\%$
- **Focusing quadrupole =**

$$\begin{bmatrix} \cos\sqrt{K}L_q & \frac{1}{\sqrt{K}}\sin\sqrt{K}L_q \\ -\sqrt{K}\sin\sqrt{K}L_q & \cos\sqrt{K}L_q \end{bmatrix}$$

- **Defocusing quadrupole =**

$$\begin{bmatrix} \cosh\sqrt{K}L_q & \frac{1}{\sqrt{K}}\sinh\sqrt{K}L_q \\ -\sqrt{K}\sinh\sqrt{K}L_q & \cosh\sqrt{K}L_q \end{bmatrix}$$

- **FODO**

$$M_{FODO} = M_{HALF\ QF}\ M_{DRIFT}\ M_{QD}\ M_{DRIFT}\ M_{HALF\ QF}$$

- **Initial FODO parameters**
  - Focusing Quadrupole strength $K_{QF} = 0.6\ m^{-2}$
  - Quadrupole length $L_Q = 0.2\ m$
  - Defocusing quadrupole strength $K_{QDF} = ?$
- **Drift parameter:**
  - Drift length $L_{drift} = 5.6m$
- **Motivation** Apply the periodicity condition on the FODO lattice to get : $\begin{bmatrix} \beta_{exit} \\ \alpha_{exit} \\ \gamma_{exit} \end{bmatrix} = \begin{bmatrix} \beta_{entrance} \\ \alpha_{entrance} \\ \gamma_{entrance} \end{bmatrix}$
  - β α and γ are the twiss parameters of the beam wich describes the behaviour of the optics along the lattice.
  - In periodic system, for stability of the equation of the motion we have :

$$|trace(M)| < 2$$

Linear beam optics

- If the FODO matrix is given by :

$$M(s_1 s_2) = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix}$$

- The transformation matrix from point s_1 to s_2 in the lattice is given by :

$$\begin{bmatrix} \beta_{s2} \\ \alpha_{s2} \\ \gamma_{s2} \end{bmatrix} = \begin{bmatrix} C^2 & -2SC & S^2 \\ -CC' & SC' + S'C & -SS' \\ C'^2 & -2S'C' & S'^2 \end{bmatrix} \begin{bmatrix} \beta_{s1} \\ \alpha_{s1} \\ \gamma_{s1} \end{bmatrix}$$

- From the stability condition:

$$|trace\ M(s_1 s_2)| = C + S' < 2$$

We get :

$$K_{QDF} = -1.096\ m^{-2}$$

- t the begining of the FODO : $\alpha = 0$, then we have $\beta = \beta max$ at the enter of the focusing quadrupole.

Linear beam optics

- The FODO matrix become :

$$M_{FODO} = \begin{bmatrix} 0.95 & 6.59 \\ -0.014 & 0.95 \end{bmatrix}$$

- With $\alpha = 0$ then we have $\beta = \beta_0$ and $\gamma = \frac{1}{\beta_0}$, then Using the transformation matrix:

$$\beta_0 = 11.6 \ m$$

- We define the phase advance matrix per cell:

$$\begin{bmatrix} \cos\phi + \alpha \sin\phi & \beta \sin\phi \\ -\gamma \sin\phi & \cos\phi - \alpha \sin\phi \end{bmatrix}$$

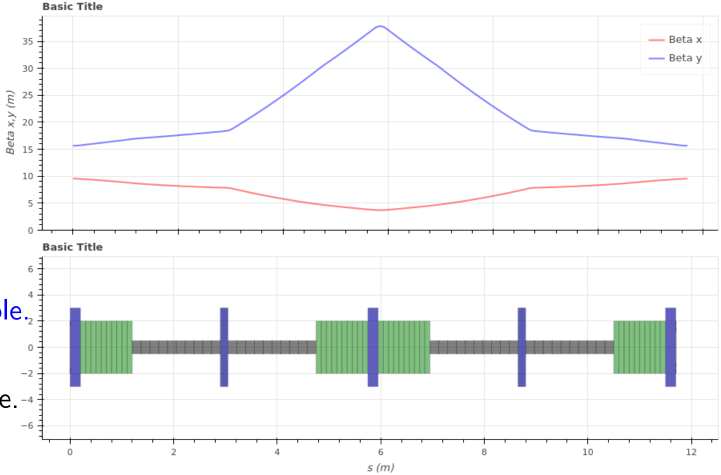
- We can immediately get the phase advance :

$$\begin{aligned} \cos\phi &= 0.95 \qquad (1) \\ \phi &= \arccos 0.95 \end{aligned}$$

Plan

ELEGANT Results

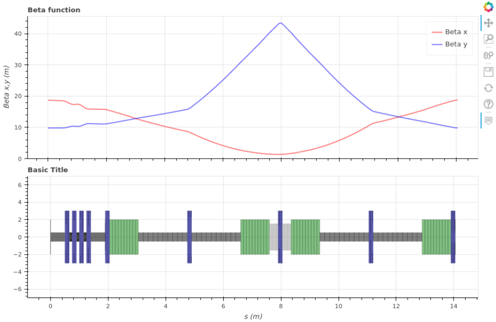
- Layout :



- Quadrupole.
- Dipole.
- Beam pipe.

ELEGANT Results with matching section

- Layout :



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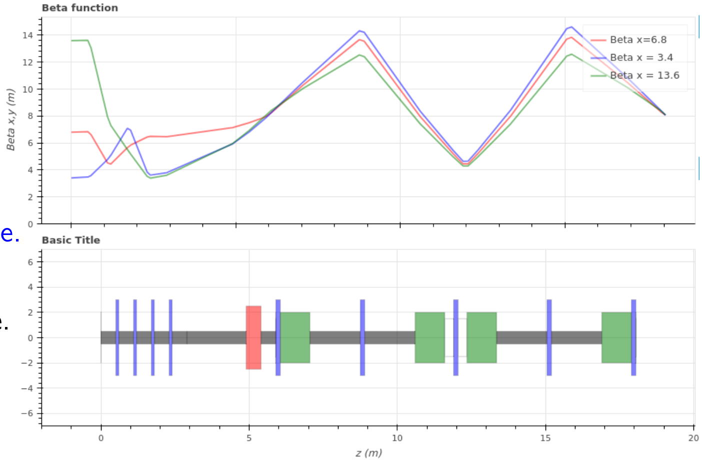
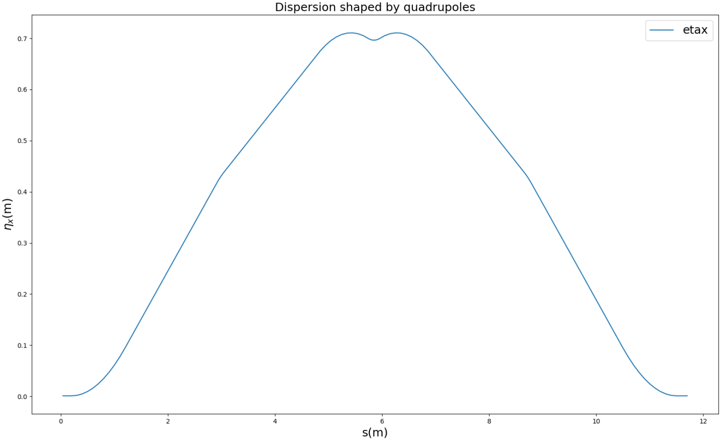


- Quadrupole.
- Dipole.
- Beam pipe.
- Cavity.

Dispersion

- Dispersion peak at the middle of the chicane.



Beam size [x,x'] plane

- The beam size at the middle of the chicane is given by :

$$\sigma = \sqrt{\epsilon \times \beta_{min}}$$

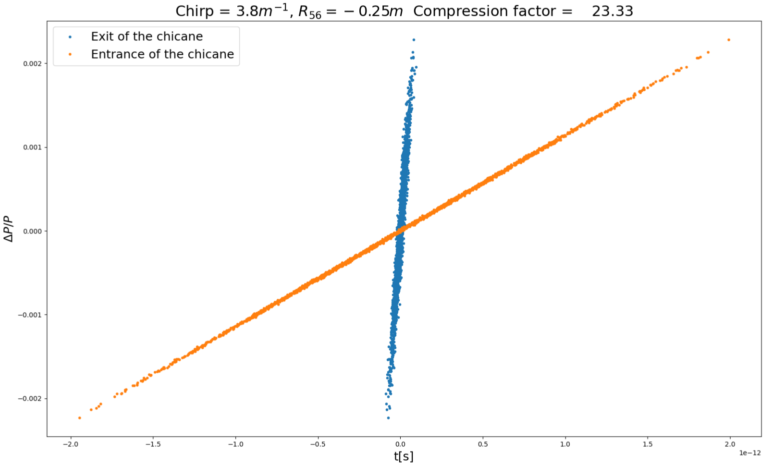
- From the positron distribution $\epsilon = 0.039\ mm\ rad$, and from the β function, we get β_{min} at the middle of the chicane:

$$\beta_{min} = 3.7\ m$$

- The beam size at the middle of the chicane is:

$$\sigma = 0.012\ m$$

Chicane exit



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Conclusion

- Fodo lattice allow us to control the beam size along the chicane.
- Optimized cavity to chirp the beam.
- Need to increase the dispersion at the middle of the chicane.
- Need an optimized matching section (quadrupoles) before the FODO lattice to match the twiss parameters.
- Mathematic calculations helps a lot for the software optimization.
- To be continued ...