GEANT4 Simulation of the Jlab MeV Mott Polarimeter

Martin McHugh
mjmchugh@jlab.org

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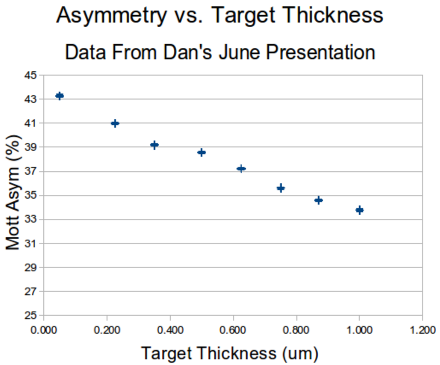
The Problem

We don't know the form of the effective Sherman function for targets of finite thickness, $S(d)$.

In order to test simulations, we look at two outputs:

- Rates
- Asymmetries



Simulation Basics

- Beam is treated as 100% polarized in y direction.
- Gaussian, circular beam profile with width of 1 mm.
- Target d and other variables determined by user.
- Two separate types of simulations:
  - Single Scattering
  - Double Scattering
- Simulation are analyzed separately and the results combined to get Rates and Asymmetries.

Why Brute Force doesn't work: 1 μA is 6.24×10^{12} e^-/s and we need $\approx$ 1000 μAs of data for a decent measurement. Can only simulate 100 million events per day...

Simulation Strategy

- Use Xavier's input to model scattering of polarized electrons from individual nuclei given E and $\vec{P}$
- Energy loss within the target is accounted for
- Simulation records scattering-vertex information:
  - Cross-sections (for first and second scattering)
  - Kinematics: E, Scattering Angles, etc
  - Position
- Events are thrown to the Left and Right detectors
- Hits are recorded
- Rates are calculated for each detector
- Asymmetry is then calculated from combined rates.

How to Generate Single-Scattering Events

1. Pick point $\vec{x}_1$ in the beam profile on the target.
2. Calculate energy loss to $\vec{x}_1$, get new energy $E^{'}$.
3. Pick angle (θ, ϕ) to throw events toward Left or Right detectors
4. Calculate $\sigma(\theta, \phi, E^{'})$.
5. Record all kinematics and dynamics of proposed scattering.
6. Throw electron *as if single scattering has occurred*.
7. Let GEANT4 handle the rest...

How to Generate Double-Scattering Events

1. Pick point $\vec{x}_1$ in the beam profile on the target
2. Calculate energy loss to $\vec{x}_1$, get new energy E_1
3. Pick point $\vec{x}_2$ in target with $|\vec{x}_2 - \vec{x}_1| < r_{\mathrm{max}}$
4. Calculate and store kinematics and dynamics of first scattering ($\vec{x}_1$ to $\vec{x}_2$)
5. Calculate energy loss to $\vec{x}_2$. Get new energy E_2
6. Pick angle (θ, ϕ) to throw events toward Left or Right detectors
7. Calculate and store kinematics and dynamics of second scattering ($\vec{x}_2$ towards detectors)
8. Throw electron *as if double scattering has occurred*
9. Let GEANT4 handle the rest...

Calculating Rates

- Need to calculate rate seen in our detectors for each process to compare to data
- Rate from one point in phase space $\vec{v}$ is:

$$d\mathcal{R}(\vec{v}) = \mathcal{L}(\vec{v})\sigma(\vec{v})\epsilon(\vec{v})dv \quad (1)$$

- Total rate is the integral:

$$\mathcal{R} = \int_V d\mathcal{R}(\vec{v}). \quad (2)$$

- Specifics of integral are different for single and double scattering

Calculating Asymmetry

- We calculate four rates: single and double scattering in Left and Right detectors.
- This allows us to calculate asymmetries as a function of target thickness:

$$A(d) = \frac{[\mathcal{R}_{L_1}(d) - \mathcal{R}_{R_1}(d)] + [\mathcal{R}_{L_2}(d) - \mathcal{R}_{R_2}(d)]}{[\mathcal{R}_{L_1}(d) + \mathcal{R}_{R_1}(d)] + [\mathcal{R}_{L_2}(d) + \mathcal{R}_{R_2}(d)]} \quad (3)$$

Single Scattering Rate

Integral approximated by Reimann sum over scattering angle, χ and azimuthal angle ψ:

$$\mathcal{R}_1 \approx \frac{N_A \rho}{A} N_B d \sum_{i=1}^{N_\chi} \sum_{j=1}^{N_\psi} \sigma_{ij} \epsilon_{ij} \sin \chi_i \Delta\chi \Delta\psi \tag{4}$$

Importantly:

$$\mathcal{R}_1 = \alpha d \tag{5}$$

Double Scattering Rate

Integral performed using Monte Carlo estimator:

$$\mathcal{R}_2 \approx \frac{2\pi^2}{9}\left(\cos\frac{\pi}{36} - \cos\frac{\pi}{18}\right) N_B \left(\frac{N_A \rho d}{A}\right)^2 \frac{1}{n}\sum_i^n \sigma_1(\vec{v}_i)\sigma_2(\vec{v}_i)\epsilon(\chi_i, \psi_i)$$

Importantly:

$$\mathcal{R}_2 = \beta d^2 \tag{6}$$

Single Scattering Rates

d [nm]	R_L [Hz/uA]	dR_L [Hz/uA]	R_R [Hz/uA]	dR_R [Hz/uA]	Avg. Rate [Hz/uA]	dR [Hz/uA]	Asym	dA
52	5.01	0.14	15.55	0.43	10.28	0.23	-51.24%	2.48%
100	9.64	0.27	29.84	0.83	19.74	0.44	-51.17%	2.48%
200	19.26	0.54	59.76	1.66	39.51	0.87	-51.24%	2.48%
215	20.70	0.58	64.21	1.79	42.45	0.94	-51.25%	2.48%
300	28.87	0.80	89.68	2.49	59.28	1.31	-51.29%	2.48%
389	37.47	1.04	115.97	3.23	76.72	1.70	-51.16%	2.48%
400	38.48	1.07	119.75	3.33	79.12	1.75	-51.36%	2.48%
487	46.85	1.30	145.54	4.05	96.19	2.13	-51.29%	2.48%
500	48.16	1.34	149.23	4.15	98.69	2.18	-51.21%	2.48%
561	54.02	1.50	167.44	4.66	110.73	2.45	-51.21%	2.48%
600	57.77	1.61	179.22	4.98	118.49	2.62	-51.25%	2.48%
700	67.38	1.88	209.29	5.82	138.33	3.06	-51.29%	2.48%
775	74.65	2.08	231.35	6.43	153.00	3.38	-51.21%	2.48%
800	77.15	2.15	239.15	6.65	158.15	3.49	-51.22%	2.48%
837	80.57	2.24	249.85	6.95	165.21	3.65	-51.23%	2.48%
900	86.64	2.41	268.98	7.48	177.81	3.93	-51.27%	2.48%
944	91.00	2.53	282.20	7.85	186.60	4.12	-51.23%	2.48%
1000	96.25	2.68	299.53	8.32	197.89	4.37	-51.36%	0.59%

- Asymmetry constant in *d* therefore $\boxed{\epsilon_1 = -51.25\% \pm 0.59\%}$
- Rate linear in *d*: $\boxed{\alpha = 0.198 \pm 0.001 \text{ Hz}/(\mu\text{A} \times \text{nm})}$

Double Scattering Rates

d [nm]	R_L [Hz/uA]	dR_L [Hz/uA]	R_R [Hz/uA]	dR_R [Hz/uA]	Avg. Rate [Hz/uA]	dR [Hz/uA]	Asym	dA
52	0.22	0.02	0.12	0.02	0.17	0.01	28.52%	9.26%
100	0.78	0.09	0.36	0.05	0.57	0.05	36.54%	9.74%
200	2.92	0.32	1.74	0.23	2.33	0.20	25.17%	8.67%
215	3.79	0.48	2.68	0.73	3.24	0.44	17.27%	13.67%
300	8.21	0.97	3.59	0.47	5.90	0.54	39.13%	9.82%
389	12.94	1.70	5.58	0.78	9.26	0.93	39.71%	10.85%
400	11.25	1.37	10.33	1.83	10.79	1.14	4.24%	10.58%
487	20.46	3.09	8.79	1.64	14.63	1.75	39.91%	12.90%
500	16.64	1.82	10.82	1.57	13.73	1.20	21.20%	8.96%
561	29.69	4.17	11.47	1.83	20.58	2.28	44.25%	12.11%
600	28.60	4.33	21.27	3.71	24.94	2.85	14.69%	11.55%
700	43.84	4.90	26.69	5.93	35.26	3.85	24.33%	11.22%
775	40.56	5.58	22.95	3.63	31.76	3.33	27.73%	10.88%
800	67.56	8.87	25.98	4.54	46.77	4.98	44.44%	11.66%
837	49.58	6.34	31.81	4.86	40.69	4.00	21.83%	10.05%
900	67.97	8.44	37.97	7.92	52.97	5.79	28.32%	11.36%
944	77.47	10.19	37.28	6.63	57.37	6.08	35.03%	11.23%
1000	76.53	9.86	49.38	8.53	62.95	6.52	21.56%	10.59%

- Asymmetry constant in d:

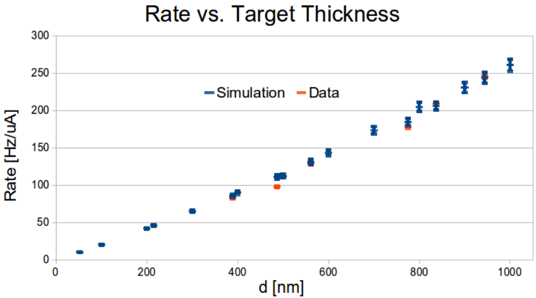
$$\epsilon_2 = 28\% \pm 11\% \text{ (std.dev.)}$$

- Rate Quadratic:

$$\beta = 6.17 \times 10^{-5} \pm 0.57 \times 10^{-5} \text{ Hz}/(\mu\text{A} \times \text{nm}^2)$$

Comparison to Data: Rates

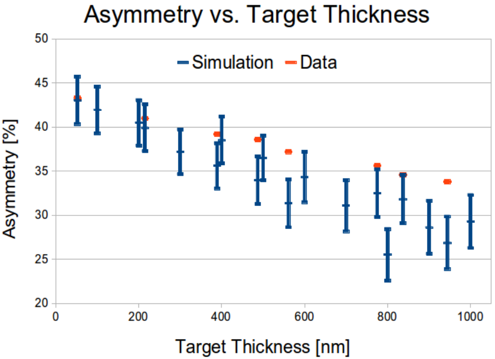
- Simulations analyzed with no energy cuts (to match data)
- We recover $\mathcal{R} = \alpha d + \beta d^2$



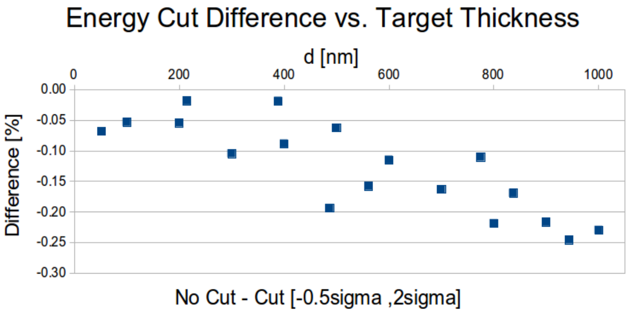
Parameter	Simulation	Data (Fit)
α [Hz/(μA $\times$ nm)]	0.198 ± 0.001	0.187 ± 0.009
β [10^{-5}Hz/(μA $\times$ nm^2)]	6.17 ± 0.57	6.59 ± 1.57

Comparison to Data: Asymmetry

- Simulations analyzed with no energy cuts
- Asymmetry calculated according to formula on slide 9
- Assume $P = 86.08\ \% \pm 0.56\ \%$
- Agrees well, particularly at low d



Asymmetry and Energy Cuts in Simulation



- Typical asymmetry error is 2.5-3 %
- Difference is insignificant
- Makes sense: no low energy “not-a-background” in simulation

Suggested Fitting Function

- Simulation agrees with data pretty well
- Well founded assumption:

$$\mathcal{R}_{L_1}(d) = \alpha d(1 + P\epsilon_1) \qquad \mathcal{R}_{R_1}(d) = \alpha d(1 - P\epsilon_1) \tag{7}$$

$$\mathcal{R}_{L_2}(d) = \beta d^2(1 + P\epsilon_2) \qquad \mathcal{R}_{R_2}(d) = \beta d^2(1 - P\epsilon_2) \tag{8}$$

- Plug into Eq. (3) gets:

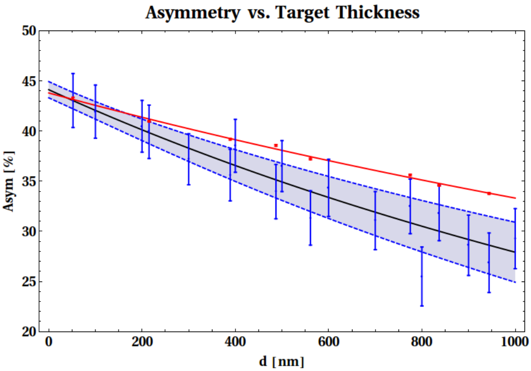
$$A(d) = P\frac{\alpha\epsilon_1 + \beta\epsilon_2 d}{\alpha + \beta d} \tag{9}$$

- Parameters known from simulation, in data we simplify to 3 parameter fit:

$$A(d) = \frac{a + bd}{1 + cd} \tag{10}$$

with $a = P\epsilon_1$, $b = P\beta\epsilon_2/\alpha$ and $c = \beta/\alpha$

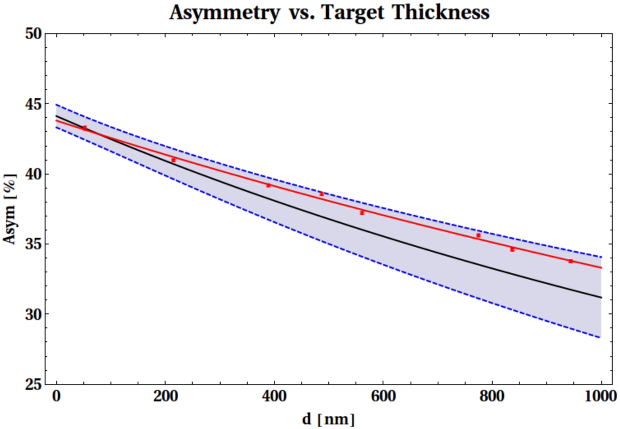
How Does This Fit Work?



- Black curve is simulation based prediction in Eq. (9)
- Red curve is fit to data using the model based fit in Eq. (10)
- Assuming data values of α and β from slide 14 we get $\epsilon_2 = -12.3\%$.

Prediction vs. Fit Discrepancy

If we change the simulation value of ϵ_2 to what we get on the previous slide, we see better agreement:

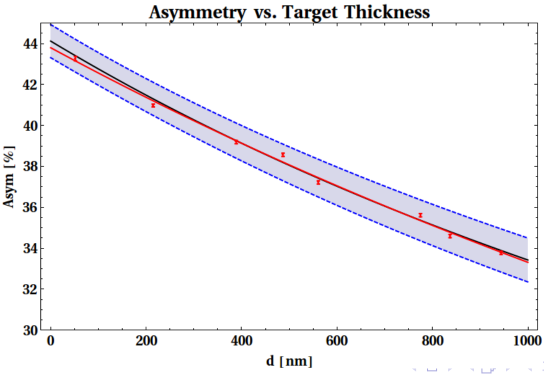


Alternate Double Scattering Asymmetry Effect

If we use the naive rejection method, $x < \sigma_1\sigma_2$, we see:

$$\epsilon_2 = \frac{\mathcal{R}_{L_2} - \mathcal{R}_{R_2}}{\mathcal{R}_{L_2} + \mathcal{R}_{R_2}} = -01.05\% \pm 0.06\%$$

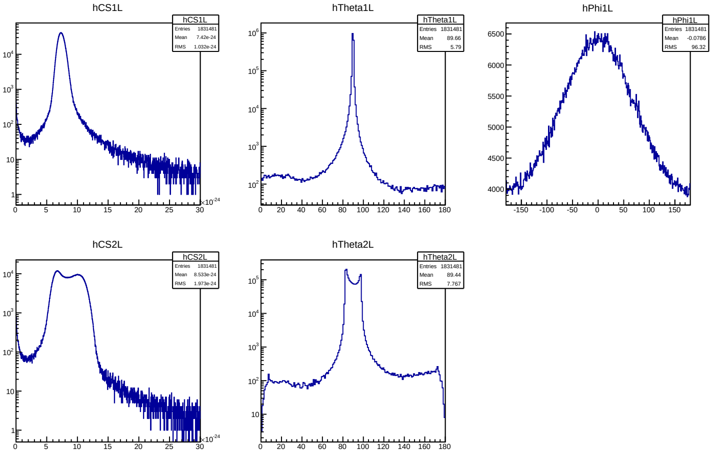
Using this gives:



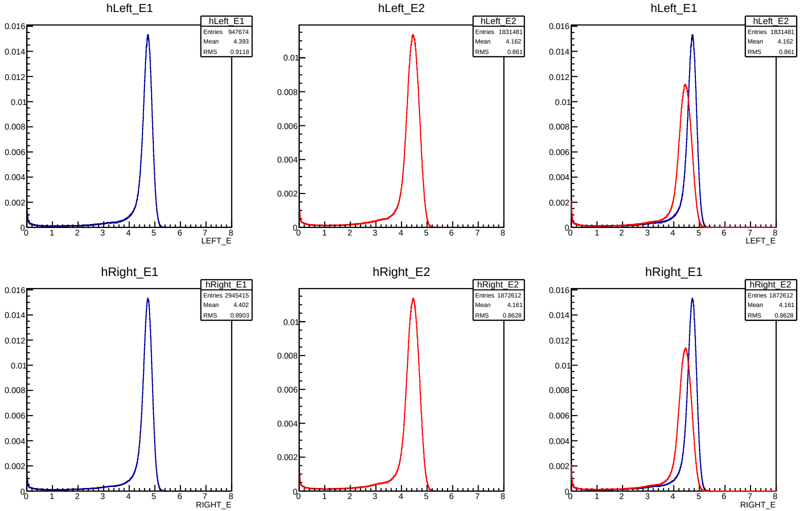
Summary

- We have *decent* agreement between simulation and data
- Simulation has indicated a physically motivated fitting function which we should use
- Rate based comparisons (i.e. α and β parameters) are solid
- Small largely due to ϵ_2 discrepancy.
- Overall, simulation effort largely complete.
- Some refinement possible for ϵ_2

Scattering into Left Detector



Spectra


hLeft_E1
hLeft_E1
Entries 947674
Mean 4.393
RMS 0.9118
LEFT_E
hLeft_E2
hLeft_E2
Entries 1831481
Mean 4.162
RMS 0.861
hLeft_E1
hLeft_E2
Entries 1831481
Mean 4.162
RMS 0.861
LEFT_E
hRight_E1
hRight_E1
Entries 2945415
Mean 4.402
RMS 0.8903
RIGHT_E
hRight_E2
hRight_E2
Entries 1872612
Mean 4.161
RMS 0.8628
hRight_E1
hRight_E2
Entries 1872612
Mean 4.161
RMS 0.8628
RIGHT_E

Energy Loss in the Gold Foils

Using the table at right, we determine the linear fit

$$\frac{dE}{dx}(E) = \frac{0.272}{\mathrm{mm}} \times E + 1.888 \frac{\mathrm{MeV}}{\mathrm{mm}}.$$

Numerical integration of the above gives us energy loss within the target. Note: A particle with initial energy of 5 MeV will only lose 3 keV/μm and will lose 500 keV in $\approx$ 200μm.

Energy [MeV]	dE/dx [MeV/mm]
1.0	2.179
2.0	2.422
3.0	2.702
4.0	2.980
5.0	3.254
6.0	3.526
7.0	3.796
8.0	4.065

Data from NIST estar database.

What Steigerwald Did

His method of calculating multiple scattering's influence was direct integration of some form:

$$N = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \int_{x_1=0}^{D} \int_{\psi=\theta_2}^{\theta_2+\Omega_\theta} \sigma_1(x_1, \theta, \phi)\sigma_2(x_2, \theta_2)E(x_1, x_2)d\psi dld\phi d\theta.$$

The problem is that his source for this integral is in German and his code is poorly documented and also in partial German.