Drift emittance vs beamline focusing solenoids

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Outline

• Case1: Simulations with non-magnetized beam

Magnetized Beam simulations:

- Case2: 0.436 mm 300 A –increased the strength of the 1st focusing solenoid
- Case3: 0.23 mm 100 A– increased $B_{\scriptscriptstyle Z}$ at the cathode by 1%
- Case4: Increased the strength of a far away solenoid
- Case5: Simple case using GPT rectcoil
- Case6: Moved the rectcoil peak 20 cm away from the cathode
- Case7: Used a new 1 mm step size GTS gun solenoid map extend up to 2.5 m
- Case8: Used a new 1 mm step size GTS gun solenoid extend up to 5 m

CASE 1 For non-magnetized beam

First two focusing solenoids are on.

Simulation parameters

- Gun HV -300 kV (3D E-field map from CST)
- Charge 1 pC
- Pulse width 50 ps (FWHM)
- Accuracy 6.5
- Space charge off
- Gun solenoid off (2D field map)
- Beam size at the cathode 0.23 mm
- First two focusing solenoids are on (1D field maps)
- Correctors are on
- Beam pipe included
- No of particles 2000

Calculated thermal emittance = 8.81E-08 m rad = 0.09 mm mrad

For non-magnetized beam



For non-magnetized beam

Beam position

First two focusing solenoids are on, but higher strengths than before.



Both cases (different focusing solenoid strengths) the x and y emittances are the same.

Original gun field map







Symmetric gun field map (I removed the inverted insulator geometry, triple point junction shield, and NEG pumps)









For non-magnetized beam

First two focusing solenoids are on, but higher strengths than before.

Using symmetric E-field map



> When we have symmetric electric field inside the gun we can have the same x and y emittance from 0-0.25 m.

Simulation parameters

- Gun HV -200 kV (3D E-field map from CST)
- Charge 1 pC
- Pulse width 50 ps (FWHM)
- Accuracy 6.5
- Space charge off
- Gun solenoid on (2D field map)
- Beam size at the cathode 0.436 mm
- First two focusing solenoids are on (1D field maps)
- Correctors are on
- Beam pipe included
- No of particles 2000

Calculated thermal emittance = 1.67E-07 m rad = 0.17 mm mrad

For magnetized beam -300 A – first two focusing solenoids are on.





Emittance_x [mm mrad]	Emittance_y [mm mrad]	Emittance_z [eVs]	Uncorrelated emittance [mm mrad]	Correlated emittance [mm mrad]
3.95E+00	3.94E+00	4.83E-11	6.91E-01	3.86E+00

For magnetized beam -300 A – first two focusing solenoids are on.



For magnetized beam -300 A – first two focusing solenoids are on.



For magnetized beam -300 A – first two focusing solenoids are on but different strengths (higher strengths).



Emittance_x [mm mrad]	Emittance_y [mm mrad]	Emittance_z [evs]	Uncorrelated emittance [mm mrad]	Correlated emittance [mm mrad]
5.39E+00	5.37E+00	4.60E-11	1.98E-01	5.43E+00

The x and y emittances increased from ~3.9 mm mrad to 5.4 mm mrad just by increasing the focusing solenoid strengths. (All the other parameters were the same.)





For magnetized beam -300 A – first two focusing solenoids are on but different strengths (higher strengths).



Simulation parameters

- Gun HV -300 kV (3D E-field map from CST)
- Charge 1 pC
- Pulse width 50 ps (FWHM)
- Accuracy 6.5
- Space charge off
- Gun solenoid on (2D field map)
- Beam size at the cathode 0.23 mm
- First two focusing solenoids are on (1D field maps)
- Correctors are on
- Beam pipe included
- No of particles 2000

Calculated thermal emittance = 8.81E-08 m rad

CASE 3 For magnetized beam -100 A – first two focusing solenoids are on



For magnetized beam -100 A – first two focusing solenoids are on



For magnetized beam -100 A – first two focusing solenoids are on





For magnetized beam -100 A – first two focusing solenoids are on but no change in strengths - increased Bz at the cathode by 1%



 \blacktriangleright Applied 1% error on the Bz without changing the focusing solenoid strengths. The error in emittance become 4%.

For magnetized beam -100 A – first two focusing solenoids are on but no change in strengths - increased Bz at the cathode by 1%



For magnetized beam -100 A – first two focusing solenoids are on but no change in strengths - increased Bz at the cathode by 1%

100 A - 0.23 mm 1% 100 A - 0.23 mm 1% 2 2 stdx [cm] avgx [cm] 0 0 -1 -1 -2 -2 -3 -3 2.5 3.5 0.0 0.5 1.0 1.5 2.0 3.0 4.0 0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 avgz [m] avgz [m] E**∳**∲ E**∮**∲∲ GPT ΞΦΦ ΞΦΦΦ GPT 100 A - 0.23 mm 1% 100 A - 0.23 mm 1% 2 2 1 stdy [cm] avgy [cm] 0 0 -1 -1 -2 -2 -3 -3 0.5 1.0 2.5 3.0 3.5 4.0 0.0 1.5 2.0 1.5 2.0 2.5 3.0 3.5 4.0 0.0 0.5 1.0 avgz [m] ΞΦΦ ΞΦΦΦ GPT avgz [m] E**●** ● E ● ● ● GPT

Beam positions

Beam size

<u>Summary</u>

- Drift emittance is very sensitive to focusing solenoid strengths.
- By increasing the focusing solenoid strength can increase the emittance and decreasing the strength can reduce the emittance.
- Looks like we can make any value for emittance by changing the focusing solenoid values.
- Tried to see whether there is an emittance exchange between the projections of the full 6D emittance but looks like its not.
- But for non-magnetized beam, the normalized emittance doesn't change with focusing solenoid strengths.

The magnetic field map of the cathode solenoid and first three focusing solenoids fields

Electric field map for -300 kV



From GPT user manual

To calculate the normalized RMS-emittances, first x_c , y_c , x'_c and y'_c are calculated by GDFA using: $x_c = x - \overline{x}$ $y_c = y - \overline{y}$ $x'_c = \beta_x - \overline{\beta}_x$ $y'_c = \beta_y - \overline{\beta}_y$

The quantities x' and y' are directly multiplied by γ to provide correct normalization for beams with high energy spread and high divergence angles. All parameters are centered around zero.

The RMS emittance for the x-x' and y-y' phase spaces are defined by:

 $nemixrms = \overline{\gamma} \sqrt{\overline{x_c^2} \cdot \overline{x_c'^2} - \overline{x_c x'_c}^2}$ $nemiyrms = \overline{\gamma} \sqrt{\overline{y_c^2} \cdot \overline{y_c'^2} - \overline{y_c y'_c}^2}$

When a cylindrical symmetric beam rotates around the z-axis, such as in a solenoidial field, strong x-y' and x'-y correlations result in very large values for the rms x- and y-emittances. This does not signal phase-space dilution or loss of beam quality in any way. It is just an artifact of the numerical method used to calculate the emittance. The emittance decreases again when the solenoidal field vanishes. This effect can be reduced by calculating the *r*-emittance, using the hypervolume in x-x'-y-y' subspace, to calculate the emittance using:

 $\texttt{nemirrms} = \overline{\gamma} \sqrt{\varepsilon_{x, rms} \cdot \varepsilon_{y, rms} - \left| \overline{x_c y_c} \cdot \overline{x'_c y'_c} - \overline{x_c y'_c} \cdot \overline{x'_c y_c} \right|}$

The units of **nemixrms**, **nemiyrms** and **nemirrms** are emittances in [m-rad]. However, the values are typically multiplied by 1,000,000 and quoted as [π mm-mrad]. Also please note that the results will be a factor of 4 less than the results produced by the now-obsolete **nemix** and **nemiy** programs.

The longitudinal emittance is calculated by:

$$\begin{split} t_c &= t - \bar{t} \\ \gamma_c &= \gamma - \bar{\gamma} \\ \texttt{nemizrms} &= \frac{mc^2}{|q_s|} \sqrt{\overline{t_c^2} \cdot \overline{\gamma_c^2} - \overline{t_c \gamma_c}^2} \end{split}$$

The factor mc^2/q_e is used to convert the units to [eV-s].

When time output is used, all particle coordinates have the same time coordinate. In this case, the time coordinate is obtained by extrapolating the centered position using $t_i = -(z_i - \overline{z})/c\beta_{z,i}$.

CASE 4 Changing the focusing solenoid far away from the gun solenoid field scaled the focusing solenoid by 0.1



Changing the focusing solenoid far away from the gun solenoid field scaled the focusing solenoid by 0.7



0.7

CASE 5 Simple case using GPT rectcoil

Simulation parameters

- Gun HV -300 kV (3D E-field map from CST) symmetric field
- Charge 1 pC
- Pulse width 50 ps (FWHM)
- Accuracy 6.5
- Space charge off
- Gun solenoid on (2D field map)
- Beam size at the cathode ~0.3 mm rms with 5 sigmas
- Beam pipe included
- No of particles 2000
- No correctors



GPT rectcoil: 2D B field



First two solenoids are at 0.7 m and 2.5 m.

Normalized emittance : $\varepsilon_{n,rms} = \beta \gamma \varepsilon$

For Gaussian beam :
$$\varepsilon_d = \frac{eB_z a^2}{2m_e c}$$
 For Flat top beam : $\varepsilon_d = \frac{eB_z a^2}{8m_e c}$

Bz = 0.113 T, a = 0.3 mm Gaussian

Drift emittance : $\varepsilon_d = \frac{1.759 \times 10^{11} \times 0.113 \times (0.3 \times 10^{-3})^2}{2 \times 299792458} = 2.9835 \times 10^{-6} m rad$

Bz = 0.113 T, a = 0.3 mm flat top

Drift emittance : $\varepsilon_d = \frac{1.759 \times 10^{11} \times 0.113 \times (0.3 \times 10^{-3})^2}{8 \times 299792458} = 7.4589 \times 10^{-7} m rad$



GPT gives particle's normalized momentum.

GBx	Particle normalized x-momentum: p_x/mc .
GBy	Particle normalized y-momentum: py/mc.
GBz	Particle normalized z-momentum: pz/mc.

These are the average normalized momentums x, y and z.





Total momentum $P = \sqrt{P_x^2 + P_y^2 + P_z^2}$



Flat top - 1st focusing solenoid off



Gaussian - 1st solenoid scaled by 0.1





Gaussian - 1st solenoid scaled by 0.5





Gaussian - 1st solenoid scaled by 0.7



Gaussian - 1st solenoid scaled by -0.2



Scaling															
factor		time	stdx	stdy	avgx	avgy	avgz	avgG	nemixrms	nemiyrms	nemizrms	nemirrms	cnemixrms	ucnemixrms	avgBz
	0	2.39E-08	3.40E-03	3.39E-03	5.09E-05	-2.34E-05	5.50E+00	1.58E+00	2.87E-06	2.87E-06	4.27E-11	3.05E-07	7 2.85E-06	3.15E-07	7.76E-01
	0.1	2.39E-08	3.07E-03	3.06E-03	4.53E-05	-3.05E-05	5.50E+00	1.58E+00	2.88E-06	2.88E-06	4.27E-11	2.19E-07	7 2.87E-06	2.24E-07	7.76E-01
	0.5	2.39E-08	4.28E-03	4.30E-03	3.56E-05	-1.28E-04	5.50E+00	1.58E+00	2.87E-06	2.87E-06	4.27E-11	1.17E-07	7 2.87E-06	5 1.17E-07	7.76E-01
	0.7	2.39E-08	1.30E-03	1.29E-03	3 -2.72E-05	9.72E-06	5.50E+00	1.58E+00	2.91E-06	2.91E-06	4.27E-11	4.98E-07	7 2.88E-06	5.05E-07	7.76E-01
	-0.2	1.92E-08	1.13E-03	1.12E-03	3 -1.58E-05	-2.51E-05	4.41E+00	1.58E+00	3.04E-06	3.02E-06	4.27E-11	6.62E-07	7 2.93E-06	6.62E-07	7.76E-01

If we have a really short gun solenoid field that doesn't overlap with the focusing solenoid fields we probably wouldn't have this problem. – Not true.

In a perfect case, drift emittance is almost a constant value even though fields get overlapped.

Simulation parameters

- Gun HV -300 kV (3D E-field map from CST) symmetric field
- Charge 1 pC
- Pulse width 50 ps (FWHM)
- Accuracy 6.5
- Space charge off
- Gun solenoid on (2D field map) moved the peak 20 cm. Bz at the cathode is 0.113 T
- Beam size at the cathode ~0.3 mm rms with 5 sigmas
- Beam pipe included
- No of particles 2000
- No correctors



Moved the peak 20 cm. Bz at the cathode is 0.113 T



Gaussian - 1st solenoid off



Gaussian - 1st solenoid scaled by 0.2



E ● ● GPT

Gaussian - 1st solenoid scaled by 0.6



0.6

	time	stdx	stdy	avgx	avgy	avgz	avgG	nemixrms	nemiyrms	nemizrms	nemirrms	cnemixrms	ucnemixrms	avgBz
0	2.39E-08	3.16E-03	3.15E-03	5.98E-05	-2.93E-05	5.50E+00	1.58E+00	2.86E-06	2.86E-06	4.36E-11	1.39E-07	2.85E-06	1.39E-07	7.76E-01
0.2	2.39E-08	3.73E-03	3.72E-03	7.24E-05	-3.63E-05	5.50E+00	1.58E+00	2.81E-06	2.81E-06	4.36E-11	1.50E-07	2.80E-06	1.53E-07	7.76E-01
0.7	2.53E-08	2.96E-03	2.96E-03	2.53E-05	-6.61E-05	5.83E+00	1.58E+00	3.08E-06	3.08E-06	4.36E-11	5.06E-07	3.04E-06	5.04E-07	7.76E-01

Bz = 0.113 T, a = 0.3 mm Gaussian

Drift emittance :
$$\varepsilon_d = \frac{1.759 \times 10^{11} \times 0.113 \times (0.3 \times 10^{-3})^2}{2 \times 299792458} = 2.9835 \times 10^{-6} m rad$$

Almost closer to the calculated value.

CASE 7 Simulations with a new 1 mm step size GTS gun solenoid extend up to 2.5 m

Simulation parameters

- Gun HV -300 kV (3D E-field map from CST) symmetric field
- Charge 1 pC
- Pulse width 50 ps (FWHM)
- Accuracy 6.5
- Space charge off
- Gun solenoid on (2D field map) new 1mm
- Beam size at the cathode ~0.3 mm rms with 5 sigmas
- Beam pipe included
- No of particles 2000
- No correctors





Bz = 0.1135 T, a = 0.3 mm Gaussian







GPT gives particle's normalized momentum.

GBx	Particle normalized <i>x</i> -momentum: p_x/mc .
GBy	Particle normalized <i>y</i> -momentum: p _y /mc.
GBz	Particle normalized z-momentum: p _z /mc.

These are the average normalized momentums x, y and z.





Total momentum
$$P = \sqrt{P_x^2 + P_y^2 + P_z^2}$$







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Gaussian - 1<sup>st</sup> solenoid scaled by 1.5
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Scaling														
factor	time	stdx	stdy	avgx	avgy	avgz	avgG	nemixrms	nemiyrms	nemizrms	nemirrms	cnemixrms	ucnemixrms	avgBz
0.5	5 1.30E-08	5.30E-03	5.32E-03	2.84E-05	-1.12E-04	2.97E+00	1.58E+00	9.92E-07	9.89E-07	4.38E-11	1.28E-07	9.98E-07	1.29E-07	7.76E-01
0.6	5 1.48E-08	5.39E-03	5.41E-03	3.19E-05	-1.13E-04	3.40E+00	1.58E+00	1.32E-06	1.31E-06	4.38E-11	1.25E-07	1.32E-06	1.26E-07	7.76E-01
0.7	7 1.43E-08	4.72E-03	4.74E-03	3.00E-05	-9.78E-05	3.27E+00	1.58E+00	1.61E-06	1.61E-06	4.38E-11	1.23E-07	1.62E-06	1.23E-07	7.76E-01
0.8	3 1.41E-08	4.20E-03	4.21E-03	2.66E-05	-8.63E-05	3.23E+00	1.58E+00	1.88E-06	1.87E-06	4.38E-11	1.21E-07	1.88E-06	1.21E-07	7.76E-01
1	L 1.55E-08	3.68E-03	3.69E-03	1.28E-05	-7.42E-05	3.55E+00	1.58E+00	2.28E-06	2.28E-06	4.38E-11	1.18E-07	2.28E-06	1.18E-07	7.76E-01
1.2	2 1.33E-08	2.82E-03	2.83E-03	1.28E-06	-5.46E-05	3.04E+00	1.58E+00	2.50E-06	2.50E-06	4.38E-11	1.17E-07	2.49E-06	1.17E-07	7.76E-01
1.5	5 1.32E-08	3.01E-03	3.01E-03	-2.28E-05	-5.01E-05	3.01E+00	1.58E+00	2.51E-06	2.50E-06	4.38E-11	1.16E-07	2.48E-06	1.19E-07	7.76E-01

Drift emittance is changed from 0.99 mm mrad to 2.5 mm mrad.

CASE 8 Simulations with a new 1 mm step size GTS gun solenoid extend up to 5 m

Simulation parameters

- Gun HV -300 kV (3D E-field map from CST) symmetric field
- Charge 1 pC
- Pulse width 50 ps (FWHM)
- Accuracy 6.5
- Space charge off
- Gun solenoid on (2D field map) new 1mm up to 5 m
- Beam size at the cathode ~0.3 mm rms with 5 sigmas
- Beam pipe included
- No of particles 2000
- No correctors





Drift emittance :
$$\varepsilon_d = \frac{1.759 \times 10^{11} \times 0.113 \times (0.3 \times 10^{-3})^2}{2 \times 299792458} = 2.9835 \times 10^{-6} m rad$$





Gaussian - 1st solenoid off

These are the average normalized momentum.









Gaussian - 1st solenoid scaled by 0.7







Scaling														
factor	time	stdx	stdy	avgx	avgy	avgz	avgG	nemixrms	nemiyrms	nemizrms	nemirrms	cnemixrms	ucnemixrms	avgBz
	0 4.13E-08	9.36E-03	9.38E-03	3 7.89E-05	-1.74E-04	9.56E+00	1.58E+00) 4.43E-06	4.42E-06	4.38E-11	8.42E-07	4.37E-06	8.40E-07	7.76E-01
0.	3 4.06E-08	9.13E-03	9.16E-03	3 2.95E-05	-1.71E-04	9.38E+00	1.58E+00) 4.14E-06	4.14E-06	4.38E-11	6.35E-07	4.10E-06	6.32E-07	7.76E-01
0.	7 4.31E-08	1.32E-02	1.33E-02	2 -5.03E-05	-2.43E-04	9.98E+00	1.58E+00	0 4.08E-06	4.09E-06	4.38E-11	6.54E-07	4.00E-06	6.55E-07	7.76E-01
0.	9 4.07E-08	1.47E-02	1.47E-02	2 -5.88E-05	-2./7E-04	9.42E+00	1.58E+00) 4.27E-06	4.28E-06	4.38E-11	8.37E-07	4.15E-06	8.37E-07	7.76E-01

With the 5m long magnetic field map drift emittance doesn't change drastically like with 2.5 m field map. But the emittance value is a bit higher.