Split Cylinder Resonant Electron Polarimeter

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Abstract

A recent proposal to measure the proton electric dipole moment (EDM) uses protons circulating in a storage ring with their spins "frozen" parallel or anti-parallel to their velocities. Polarimetry is required both to stabilize the frozen spin operation and to measure the EDM-induced precession. This paper proposes a test of resonant electron (rather than proton) polarimetry using a polarized 0.5 MeV kinetic energy, 500 MHz bunch frequency linac electron beam at Jefferson Laboratory. The resonator is a 5 cm long copper cylinder, sliced longitudinally by a single 1 mm gap that serves as the capacity C of a high frequency LC or "whispering gallery" microwave oscillator; the inductance L is provided by the conducting cylinder serving as a single turn solenoid. As a longitudinally polarized electron bunch passes through the resonator its magnetization excites the fundamental oscillation mode of the resonator. The polarimeter detects and measures the longitudinal component of polarization by a kind of inverse NMR in which the nuclear magnetic moments excite an external cavity, rather than the other way round. Successive bunches are arranged to have alternating forward and backward polarizations. This moves the beam polarization frequency to odd harmonics of 250 MHz, away from the direct beam charge frequency harmonics. This greatly suppresses the "background" response to be beam charge relative to the "foreground" polarization response. The resonator response is read out by transformer coupling to an external cavity resonant that surrounds the split cylinder and is tuned to the same frequency.

Contents

1 Introduction

A proposed experiment to measure the proton electric dipole moment (EDM) uses protons stored in a fully-electrostatic storage ring. The main bending field is produced by applying a voltage between inner and outer chamber walls. The full ring consists of repetitions of these sector bends separated by drifts, quadrupoles, RF cavities and so on. The nominal design orbit consists of circular arcs of radius r_0 joined by straight lines through straight sections.

To reduce systematic errors there will be two counter-circulating polarized beams, as nearly identical as possible. The polarizations of both beams will be "frozen", parallel or anti-parallel to the beam directions. Polarimetry (i.e. measuring the polarization of each circulating bunch) is required to monitor and stabilize this frozen spin operation.

Acting on whatever EDM the protons possess, the (dominant) radial electric field tends to tip the beam polarizations up or down. It is this tipping that is to be measured to obtain the proton EDM. Ability to perform this measurement sets stringent polarimetry requirements.

This paper proposes a test of resonant electron (rather than proton) polarimetry using a polarized 0.5 MeV kinetic energy, 500 MHz bunch frequency linac electron beam at Jefferson Laboratory.

2 Resonant polarimetry

2.1 Apparatus

Consider a single, longitudinally polarized bunch of electrons in a linac beam that passes through the split-cylinder resonator shown in Figure 1. The split cylinder can be regarded as a one turn solenoid.

The bunch polarizations will toggle, bunch-to-bunch, between directly forward and directly backward. This is achieved by having two symmetrically interleaved beams, an A beam and a B beam, each having bunch repetition frequency $f_0 = 0.25\,\mathrm{GHz}$ (4 ns bunch separation). The resonator harmonic number, relative to f_0 is an odd number, tentatively it is $h_c = 11$. Irrespective of polarization, the charged bunch frequency will be $2f_0 = 0.5\,\mathrm{GHz}$. Treated as an LC circuit, the split cylinder inductance is L_c and the gap capacity is C_c. In practice the bunches will be only partially polarized but, for estimating the signal strength and signal to noise ratio we assume the bunches are 100%, longitudinally polarized.

2.2 Resonator parameters

The highly conductive split-cylinder can be treated as a one-turn solenoid. In terms of its current I, its magnetic field B is given by

$$B = \mu_0 \frac{I}{l_c}, \tag{1}$$

and its magnetic energy W_m can be expressed either in terms of B or I;

$$W_m = \frac{1}{2}\frac{B^2}{\mu_0}\pi r_c^2 l_c = \frac{1}{2} L_c I^2. \tag{2}$$

Its self-inductance is therefore

$$L_c = \mu_0 \frac{\pi r_c^2}{l_c}. \tag{3}$$

The gap capacitance (with gap g_c reckoned for vacuum dielectric and fringing neglected) is

$$C_c = \epsilon_0 \frac{w_c l_c}{g_c}. \tag{4}$$

Other resonator parameters, with proposed values, are given in Table 1.

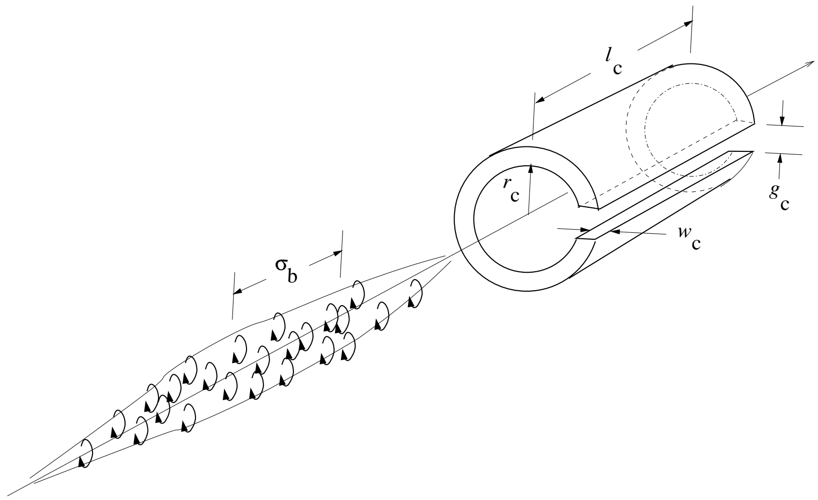

Figure 1: Perspective view of polarized beam bunch passing through the polarimeter. Dimensions are shown for the polarized proton bunch and the split-cylinder copper resonator, and listed in Table 1. For the proposed test using a polarized electron beam at Jefferson Lab, the bunch will actually be somewhat shorter than the cylinder, and have a beer can shape.

3 "Local" Lenz law (LLL) approximation

A "local" Lenz law approximation for calculating the current induced in our split cylinder by a passing polarized beam bunch is introduced by Figure 3. The split cylinder resonator is treated as a one turn solenoid and, for simplicity, the electron bunch is assumed to have a beer can shape, with length l_b and radius r_b. The magnetization $\mathbf{M}$ within length Δz of a beam bunch (due to all electron spins in the bunch pointing, say, forward) is ascribed to azimuthal Ampẽrian current $\Delta I_b = i_b \Delta z$. In other words, in the volume within the beam bunch the magnetic field is also a perfect solenoid (with end fields being neglected).

For sufficiently short cylinder lengths, the bunch transit time will be short compared to the oscillation period of the split cylinder and the presence of the gap in the cylinder produces negligible suppression of the Lenz's law current induced by the passing bunch (because the charge piles up harmlessly on the capacitance of the gap). Define i_{LL} to be the Lenz law current per longitudinal length. Then $\Delta I_{LL} = i_{LL} \Delta z$ is the induced azimuthal current shown in the (inner skin depth) of the cylinder, in the "local region" of the figure. To prevent any net flux from being present locally within the section of length Δz, the flux due to the induced Lens law current must cancel the Ampère flux.

The Lenz law magnetic field is $B_{LL} = \mu_0 i_{LL}$ and the magnet flux through the cylinder is

$$\phi_{LL} = \mu_0 \pi r_c^2 i_{LL}. \tag{5}$$

According to Jackson's[2] section 5.10, the magnetic field $\mathbf{B_b}$ within the polarized beam bunch is equal to $\mu_0 \mathbf{M_b}$ which is the magnetization (magnetic moment per unit volume) due to the polarized electrons.

$$B_b = \mu_0 M_B = \mu_0 \frac{N_e \mu_B}{\pi r_b^2 l_b}, \tag{6}$$

where N_e is the total number of electrons in each bunch. The flux through Δz due to this interval

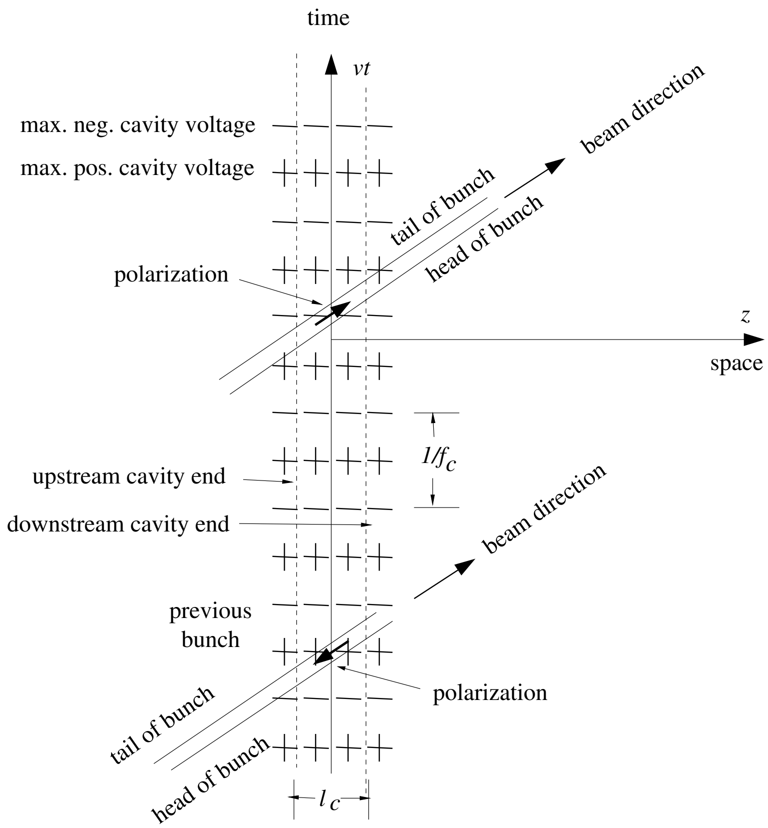

Figure 2: Space-time plot showing entry of front, followed by exit of back of one bunch, followed by the entrance and exit of the following bunch. Bunch separations and cavity length are arranged so that cavity excitations from all four beam magnetization exitations are perfectly constructive, but direct excitation by bunch charge is perfectly destructive.

of the beam bunch is therefore

$$\phi_b = B_b \pi r_b^2 = \mu_0 \, \frac{N_e \mu_B}{l_b}. \tag{7}$$

Since the Lenz law and bunch fluxes have to cancel we obtain

$$i_{LL} = -\frac{N_e \mu_B}{l_b} \, \frac{1}{\pi r_c^2}. \tag{8}$$

For a bunch that is longitudinally uniform (as we are assuming) we can simply take $\Delta z = l_b$ and obtain

$$I_{LL} = i_{LL} l_b = -\frac{N_e \mu_B}{\pi r_c^2} \, \frac{\Delta z}{l_b}. \tag{9}$$

Once the bunch is fully within the cylinder, I_{LL} "saturates", no longer increasing proportional to Δz.

We now make the further assumption (somewhat contradicting the figure, but consistent with the proposed J-LAB test) that the bunch is sufficiently shorter than the cylinder (i.e. $l_b << l_c$) that the linear build up of I_{LL} can be ascribed to the constant applied voltage V_{LL} required to satisfy Faraday's law.

parameter name	parameter symbol	formula	unit	value
length	l_c		m	0.04733
radius	r_c		m	0.01
gap	g_c		m	0.00103943
thickness	w_c		m	0.001
capacity*	C_c	$\epsilon_0 \frac{w_c l_c}{g_c/\epsilon_r}$	pF	0.40317
inductance	L_c	$\mu_0 \frac{\pi r_c^2}{l_c}$	nH	8.3411
resonant freq.	f_c	$1/(2\pi\sqrt{L_c C_c})$	GHz	2.7445
resonator wavelength	λ_c	c/f_c	m	0.10923
copper resistivity	$\rho_{\rm Cu}$		ohm-m	1.68e-8
skin depth	δ_s	$\sqrt{\rho_{\rm Cu}/(\pi f_c \mu_0)}$	μm	1.2452
eff. resist.	R_c	$2\pi r_c \rho_{\rm Cu}/(\delta_s l_c)$	ohm	0.017911
quality factor	Q_c			8030.7
bunch frequency	$f_A = f_B = f_0$		GHz	0.2495
cavity harm. number	h_c	f_c/f_0		11
electron velocity	v_e	$c\sqrt{1-1/2^2}$	m/s	2.5981e8
cavity transit time	Δt	l_c/v_e	ns	0.18230
transit phase advance	$2\pi f_0 \Delta t$	$2\pi f_0 l_c/v_e$		0.28578
transit cycle advance	$\Delta\phi_c$	$f_c \Delta t$		0.50032
electrons per bunch	N_e			4e6
bunch length	l_b		m	0.01
bunch radius	r_b		m	0.005
entry cycle advance		$\Delta\phi_c l_b/l_c$		0.15011

Table 1: Resonator and beam parameters. The capacity has been calculated using the parallel plate formula. The true capacity will probably be somewhat greater, and the the gap g_c will have to be adjusted to tune the natural frequency. When the A and B beam bunches are symmetrically interleaved, the bunch repetition frequency (with polarization ignored) is $2f_0$.

For a CEBAF 160 μA, 0.5 GHz bunch frequency beam the number of electrons per bunch is approximately 2×10^6. Using parameters from Table 1 we obtain the saturation level Lenz law current to be

$$I_{LL}^{\rm sat.} = -\frac{N_e \mu_B}{\pi r_c^2} \quad \Big(\overset{\rm e.g.}{=} -5.9078 \times 10^{-14}\,{\rm A} \Big). \tag{10}$$

The charge that has flowed onto the capacitor during the linear current entrance build up, at the instant the bunch is fully within the cylinder is

$$Q_1^{\rm max.} = \frac{1}{2}\, I_{LL}^{\rm sat.} \frac{l_b}{v_e} \quad \Big(\overset{\rm e.g.}{=} -1.6156 \times 10^{-24}\,{\rm C.} \Big). \tag{11}$$

The meaning of the superscript "max" is that, if there were no further resonator excitations, the charge on the capacitor would oscillate between $-Q_1^{\rm max.}$ and $Q_1^{\rm max.}$.

Except for the back voltage due to charge accumulating on the capacitor, $I_{LL}^{\rm sat.}$ is the constant current that would flow in the inductance while the first bunch remains within the cylinder. But, because the resonator natural frequency is so high, it is not legitimate to neglect the back voltage. As Figure 2 indicates, by the time the bunch exits the cylinder, the capacitor voltage is supposed to be just reversed. The transit time is

$$\Delta t = \frac{l_c}{v_e} \quad \overset{\rm e.g.}{=} \frac{0.04733}{2.598 \times 10^8} = 0.1825\,{\rm ns}, \tag{12}$$

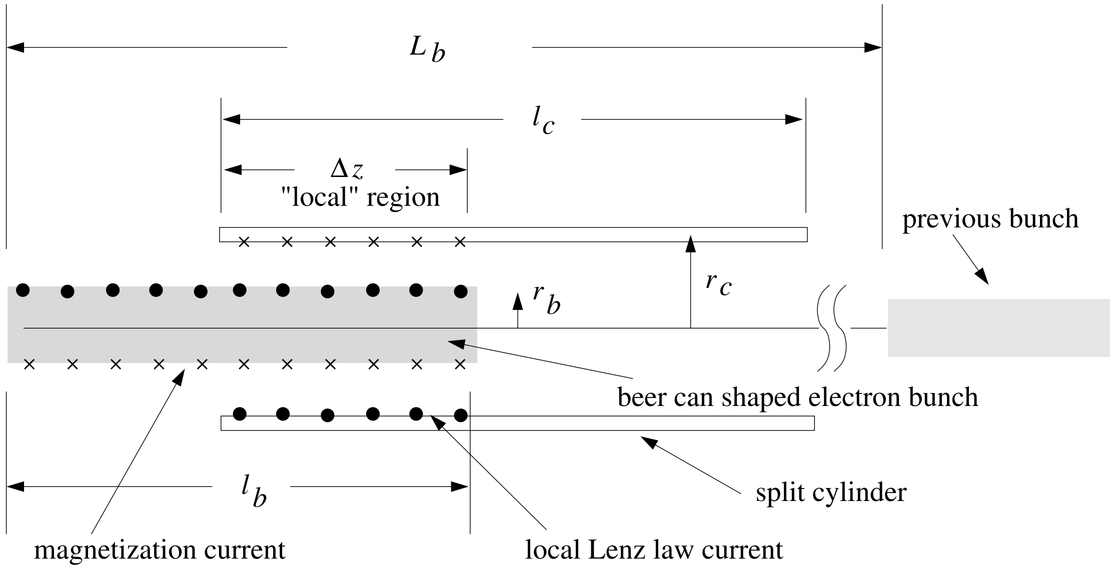

Figure 3: Schematic of beer-can-shaped electron bunch entering the split-cylinder resonator, which is longer than the bunch. Lenz's law is applied to the local overlap region of length Δz. Flux due to the induced Lenz law current is assumed to exactly cancel locally the flux due to the Ampère bunch polarization current.

for which, $f_c \Delta t = 0.5$. As a result the (now negative) Lenz e.m.f. during the exit effectively doubles the amount of charge that, in effect, has been allowed to bypass the inductance, to appear on the capacitor.

In a lumped constant circuit model $Q^{\text{max.}} = 2Q_1^{LL}$ is the excess (maximum) charge on the capacitor due to the passage of the first bunch. Without subsequent bunch passages this maximum charge would decay exponentially with time constant $\tau_c = 2Q/\omega_0$, where Q is the resonator "quality factor".

As Figure 2 also indicates, the parameters have been adjusted so that all bunch entrances and exits contribute constructively to $Q^{\text{max.}}$. On subsequent bunch passages there will already be current flowing due to previous bunch passages. Eventually a steady state will be achieved, in which the resonator energy gained during each bunch passage exactly cancels the ohmic energy lost during the interval between bunch passages.

4 Resonant excitation

When a longitudinally polarized bunch enters the conducting cylinder its magnetization tries to change the flux linking the cylinder. By Lenz's law this change in flux is opposed by azimuthal current flowing in the cylinder. The resulting voltage due to charge on the capacitor opposes and, after many cycles, establishes a steady state in which the induced response each cycle just matches the resistive decay of the resonator.

In any case the Lenz law current is present only while the bunch is passing through the cylinder. It is a quite good approximation to treat the applied voltage as having a square "top hat" shape, with one sign on entry and the opposite sign on exit. For the circuit to respond to beam magnetization, but not to the charge itself, the bunch magnetizations alternate, pulse-to-pulse. This is accomplished by tuning the resonator to an odd harmonic of the bunch frequency divided by 2.

The effect of the pulse-to-pulse toggling of the polarization is the reduce the (current-weighted) polarization frequency from 0.5 GHz to 0.25 GHz. Odd harmonics of 0.25 GHz that are excited by the beam polarization will therefore be isolated in the frequency domain by direct current excitation at harmonics of 0.5 GHz.

In actual practice, as well as having alternating polarization, the bunch charge will also have slightly different charges, which will cause some direct current excitation to leak into odd harmonics. However this spurious signal will also be reduced by the symmetry of the split-cylinder configuration. Even for imperfect alignment and positioning the direct charge excitation will therefore be further reduced.

In a MAPLE program used to calculate the response, the excitation is modeled using "piecewise defined" train of pulses. The bipolar pulses modeling entry to and exit from the resonator are obtained as the difference between two "top hat" pulse trains, one slightly displaced from the other in time. Here is a fragment of this code:

```
TopHatAltWave0p3 := t-> piecewise(
                         0<(t-0.3) and t<  0+(1+0.3),  1,
                        11<(t-0.3) and t< 11+(1+0.3), -1,
                        22<(t-0.3) and t< 22+(1+0.3),  1,
                        33<(t-0.3) and t< 33+(1+0.3), -1,
                          .................
                       572<(t-0.3) and t<572+(1+0.3),  1,
                       583<(t-0.3) and t<583+(1+0.3), -1,
                       594<(t-0.3) and t<594+(1+0.3),  1,
                       605<(t-0.3) and t<605+(1+0.3), -1,
                       616<(t-0.3) and t<616+(1+0.3),  1, 0):

                          .................
TopHatAltWaveDiff := t-> TopHatAltWave0(t) - TopHatAltWave0p3(t):
```

The last line shows the subtraction of a wave displaced by 0.3 time units (the earlier instructions show a few lines) from an identical, but undisplaced train.

In this form the bipolar pulse separations are 1 unit and the bunch-to-bunch separations are 11 units. (The choice of 11 is based on the tentatively adopted harmonic number $h_c = 11$, which is the ratio between resonator frequency and (same polarity) bunch frequency.) Two short sections of the pulse train is shown in Figure 4.

The bunch train terminates after 56 pulses, by which time a steady state has almost been achieved. This enables the complete analysis, including transients, to be performed by Laplace transformation. An alternate approach, that would describe only the steady-state response, would be to represent the bunch train by a Fourier series and to use the complex impedance formalism.

As explained in a later figure caption, in order to reduce the computation time (and avoid saturating the figure data sets) the circuit resistance has been artificially increased by a factor of 10, $r_c \rightarrow 10r_c$. This only affects the figures. The actual excitation is obtained from the analytic formulas described next.

Lumped constant representation of the split-cylinder resonator as a parallel resonant circuit is shown in Figure 5. The resistor symbol is lower case r as mnemonic reminder that we are dealing with a circuit for which inductance L and capacitance C are dominant. The resistor r is taken in series with the inductance under the assumtion that its resistance dominates all other circuit resistances.

The element impedances are given in the figure. The exitation caused by polarized beam passing through the split-cylinder is represented by Lenz law voltage source $\bar{V}_{LL}$, which is the alternating

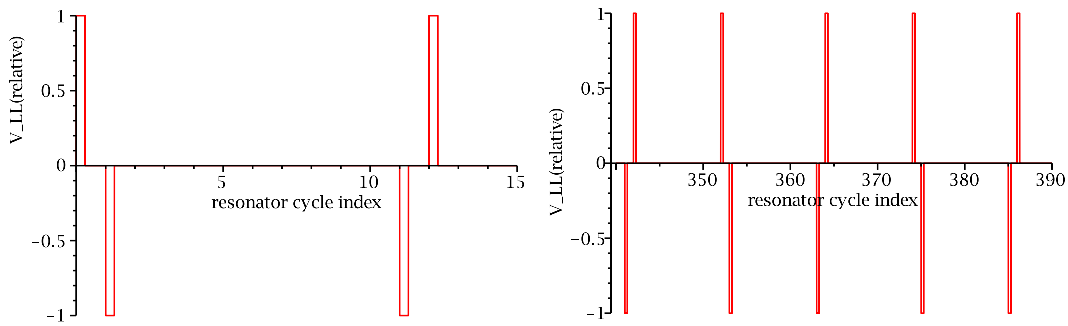

Figure 4: Pulsed excitation voltage pulses caused by successive polarized bunch passages through the resonator. A few initial pulses are shown on the left. The units of the horizontal time scale are such that, during one unit along the horizontal time axis, the natural resonator oscillation phase advances by π. The second pulse starts exactly at 1 in these units, because the resonator length l_c has been arranged so that this time interval is also equal to the bunch transit time through the split-ring. Also, h_c=11 units of horizontal scale advance corresponds to a phase advance of π at the $f_A = f_B = f_0 = 0.2495\,\mathrm{GHz}$ "same-polarization repetition frequency". In other words, 1 unit corresponds almost exactly to $2/11\,\mathrm{ns}$ time duration and is a phase advance of π at the $h_c f_0$ polarization repetition frequency and 2π at the $2h_c f_0$ charge repetition frequency. The interval exhibited on the right is a section of the same pulse train plotted with a different horizontal scale, and runs from 340 units to 440 units.

bunch train already described. Voltage division in this series circuit produces capacitor voltage transform $\bar{V}_C(s)$;

$$\bar{V}_C(s) = \frac{1/(Cs)}{1/(Cs) + r + Ls}\,\bar{V}_{LL}(s) = \frac{\bar{V}_{LL}(s)}{1 + rs + CLs^2}. \tag{13}$$

For excitation voltage $V_{LL}(t)$ as shown in Figure 4, MAPLE has been used to determine the Laplace transform $\bar{V}_{LL}(s)$ for substitution into this equation, to obtain $\bar{V}_C(s)$. MAPLE is then used to invert this transform to obtain the capacitor voltage $V_C(t)$, which is plotted in Figures 7 and 8. A short section of the output, superimposed on the input is plotted in Figure 9. Comparing this figure with the early time relation between resonator amplitude and excitation in Figure 6 shows that the response is very nearly in phase with the excitation.

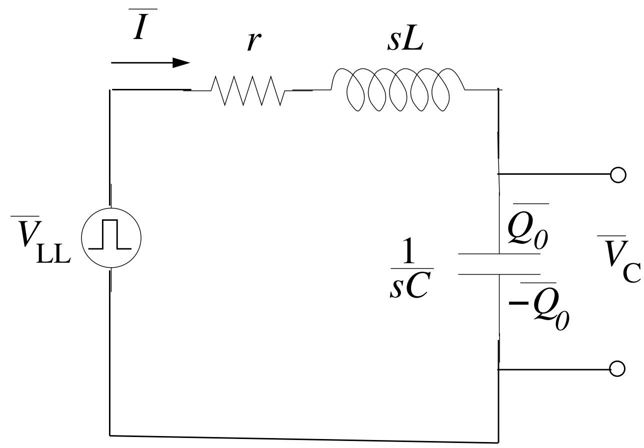

Figure 5: Circuit model for excitation voltage division between capacitance C and inductance L of the resonant LC. The overhead bar on the $\bar{I}$ symbols indicate they represent Laplace-transformed currents.

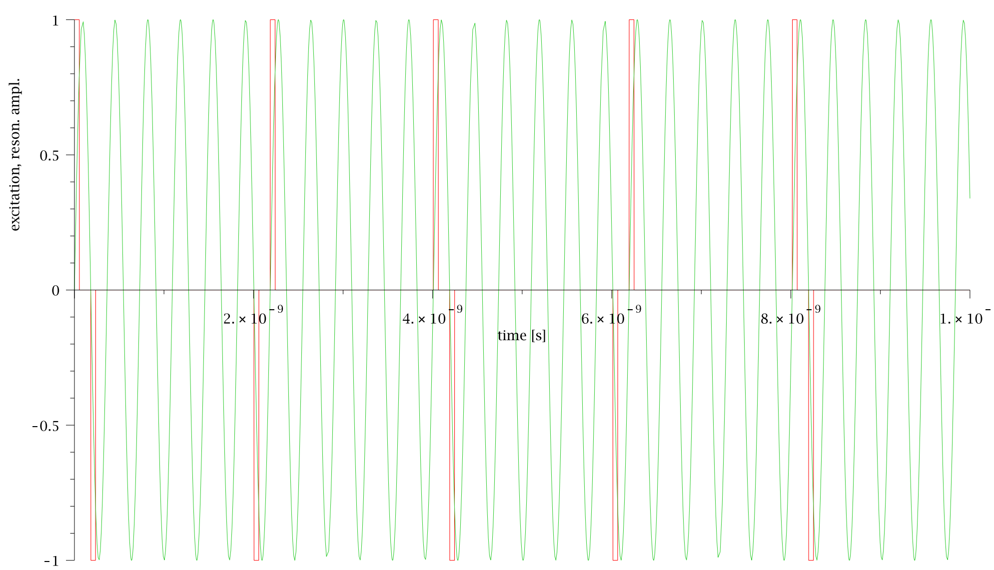

Figure 6: Alternating polarization excitation pulses superimposed on resonator amplitude and plotted against time. Bunch separations are 2 ns, bunch sepraration between same polarization pulses is 4 ns.

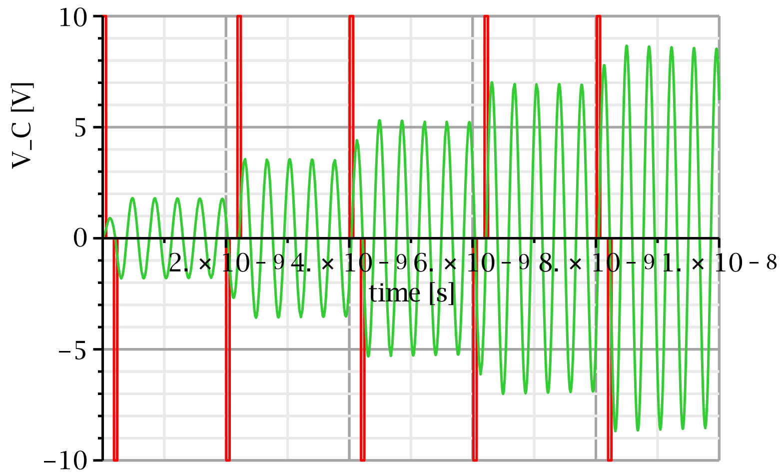

Figure 7: Accumulating capacitor voltage response V_C while the first five linac bunches pass the resonator. The accumulation factor relative to a single passage, is plotted.

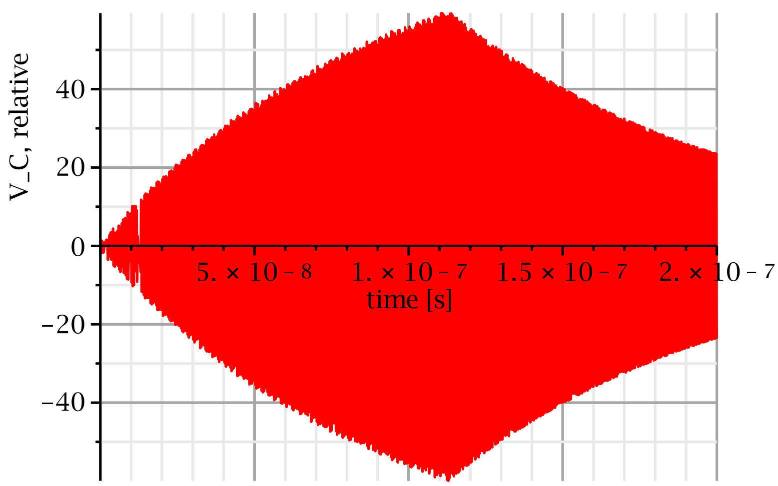

Figure 8: Relative resonator response to a train of beam pulse that terminates after about 110 ns. (The Laplace transform formalism requires the time duration of the excitation to be finite.) After this time the resonator rings down at roughly the same rate as the build-up. With just one exceptions the circuit parameters are those given in Table 1. The exception is that the resistance for the plot is $r = 10r_c$. The true response build up would be greater by a factor of 10, over a factor of 10 greater build-up time.

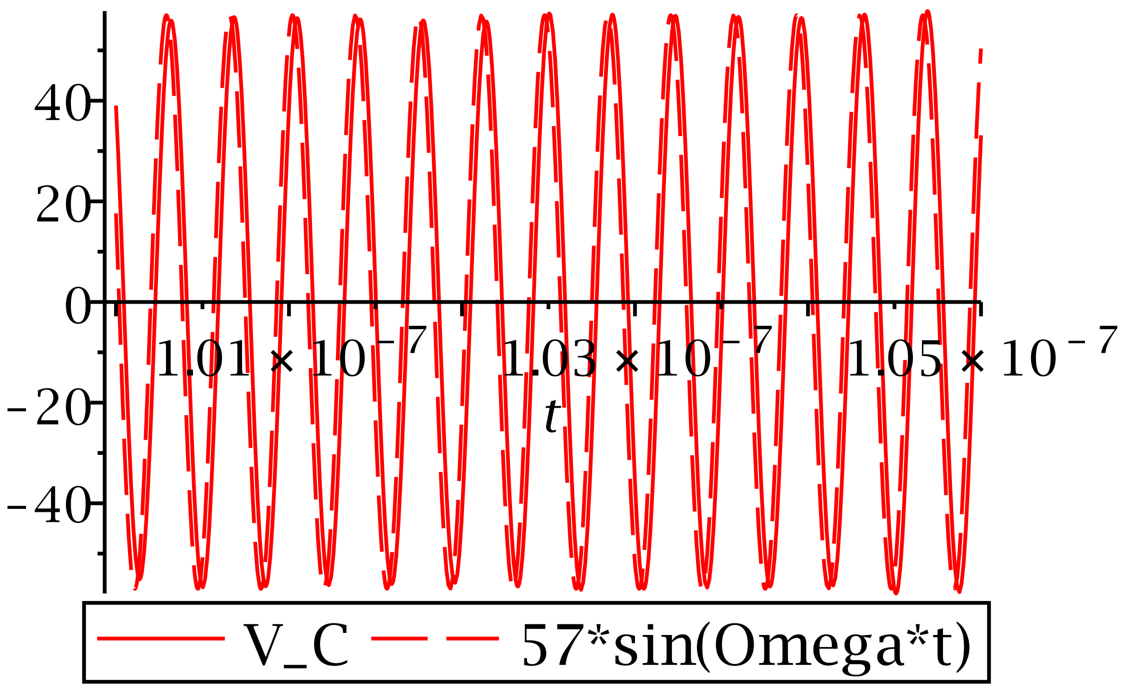

Figure 9: Phases of drive and response after 100 ns. Surprisingly, the response is almost in phase with the excitation. This is presumeably because the entrance and exit excitations are separated in phase by π.

5 Tentative conclusions

For resonator parameters shown in Table 1, the maximum charge $Q_1^{\rm max.}$ residing on the resonator capacitor when the first bunch has just entered the resonator was determined in Eq. (11). Figure 7 shows the capacitor charge building up constructively over the next several excitation pulses. The synchronism has been arranged so that every entrance and exit Lenz law excitation is constructive. Multiplying V_C by 10 in Figure 8 (to correct for the actual circuit resistance r_c having been artificially increased by a factor of 10 to reduce the computation time) the capacitor build-up factor when steady state has been reached is approximately 700. This is less than the resonator Q value of 8031 by a factor more or less equal to the $h_c = 11$ resonator harmonic number. At that time the capacitor charge is $700\,Q_1^{\rm max.} = 1.6156 \times 10^{-24} = 1.1309 \times 10^{-21}\,$C, and the maximum capacitor voltage is

$$V_c^{\rm max} = \frac{(Q_c/h_c)\,Q_1^{\rm max.}}{C_c} = \frac{(8031/11)\cdot(1.1309\times 10^{-21})}{0.4032\times 10^{-12}} = 2.926\times 10^{-9}\,{\rm V}. \tag{14}$$

As an aside, one can comment that, since the electron velocity of $0.866\,c$ is almost fully relativistic, this voltage is very nearly independent of γ. As far as I am concerned this lays to rest a decade old controversy concerning the γ-dependence of cavity excitation by a passing bunch of polarized particles. This paper has shown that, once the particles have become fully relativistic there is no further dependence on γ of the resonator excitation.

References

[1] Storage Ring EDM Collaboration, *A Proposal to Measure the Proton Electric Dipole Moment with* 10^{-29} *e-cm Sensitivity,* October, 2011

[2] J. Jackson, *Classical Electrodynamics,* 3rd edition, John Wiley, 1998