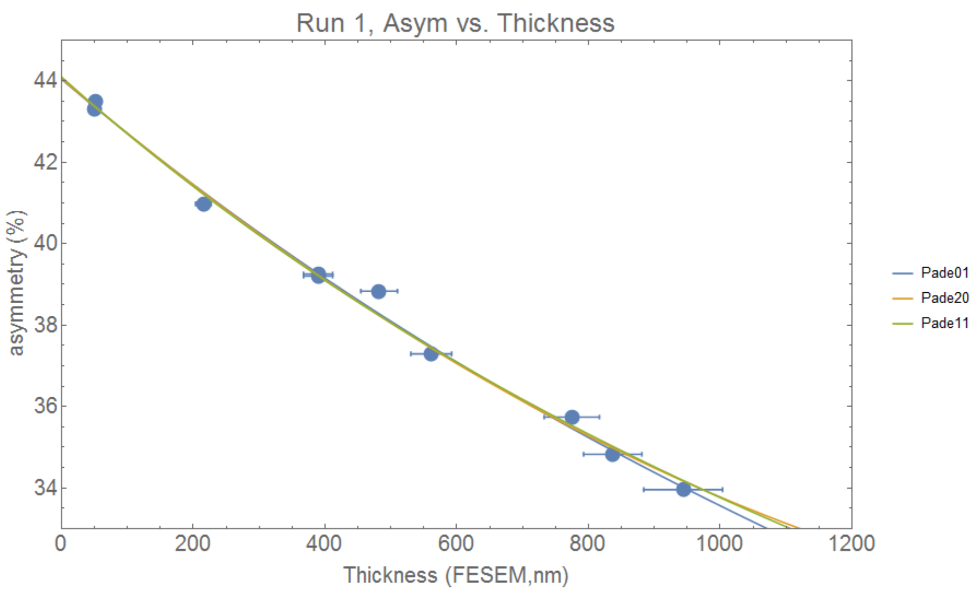
Mathematica fits vs. Root

17Jan 2017

Asym vs. T, Run 1



Pade01	$\frac{44.0395}{1+0.000312289\,x}$	Reduced $\chi 2$	1.62572
Pade20	$44.0492 - 0.0137301\,x + 3.45486\times10^{-6}\,x^2$	Reduced $\chi 2$	1.36746
Pade11	$\frac{44.0928+0.00336456\,x}{1+0.000405728\,x}$	Reduced $\chi 2$	1.27408

In[137]:= `PlotPade01[{"ParameterTable"}]`

Out[137]=

	Estimate	Standard Error	t- Statistic	P- Value
a0	44.0395	0.101186	435.231	8.69741×10^{-19}
b1	0.000312289	9.62375×10^{-6}	32.4498	8.86513×10^{-10}

In[138]:= `PlotPade20[{"ParameterTable"}]`

Out[138]=

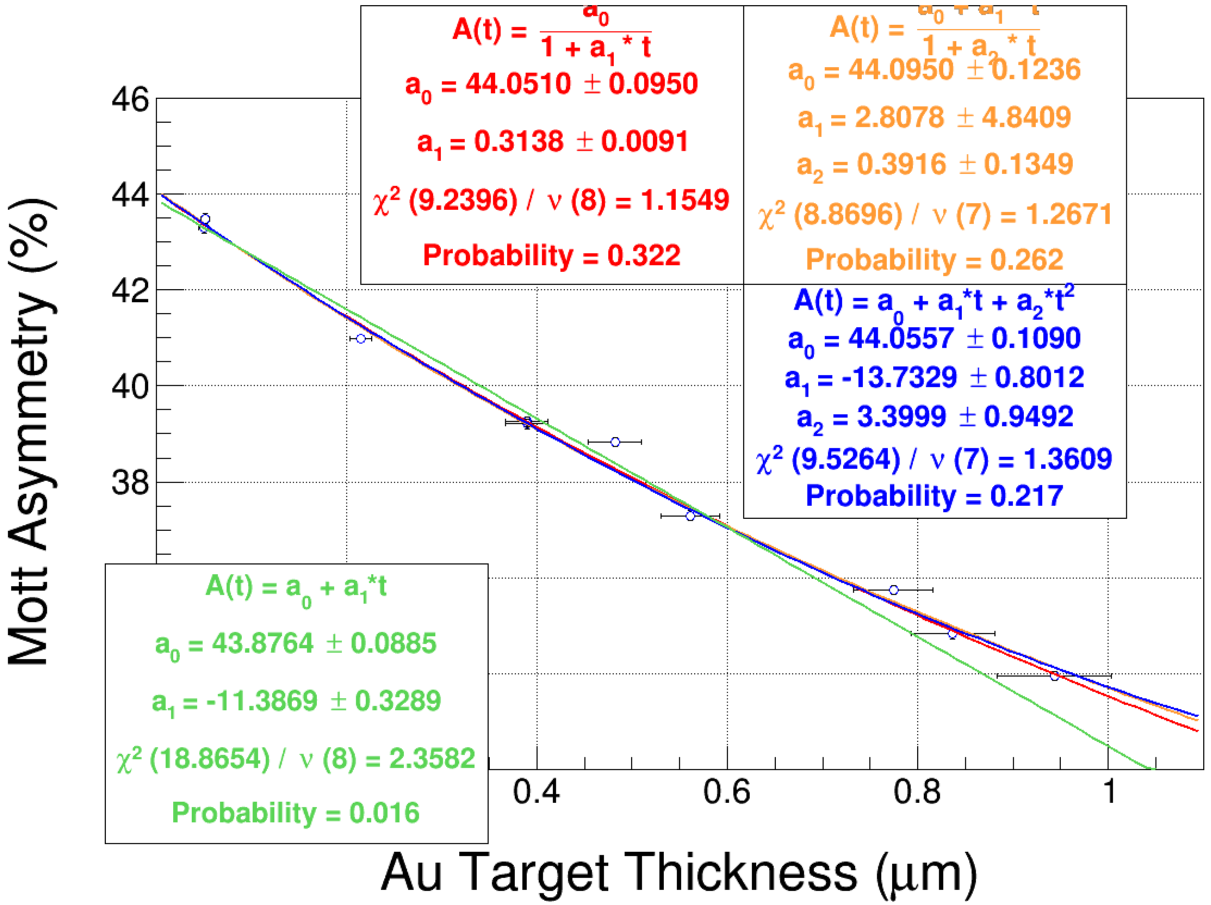
	Estimate	Standard Error	t- Statistic	P- Value
a0	44.0492	0.129462	340.247	5.00191×10^{-16}
a1	−0.0137301	0.00101446	−13.5344	2.825×10^{-6}
a2	3.45486×10^{-6}	1.22603×10^{-6}	2.81792	0.0258504

In[139]:= `PlotPade11[{"ParameterTable"}]`

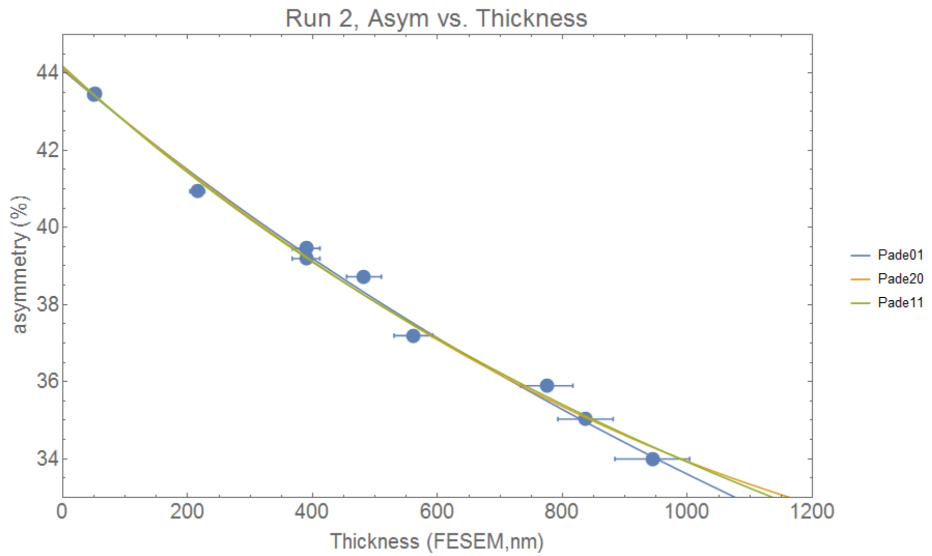
Out[139]=

	Estimate	Standard Error	t- Statistic	P- Value
a0	44.0928	0.139491	316.097	8.37404×10^{-16}
a1	0.00336456	0.00567884	0.592474	0.572167
b1	0.000405728	0.000158142	2.56559	0.0372469

Asym vs. T, Run 1



Asym vs. T, Run 2



Pade01	$\frac{44.0809}{1+0.000312034\,x}$	Reduced $\chi 2$	1.62572
Pade20	$44.1172 - 0.0140333\,x + 3.84757\times10^{-6}\,x^2$	Reduced $\chi 2$	1.29458
Pade11	$\frac{44.1759+0.00564007\,x}{1+0.000468957\,x}$	Reduced $\chi 2$	1.17529

In[183]:= `PlotPade01[{"ParameterTable"}]`

Out[183]=

	Estimate	Standard Error	t- Statistic	P- Value
a0	44.0809	0.113083	389.809	2.1005×10^{-18}
b1	0.000312034	9.96072×10^{-6}	31.3265	1.17282×10^{-9}

In[184]:= `PlotPade20[{"ParameterTable"}]`

Out[184]=

	Estimate	Standard Error	t- Statistic	P- Value
a0	44.1172	0.141336	312.144	9.14522×10^{-16}
a1	-0.0140333	0.00102563	-13.6825	2.62417×10^{-6}
a2	3.84757×10^{-6}	1.20724×10^{-6}	3.18706	0.0153391

In[185]:= `PlotPade11[{"ParameterTable"}]`

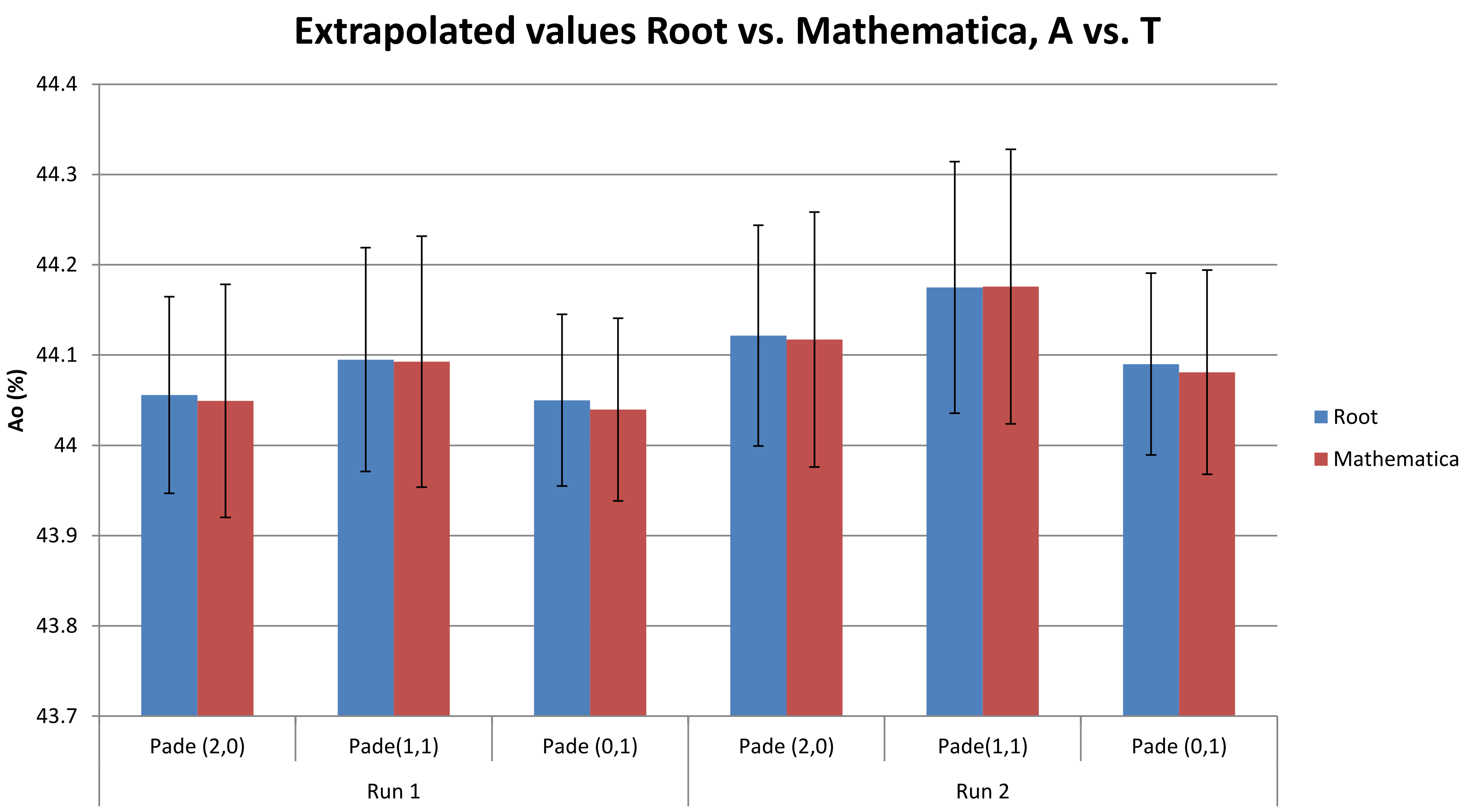
Out[185]=

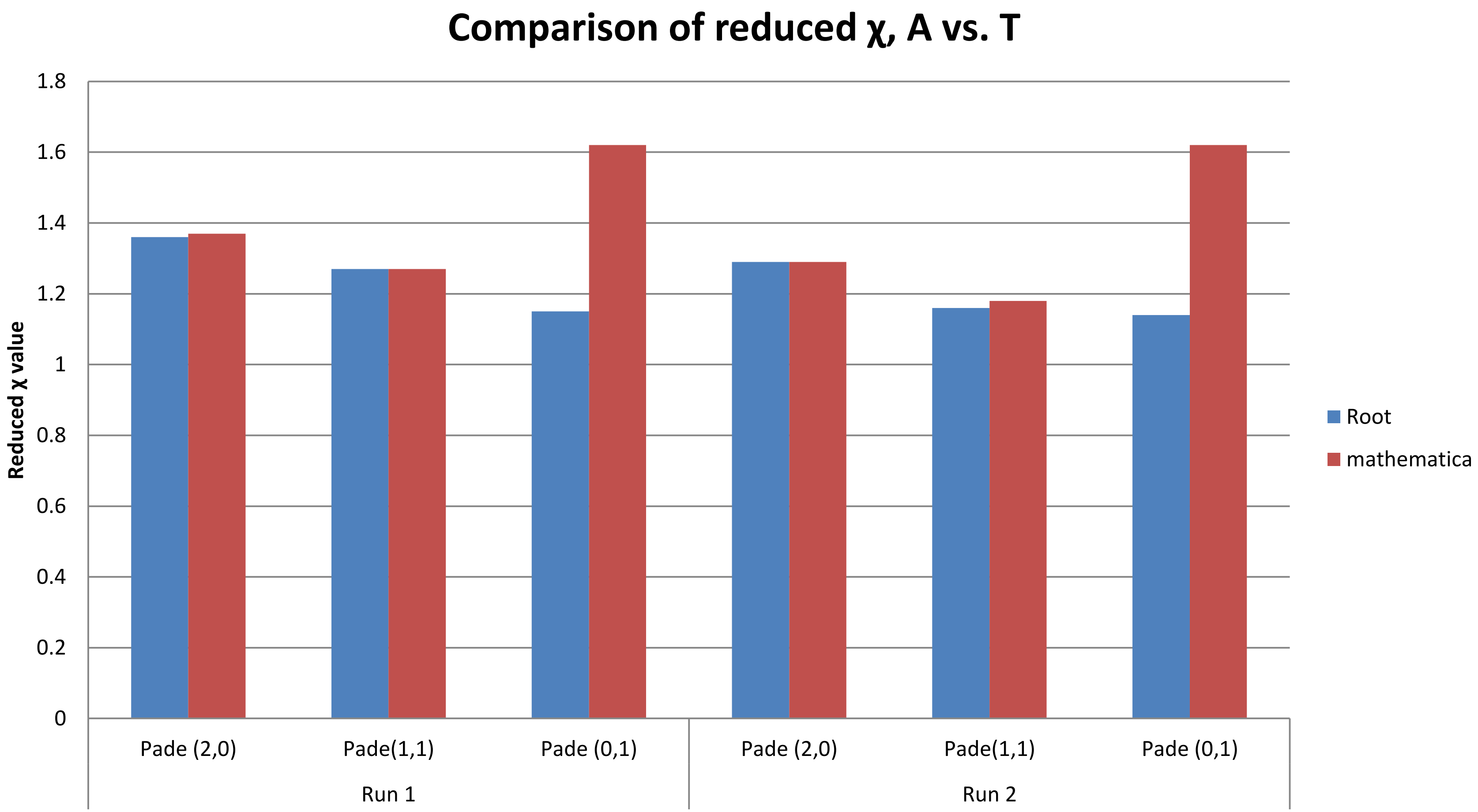
	Estimate	Standard Error	t- Statistic	P- Value
a0	44.1759	0.151864	290.891	1.49814×10^{-15}
a1	0.00564007	0.00583423	0.966719	0.365888
b1	0.000468957	0.000162807	2.88046	0.0236362

Root vs. Mathematica

A vs. T	ao	Δao	a1	a2	b1	Chi sq	
Pade (2,0)	44.0557	.1090	-13.76	3.4		1.36	Root
Pade(1,1)	44.095	.124	2.8		0.39	1.27	Root
Pade (0,1)	44.05	0.095			0.31	1.15	Root
Pade (2,0)	44.1216	.1222	-13.99	3.74		1.29	Rt run2
Pade(1,1)	44.175	.1393	4.8		0.45	1.16	Rt run2
Pade (0,1)	44.09	0.1006	-11.36		0.3136	1.14	Rt run2

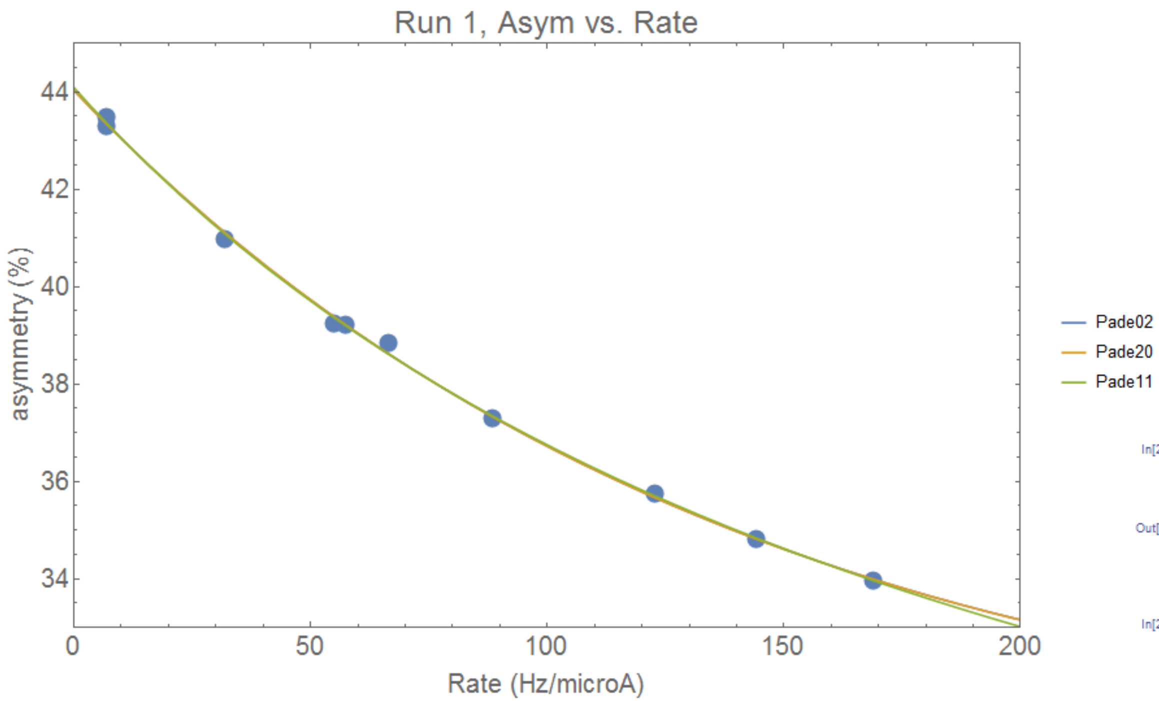
A vs. T	ao	Δao	a1	a2	b1	Chi sq	
Pade (2,0)	44.0492	0.129	-0.0137	3.45e-6		1.37	MM run 1
Pade(1,1)	44.0928	0.139	0.00336		0.0002747	1.27	MM run 1
Pade (0,1)	44.0395	0.1012			0.0003122	1.62	MM run 1
Pade (2,0)	44.1172	0.1413	-0.014	3.85e-6		1.29	MM run 2
Pade(1,1)	44.1759	0.152	0.00564		0.000469	1.18	MM run 2
Pade (0,1)	44.0809	0.1131			0.000312	1.62	MM run 2


Extrapolated values Root vs. Mathematica, A vs. T
44.4
44.3
44.2
44.1
44
43.9
43.8
43.7
Ao (%)
Root
Mathematica
Pade (2,0)
Pade(1,1)
Pade (0,1)
Pade (2,0)
Pade(1,1)
Pade (0,1)
Run 1
Run 2



Comparison of reduced χ, A vs. T
Reduced χ value
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Pade (2,0)
Pade(1,1)
Pade (0,1)
Pade (2,0)
Pade(1,1)
Pade (0,1)
Run 1
Run 2
Root
mathematica

Asym vs. Rate run 1



Pade02	$\frac{44.0279}{1+0.00233996\,x-3.50579\times10^{-6}\,x^2}$	Reduced $\chi 2$	1.62572
Pade20	$\frac{44.0278}{1+0.00233989\,x-3.50544\times10^{-6}\,x^2}$	Reduced $\chi 2$	1.62749
Pade11	$\frac{44.0829+0.103973\,x}{1+0.00482485\,x}$	Reduced $\chi 2$	1.38485

In[204]:= PlotPade02[{"ParameterTable"}]

Out[204]=

	Estimate	Standard Error	t- Statistic	P- Value
a0	44.0279	0.105428	417.611	1.19212×10^{-16}
b1	0.00233996	0.000082182	28.4729	1.69487×10^{-8}
b2	-3.50579×10^{-6}	4.74043×10^{-7}	−7.39551	0.000149995

In[205]:= PlotPade20[{"ParameterTable"}]

Out[205]=

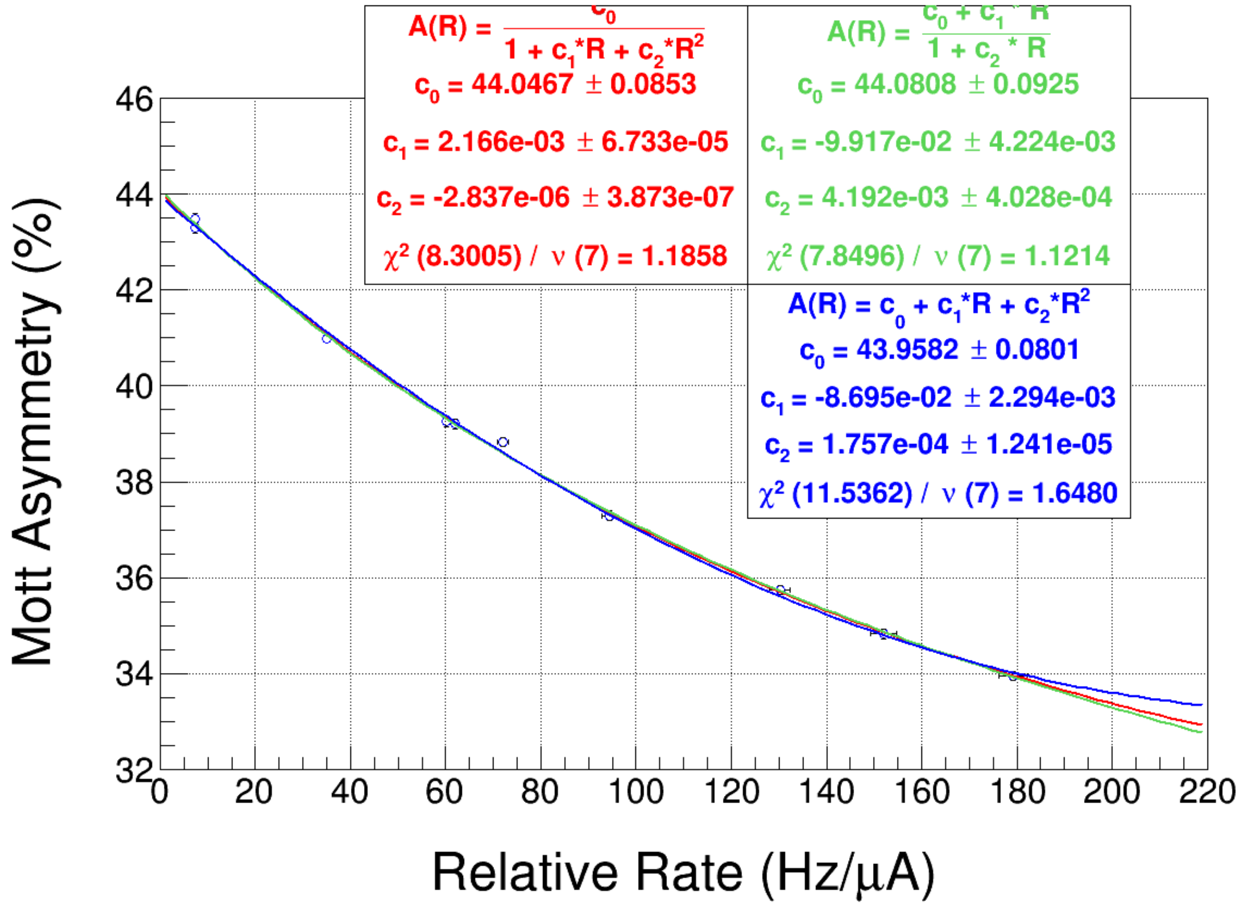
	Estimate	Standard Error	t- Statistic	P- Value
a0	44.0278	0.105471	417.439	1.19556×10^{-16}
a1	0.00233989	0.0000821909	28.469	1.69649×10^{-8}
a2	-3.50544×10^{-6}	4.74022×10^{-7}	−7.3951	0.000150047

In[206]:= PlotPade11[{"ParameterTable"}]

Out[206]=

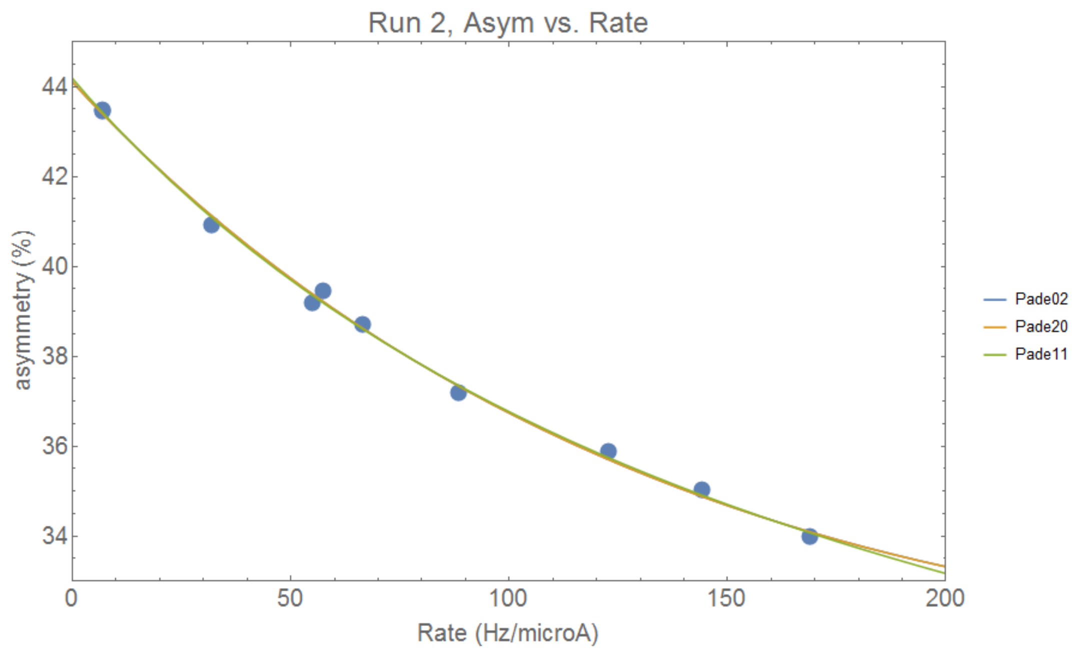
	Estimate	Standard Error	t- Statistic	P- Value
a0	44.0829	0.105288	418.691	1.17076×10^{-16}
a1	0.103973	0.0149856	6.9382	0.00022354
b1	0.00482485	0.000437076	11.0389	0.0000111116

A vs. R run 1



A(R) = c_0 / (1 + c_1*R + c_2*R^2)
c_0 = 44.0467 ± 0.0853
c_1 = 2.166e-03 ± 6.733e-05
c_2 = -2.837e-06 ± 3.873e-07
χ² (8.3005) / ν (7) = 1.1858
A(R) = (c_0 + c_1 * R) / (1 + c_2 * R)
c_0 = 44.0808 ± 0.0925
c_1 = -9.917e-02 ± 4.224e-03
c_2 = 4.192e-03 ± 4.028e-04
χ² (7.8496) / ν (7) = 1.1214
A(R) = c_0 + c_1*R + c_2*R^2
c_0 = 43.9582 ± 0.0801
c_1 = -8.695e-02 ± 2.294e-03
c_2 = 1.757e-04 ± 1.241e-05
χ² (11.5362) / ν (7) = 1.6480
Mott Asymmetry (%)
46
44
42
40
38
36
34
32
0
20
40
60
80
100
120
140
160
180
200
220
Relative Rate (Hz/μA)

Asym. vs rate run 2



Pade02	$\frac{44.0948}{1+0.00238883\,x-3.86058\times10^{-6}\,x^2}$	Reduced $\chi 2$	2.95104
Pade20	$\frac{44.0947}{1+0.00238879\,x-3.86036\times10^{-6}\,x^2}$	Reduced $\chi 2$	2.95485
Pade11	$\frac{44.1744+0.120627\,x}{1+0.00529568\,x}$	Reduced $\chi 2$	2.54385

```
PlotPade02[{"ParameterTable"}]
```

	Estimate	Standard Error	t-Statistic	P-Value
a0	44.0948	0.167453	263.327	3.00729×10^{-15}
b1	0.00238883	0.000121298	19.6938	2.1742×10^{-7}
b2	-3.86058×10^{-6}	6.70129×10^{-7}	-5.76095	0.000690605

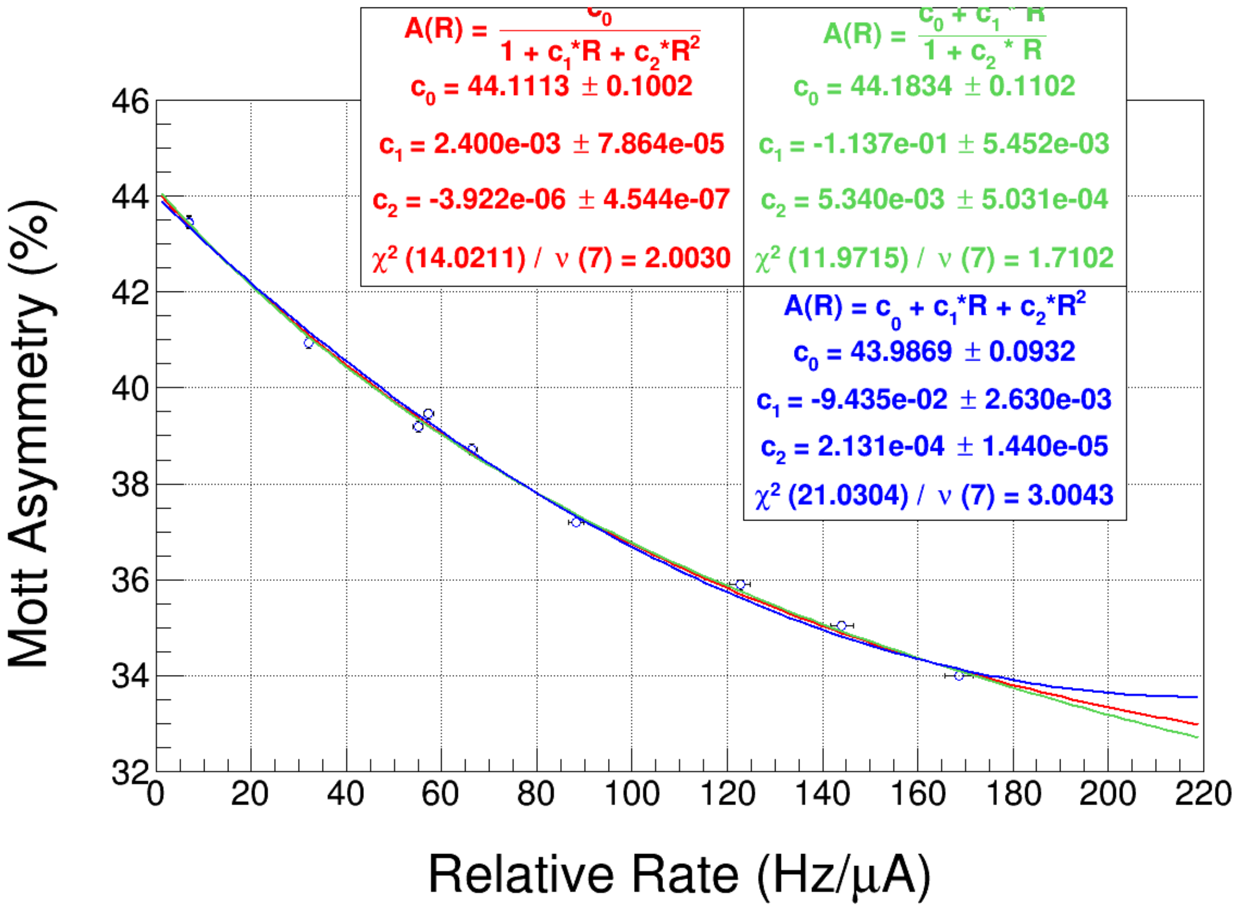
```
PlotPade20[{"ParameterTable"}]
```

	Estimate	Standard Error	t-Statistic	P-Value
a0	44.0947	0.167542	263.186	3.01854×10^{-15}
a1	0.00238879	0.000121327	19.6889	2.17792×10^{-7}
a2	-3.86036×10^{-6}	6.70174×10^{-7}	-5.76023	0.000691115

```
PlotPade11[{"ParameterTable"}]
```

	Estimate	Standard Error	t-Statistic	P-Value
a0	44.1744	0.169683	260.335	3.25766×10^{-15}
a1	0.120627	0.0227844	5.29428	0.00113037
b1	0.00529568	0.000666788	7.94208	0.0000954748

A vs. R run 2

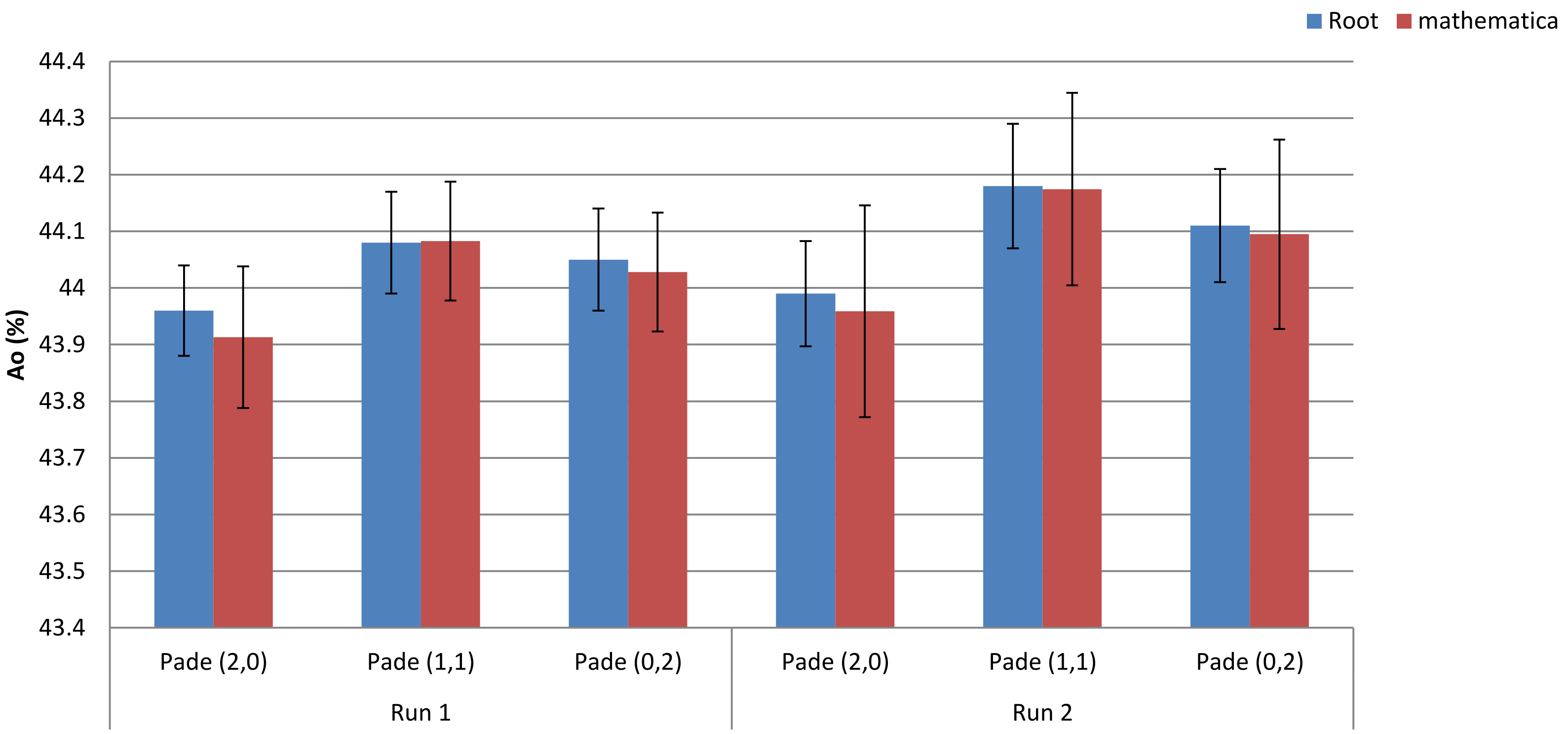


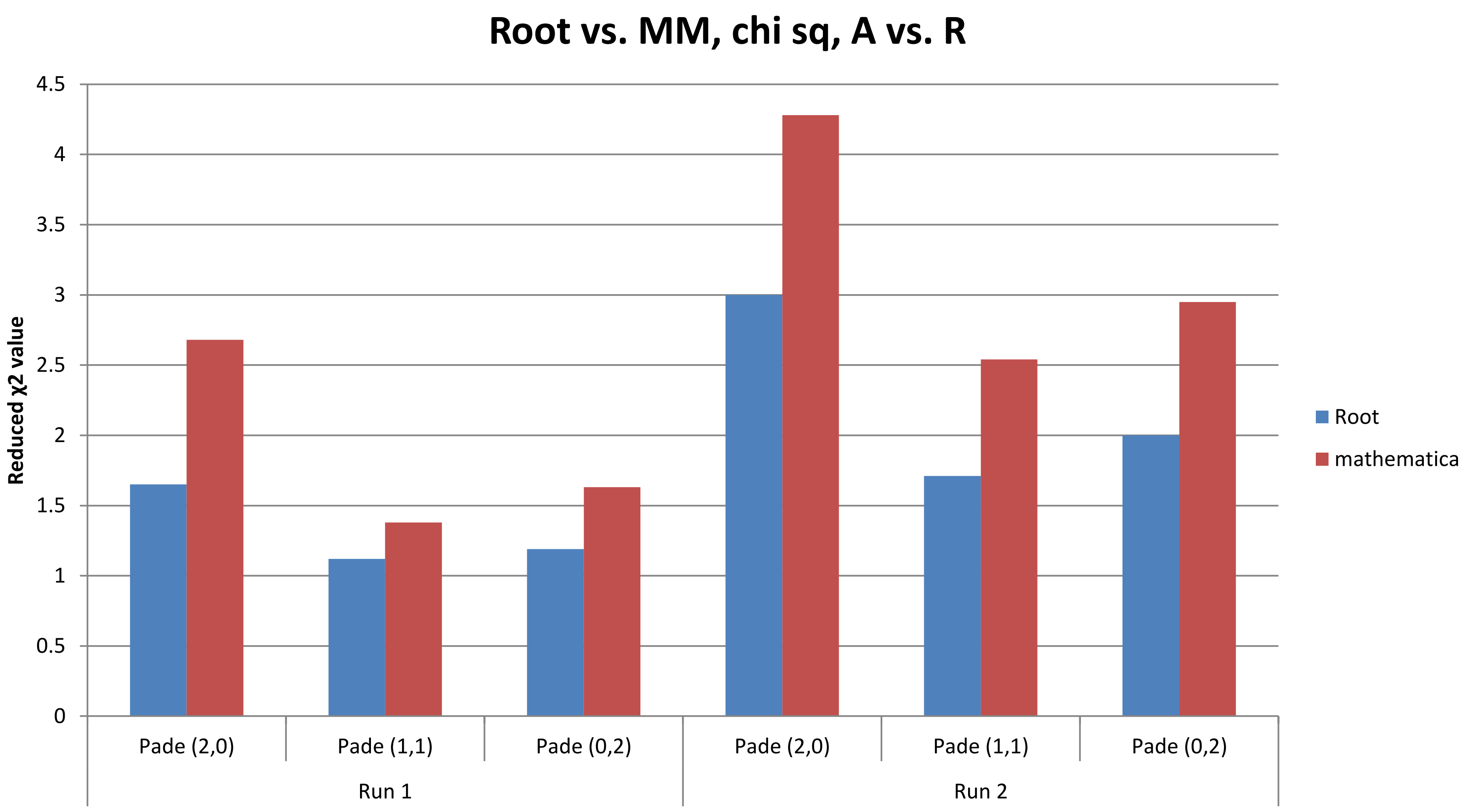
A(R) = c0 / (1 + c1*R + c2*R2)
c0 = 44.1113 ± 0.1002
c1 = 2.400e-03 ± 7.864e-05
c2 = -3.922e-06 ± 4.544e-07
χ2 (14.0211) / ν (7) = 2.0030
A(R) = (c0 + c1 * R) / (1 + c2 * R)
c0 = 44.1834 ± 0.1102
c1 = -1.137e-01 ± 5.452e-03
c2 = 5.340e-03 ± 5.031e-04
χ2 (11.9715) / ν (7) = 1.7102
A(R) = c0 + c1*R + c2*R2
c0 = 43.9869 ± 0.0932
c1 = -9.435e-02 ± 2.630e-03
c2 = 2.131e-04 ± 1.440e-05
χ2 (21.0304) / ν (7) = 3.0043
Mott Asymmetry (%)
Relative Rate (Hz/μA)
32
34
36
38
40
42
44
46
0
20
40
60
80
100
120
140
160
180
200
220

A vs. R	Ao	dAo	Chi squared	Run
Pade (2,0)	43.96	0.08	1.65	Root, 1
Pade (1,1)	44.08	0.09	1.12	Root, 1
Pade (0,2)	44.05	0.09	1.19	Root, 1
Pade (2,0)	43.99	0.093	3.00	Root, 2
Pade (1,1)	44.18	0.11	1.71	Root, 2
Pade (0,2)	44.11	0.100	2.00	Root, 2

A vs. R	Ao	dAo	Chi squared	Run
Pade (2,0)	43.913	0.125	2.68	MM, 1
Pade (1,1)	44.0829	0.105	1.38	MM, 1
Pade (0,2)	44.0279	0.105	1.63	MM, 1
Pade (2,0)	43.959	0.187	4.28	MM, 2
Pade (1,1)	44.1744	0.170	2.54	MM, 2
Pade (0,2)	44.0948	0.167	2.95	MM, 2

Root vs. MM, A vs. R




Root vs. MM, chi sq, A vs. R
Reduced χ2 value
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Pade (2,0)
Pade (1,1)
Pade (0,2)
Pade (2,0)
Pade (1,1)
Pade (0,2)
Run 1
Run 2
Root
mathematica

Summary

- Extrapolated Ao in agreement with two methods (mathematica and root analysis of same data set)
- Some differences in the Chi squared values may come from slightly different methods for calculation, x-error bar handling
- Pick one, pick thickness units, proceed.

Math in mathematica

- Get y(x+dx), y(x-dx) for fitting function, add this dy due to x error bar in quadrature
- Use weighting function: w=(1/dy)^2 at each pt
- Use the mathematica “NonlinearModelFit” function – least squares, using weighted data points, standard error on Ao term
- Define χ2 (sum over all data pts)

  SUM{[(data pt – fit(xi))/Δyi]^2}
- Reduced χ2 :

  χ2 /(# data points - fit parameters)