Magic Energies for the 12 GeV Upgrade

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Abstract

Only specific final beam energies at CEBAF allow the beam polarization to be longitudinal in two experimental halls simultaneously. Those which provide simultaneous longitudinal polarization in two experimental halls for final beam energies between 2 and 6 GeV have been previously reported[1]. The upgrade of CEBAF to 12 GeV will augment the combination of such energies; the summary of combinations which span the upper 50% of maximum beam energy (6 to 12 GeV) are reported in this note. The impact of the location of the new Hall D on the results is also discussed.

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1 Introduction

Because a large fraction of the nuclear physics experiments approved at CEBAF require a polarized electron beam it is desirable to distribute the polarized beam to more than one of the experimental halls simultaneously. Because the beam polarization varies as it traverses the accelerator, it is only possible for the beam polarization to be simultaneously longitudinal in two experimental halls at specific final beam energies. These specific beam energies are often referred to as "magic energies". The magic energies between 2 and 6 GeV have been previously reported[1].

The upgrade of CEBAF to 12 GeV will augment the possible combinations. A new complete set of magic energies for the upgrade are reported in this note. The impact of the location of the new Hall D on these calculations is also discussed.

2 Spin Precession Calculations and the The Location of Hall D

Hall D will utilize the energy, not the polarization, of the electron beam. Consequently, the calculations below consider achieving longitudinal beam

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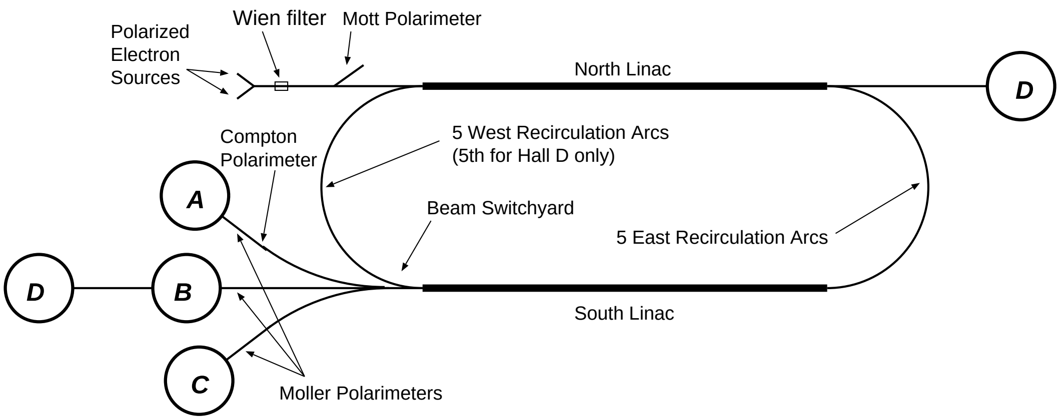

Fig. 1. Author's depiction of CEBAF 12 GeV layout with two locations for Hall D as discussed in this note.

polarization in Halls A, B and C, while using a linac gradient which achieves a final beam energy of 12.1 GeV in Hall D.

The pre-conceptual design report[2] states that the final beam energy will be achieved in 5.5 passes of the accelerator. An alternative approach[3] suggests achieving the final beam energy using the present 5-pass accelerator configuration and placing Hall D directly behind Hall B. These two options are shown in Fig.1.

2.1 Spin Precession to Halls A, B, C

The beam polarization precesses between the injector and either Halls A, B, or C while traversing the accelerator. The precession can be exactly calculated. For n recirculations through the accelerator to Hall h the total precession, Ψ_n^h, can be summed and written as,

$$\Psi_n^h = \left(\frac{g-2}{2m_e}\right)[(n\theta_1 + (n-1)\theta_2)E_0 + \frac{n}{2}((n+1)\theta_1 + (n-1)\theta_2)E_1 \\ + \frac{n(n-1)}{2}(\theta_1+\theta_2)E_2 + (E_0 + n(E_1+E_2)\theta_h)], \quad (1)$$

where E_0, E_1, and E_2 are the energy gains of the injector, north linac, and south linac, θ_1 and θ_2 are the bend angles of the east and west recirculation arcs, and $\theta_h (h \in A, B, C)$ is the bend angle of the respective experimental hall transport arc. The physical constants used are the electron mass ($m_e = 0.51099906\,\mathrm{MeV/c^2}$) and the gyromagnetic factor of the electron (g=2.002319). Note that Eqn. 1 assumes the energy gain on each pass through each linac is the same. In practice, this is assurred by measuring and correcting the path length of each recirculation pass.

Eqn. 1 can be simplified for the purpose of these calculations by assuming that the north and south linacs energy gains are equal (E_1=E_2=E_{linac}), the injector energy is 11.25% of the linac energy gain (E_0=$0.1125 \cdot E_{linac}$), and the recirculation arcs are equal and perfect (θ_1=θ_2=π). Defining $\alpha = 0.1125$, this yields

$$\Psi_n^h = E_{linac}\left(\frac{g-2}{2m_e}\right)\left[2n^2 - n\left(1 - 2\alpha - \frac{2\theta_h}{\pi}\right) - \alpha\left(1 - \frac{\theta_h}{\pi}\right)\right]\pi. \qquad (2)$$

The difference in precession between two halls is then written as

$$\Psi_{n1}^{h1} - \Psi_{n2}^{h2} = E_{linac} \cdot \left(\frac{g-2}{2m_e}\right) \cdot f(h1, n1, h2, n2) \cdot \pi, \qquad (3)$$

where $h1, n1$ and $h2, n2$ refer to the hall and pass number, respectively, for two halls and $f(h1, n1, h2, n2)$ is the grouping of terms that depend upon the choice of hall and pass. When the quantity $E_{linac} \cdot (\frac{g-2}{2m_e}) \cdot f(h1, n1, h2, n2)$ is an integer this assures that the difference in spin precession between the two halls is a multiple of π. For example, considering Hall A at 2-pass and Hall C at 5 pass one would look for solutions where $(\Psi_2^A - \Psi_5^C)/\pi$ is an integer. With this condition met, the Wien filter spin rotator, located in the injector and common to all electron beams, can be used to rotate the beam polarization for both halls in unison so they are both longitudinal at the hall targets. Note, each hall's polarization may be either parallel or anti-parallel to the beam momentum because the stated condition is that $E_{linac} \cdot (\frac{g-2}{2m_e}) \cdot f(h1, n1, h2, n2)$ is a multiple of π, not 2π. With this in hand, the calculations are straightforward.

2.2 Hall D at 5.5 Passes

In this case, the maximum linac energy is 1088.86 MeV. The method for beam extraction is such that no two halls may receive the same final beam energy, so $n1 \neq n2$. Solutions which span 50-100% of the linac energy are considered.

Table 1
Results for Hall D at 5.5-pass, $E_{linac} = 1088.86$ MeV.

Pair of Halls	A & B	B & C	A & C
Combinations	565	543	552
Numerical Values (see)	Table 3	Table 4	Table 5
Graphical Plot (see)	Fig. 2	Fig. 3	Fig. 4

2.3 Hall D at 5.0 Passes

In this case, the maximum linac energy is 1096.54 MeV. The method for beam extraction is such that two halls may receive the same final beam energy only at fifth pass, so when $n1 = n2 = 5$. Solutions which span 50-100% of the linac energy are considered.

Table 2
Results for Hall D at 5-pass, $E_{linac} = 1096.54$ MeV.

Pair of Halls	A & B	B & C	A & C
Combinations	622	603	616
Numerical Values (see)	Table 6	Table 7	Table 8
Graphical Plot (see)	Fig. 5	Fig. 6	Fig. 7

3 Conclusion

In summary, there are a suitably large number of combinations to deliver longitudinal beam polarization to two halls simultaneously. The number of combinations which span the upper 50% of maximum beam energy (6 to 12 GeV) number more than 500 between any two of Halls A, B, or C, and for either scenario for the location of Hall D. Generally, there are about 10% more combinations when 12.1 GeV is obtained in 5 passes (about 610) rather than 5.5 passes (about 550), owing to the fact that the final beam energy is achieved in 10% fewer linac passes. Beam depolarization due to synchrotron radiation processes at higher beam energy in the recirculation arcs is not addressed in this note, however, it may be useful to compare the two scenarios in this regard.

References

[1] C.K. Sinclair, Jefferson Laboratory Tech Note TJNAF-TN-97-021 (1997).

[2] *Pre-Conceptual Design Report (pCDR) for the Science and Experimental Equipment for the 12 GeV Upgrade of CEBAF*, June 11, 2004.

[3] J. Benesch, private communication.

Table 3
Numerical results for Halls A & B, 5.5 pass, $E_{linac} = 1088.86\,\text{MeV}$. Note: each entry shows $f(h1, n1, h2, n2)$ the integer solutions and the number of solutions.

	n_a=1	n_a=2	n_a=3	n_a=4	n_a=5
$n_b = 1$	Condition	6.0818	15.7234	29.3651	47.0068
	Not	8 to 15	20 to 38	37 to 72	59 to 116
	Allowed	8 cases	19 cases	36 cases	58 cases
$n_b = 2$	-4.7849	Condition	10.4984	24.1401	41.7818
	-6 to -11	Not	13 to 25	30 to 59	59 to 116
	6 cases	Allowed	13 cases	30 cases	52 cases
$n_b = 3$	-14.0099	-8.3682	Condition	14.9151	32.5568
	-18 to -34	-11 to -20	Not	19 to 36	41 to 80
	17 cases	10 cases	Allowed	18 cases	40 cases
$n_b = 4$	-27.2349	-21.5932	-11.9516	Condition	19.3318
	-34 to -67	-27 to -53	-15 to -29	Not	24 to 47
	34 cases	27 cases	15 cases	Allowed	24 cases
$n_b = 5$	-44.4599	-38.8182	-29.1766	-15.5349	Condition
	-55 to -109	-48 to -95	-37 to -72	-27 to -38	Not
	55 cases	48 cases	36 cases	19 cases	Allowed

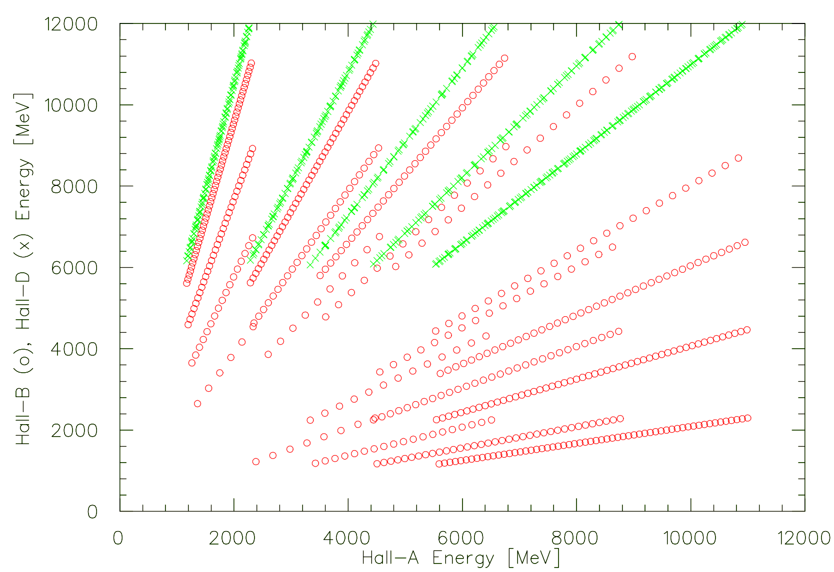

Fig. 2. (Color) Graphical results for Halls A & B, 5.5 pass, $E_{linac} = 1088.86\,\text{MeV}$. Red circles indicate solutions. Green crosses indicate final Hall D beam energy.

Table 4
Numerical results for Halls B & C, 5.5 pass, $E_{linac} = 1088.86$ MeV. Note: each entry shows $f(h1, n1, h2, n2)$ the integer solutions and the number of solutions.

	n_b=1	n_b=2	n_b=3	n_b=4	n_b=5
$n_c = 1$	Condition	5.6651	14.8901	28.1151	45.3401
	Not	7 to 13	19 to 36	35 to 69	57 to 112
	Allowed	7 cases	18 cases	35 cases	56 cases
$n_c = 2$	-4.3682	Condition	10.0818	23.3068	40.5318
	-6 to -10	Not	13 to 24	29 to 57	59 to 110
	5 cases	Allowed	12 cases	29 cases	50 cases
$n_c = 3$	-13.1766	-7.9516	Condition	14.4984	31.7234
	-17 to -32	-10 to -19	Not	18 to 35	40 to 78
	16 cases	10 cases	Allowed	18 cases	39 cases
$n_c = 4$	-25.9849	-20.7599	-11.5349	Condition	18.9151
	-33 to -64	-26 to -51	-15 to -28	Not	24 to 46
	32 cases	26 cases	14 cases	Allowed	23 cases
$n_c = 5$	-42.7932	-37.5682	-28.3432	-15.1182	Condition
	-53 to -105	-47 to -92	-36 to -70	-19 to -37	Not
	53 cases	46 cases	35 cases	19 cases	Allowed

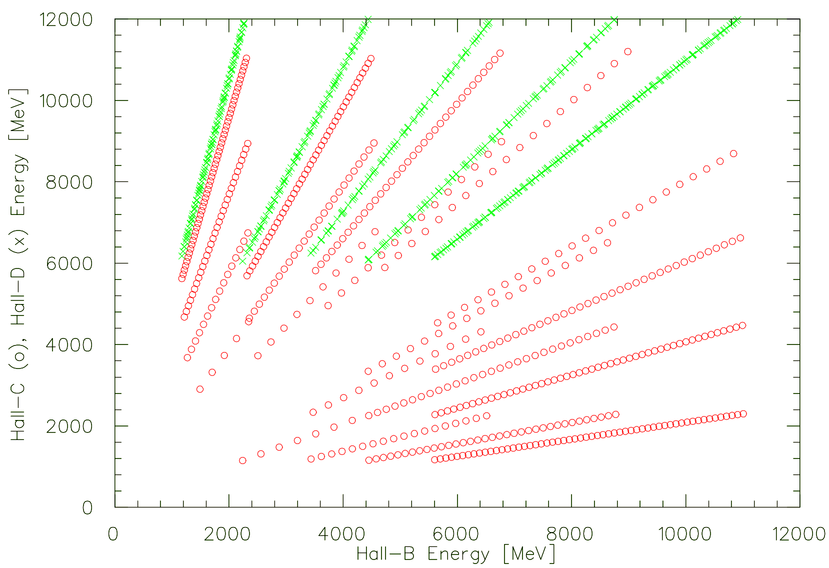

Fig. 3. (Color) Graphical results for Halls B & C, 5.5 pass, $E_{linac} = 1088.86$ MeV. Red circles indicate solutions. Green crosses indicate final Hall D beam energy.

Table 5
Numerical results for Halls A & C, 5.5 pass, $E_{linac} = 1088.86$ MeV. Note: each entry shows $f(h1, n1, h2, n2)$ the integer solutions and the number of solutions.

	n_a=1	n_a=2	n_a=3	n_a=4	n_a=5
$n_c = 1$	Condition	6.5219	16.1635	29.8052	47.4469
	Not	9 to 16	20 to 39	37 to 73	59 to 117
	Allowed	8 cases	20 cases	37 cases	59 cases
$n_c = 2$	-3.9281	Condition	11.3552	24.9969	42.6385
	-5 to -9	Not	15 to 28	31 to 61	53 to 105
	5 cases	Allowed	14 cases	31 cases	59 cases
$n_c = 3$	-12.7365	-7.0948	Condition	16.1885	33.8302
	-16 to -31	-9 to -17	Not	21 to 40	42 to 83
	16 cases	9 cases	Allowed	20 cases	42 cases
$n_c = 4$	-25.5448	-19.9031	-10.2615	Condition	21.0219
	-32 to -63	-25 to -49	-13 to -25	Not	26 to 51
	32 cases	25 cases	13 cases	Allowed	26 cases
$n_c = 5$	-42.3531	-36.7115	-27.0698	-13.4281	Condition
	-53 to -104	-46 to -90	-34 to -66	-17 to -33	Not
	52 cases	45 cases	33 cases	17 cases	Allowed

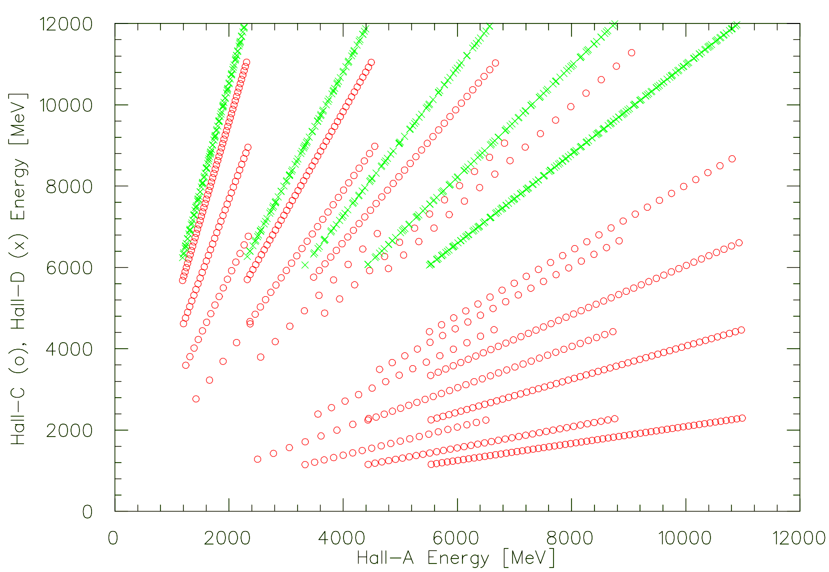

Fig. 4. (Color) Graphical results for Halls A & C 5.5 pass, $E_{linac} = 1088.86$ MeV. Red circles indicate solutions. Green crosses indicate final Hall D beam energy.

Table 6
Numerical results for Halls A & B, 5.0 pass, $E_{linac} = 1096.54$ MeV. Note: each entry shows $f(h1, n1, h2, n2)$ the integer solutions and the number of solutions.

	n_a=1	n_a=2	n_a=3	n_a=4	n_a=5
$n_b = 1$	Condition	6.0818	15.7234	29.3651	47.0068
	Not	9 to 16	22 to 42	40 to 79	64 to 127
	Allowed	8 cases	21 cases	40 cases	64 cases
$n_b = 2$	-4.7849	Condition	10.4984	24.1401	41.7818
	-7 to -12	Not	15 to 28	33 to 65	57 to 113
	6 cases	Allowed	14 cases	33 cases	57 cases
$n_b = 3$	-14.0099	-8.3682	Condition	14.9151	32.5568
	-20 to -38	-12 to -22	Not	21 to 40	45 to 88
	19 cases	11 cases	Allowed	20 cases	44 cases
$n_b = 4$	-27.2349	-21.5932	-11.9516	Condition	19.3318
	-37 to -73	-30 to -58	-17 to -32	Not	27 to 52
	37 cases	29 cases	16 cases	Allowed	26 cases
$n_b = 5$	-44.4599	-38.8182	-29.1766	-15.5349	2.1068
	-61 to -120	-53 to -105	-40 to -79	-22 to -42	3 to 5
	60 cases	53 cases	40 cases	21 cases	3

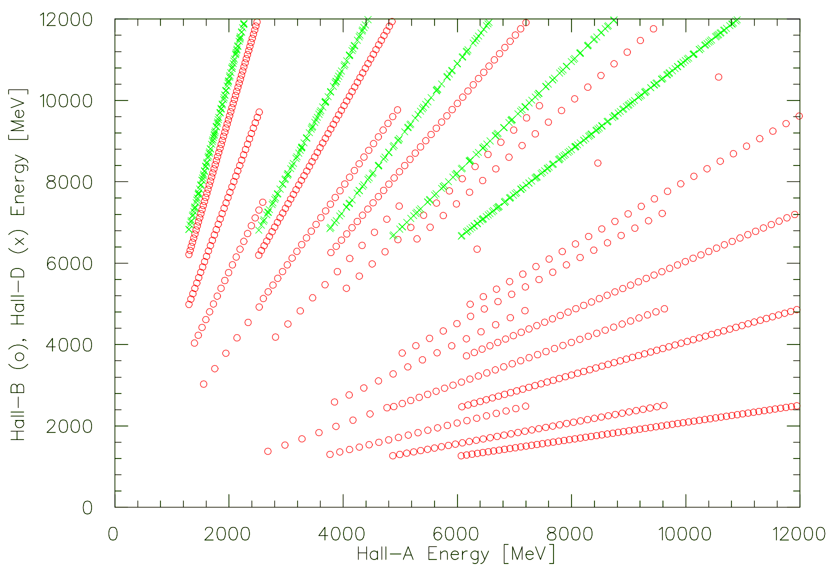

Fig. 5. (Color) Graphical results for Halls A & B, 5.0 pass, $E_{linac} = 1096.54$ MeV. Red circles indicate solutions. Green crosses indicate final Hall D beam energy.

Table 7
Numerical results for Halls B & C, 5.0 pass, $E_{linac} = 1096.54\,\mathrm{MeV}$. Note: each entry shows $f(h1, n1, h2, n2)$ the integer solutions and the number of solutions.

	n_b=1	n_b=2	n_b=3	n_b=4	n_b=5
$n_c = 1$	Condition	5.6651	14.8901	28.1151	45.3401
	Not	8 to 15	21 to 40	39 to 76	62 to 123
	Allowed	8 cases	20 cases	38 cases	62 cases
$n_c = 2$	-4.3682	Condition	10.0818	23.3068	40.5318
	-6 to -11	Not	14 to 27	32 to 63	56 to 110
	6 cases	Allowed	14 cases	32 cases	55 cases
$n_c = 3$	-13.1766	-7.9516	Condition	14.4984	31.7234
	-18 to -35	-11 to -21	Not	20 to 39	44 to 86
	18 cases	11 cases	Allowed	20 cases	43 cases
$n_c = 4$	-25.9849	-20.7599	-11.5349	Condition	18.9151
	-36 to -70	-29 to -56	-16 to -31	Not	26 to 51
	35 cases	28 cases	16 cases	Allowed	26 cases
$n_c = 5$	-42.7932	-37.5682	-28.3432	-15.1182	2.1068
	-59 to -116	-52 to -102	-39 to -76	-21 to -41	3 to 5
	58 cases	51 cases	38 cases	21 cases	3

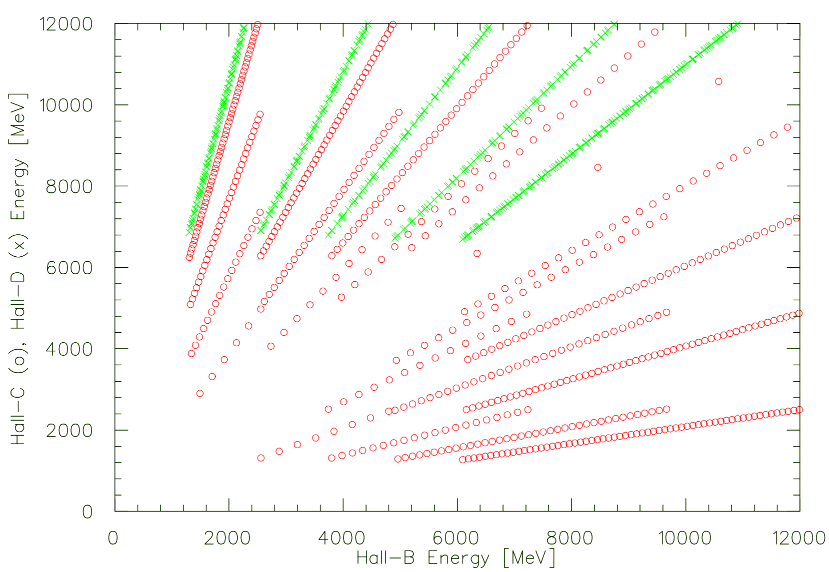

Fig. 6. (Color) Graphical results for Halls B & C, 5.0 pass, $E_{linac} = 1096.54\,\mathrm{MeV}$. Red circles indicate solutions. Green crosses indicate final Hall D beam energy.

Table 8
Numerical results for Halls A & C, 5.0 pass, $E_{linac} = 1096.54$ MeV. Note: each entry shows $f(h1, n1, h2, n2)$ the integer solutions and the number of solutions.

	n_a=1	n_a=2	n_a=3	n_a=4	n_a=5
$n_c = 1$	Condition	6.5219	16.1635	29.8052	47.4469
	Not	9 to 17	22 to 43	41 to 80	65 to 128
	Allowed	9 cases	22 cases	40 cases	64 cases
$n_c = 2$	-3.9281	Condition	11.3552	24.9969	42.6385
	-6 to -10	Not	16 to 30	34 to 67	58 to 115
	5 cases	Allowed	15 cases	34 cases	58 cases
$n_c = 3$	-12.7365	-7.0948	Condition	16.1885	33.8302
	-18 to -34	-10 to -19	Not	22 to 43	46 to 91
	17 cases	10 cases	Allowed	22 cases	46 cases
$n_c = 4$	-25.5448	-19.9031	-10.2615	Condition	21.0219
	-35 to -69	-28 to -54	-14 to -27	Not	29 to 57
	35 cases	27 cases	14 cases	Allowed	29 cases
$n_c = 5$	-42.3531	-36.7115	-27.0698	-13.4281	4.2135
	-58 to -115	-50 to -99	-37 to -73	-19 to -36	6 to 11
	58 cases	50 cases	37 cases	18 cases	6

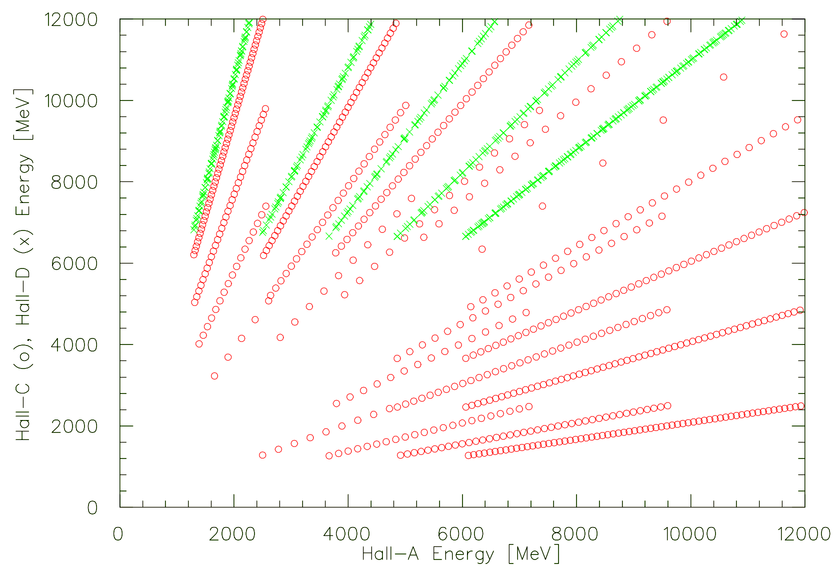

Fig. 7. (Color) Graphical results for Halls A & C, 5.0 pass, $E_{linac} = 1096.54$ MeV. Red circles indicate solutions. Green crosses indicate final Hall D beam energy.