Weekly Meeting Presentation

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August 29, 2018

Meeting Topics

- Updated IPR vs beam energy graph
- Electron-Ion beam dynamics - what I learned from Fast Ion Instability by Alex Chao
- Charge Neutralization
- GPT
- Other?

Ion production rate

- The ion production rate for a electron beam ionizing a certain gas is given by Reiser[4]

$$\frac{dn}{dt} = n_b n_g \sigma_i v$$

where n is the ion density, n_b is the electron beam density, n_g is the neutral gas density, σ_i is the ionization cross section for a given gas species, and v is the velocity of the electrons.

Ionization Cross Section

- The form for the ionization cross section σ_i for gas species i follows from Bethe's theory[1]. The general form used by Reiser is in the form from Slinker, Tayler and Ali's paper[6] shown below:

$$\begin{aligned}\sigma_i &= \frac{8a_0^2\pi I_R A_1}{m_e c^2\beta^2} f(\beta)\left(\ln\frac{2A_2 m_e c^2\beta^2\gamma^2}{I_R} - \beta^2\right)\\ &= \frac{1.872\times10^{-24}A_1}{\beta^2} f(\beta)\left[\ln\left(7.515\times10^4 A_2\beta^2\gamma^2\right) - \beta^2\right]\end{aligned}$$

$$f(\beta) = \frac{I_i}{T_e}\left(\frac{T_e}{I_i} - 1\right) = \frac{2I_i}{m_e c^2\beta^2}\left(\frac{m_e c^2\beta^2}{2I_i} - 1\right)$$

Here, a_0 is the Bohr radius, $I_R = 13.6$ eV is the Rydberg energy, $\beta = \frac{v}{c}$, $\gamma = \left(1-\beta^2\right)^{-\frac{1}{2}}$, m_e and T_e are the electron's mass and kinetic energy respectively, I_i is the ionization energy of gas species i, $f(\beta)$ is a correction function for fitting the velocity data at low energies ($T_e \approx I_i$), and $A_1 = M^2$ & $A_2 = \frac{e^{C/M^2}}{7.515\times10^4}$ are emperical constants that depend on the gas species. These constants are given by Rieke and Prepejchal[5].

Ionization Cross Section vs Beam Energy

- Rewriting σ_i as a function of beam energy T_e, we can plot σ_i for various gas species that are common in the accelerator vacuum:

$$\sigma_i = \frac{1.872 \times 10^{-24} A_1}{1 - \left(\frac{m_e c^2}{m_e c^2 + T_e}\right)^2} \frac{I_i}{T_e} \left(\frac{T_e}{I_i} - 1\right)$$
$$\times \left[\ln\left(7.515 \times 10^4 A_2 \left(1 - \left(\frac{m_e c^2}{m_e c^2 + T_e}\right)^2\right)\left(1 + \frac{T_e}{m_e c^2}\right)\right) - \left(1 - \left(\frac{m_e c^2}{m_e c^2 + T_e}\right)^2\right)\right]$$

Gas Species	$A_1 = M^2$	C	A_2	I_i(eV)
H_2	0.695	8.115	1.5668	15.4
CH_4	4.23	41.85	0.2635	12.6
N_2	3.74	34.84	0.1478	15.6
CO_2	5.75	55.92	0.2227	13.8

Table: Values for C, $M^2 = A_1$, and A_2 given by Rieke and Prepejchal and I_i given by NIST for some common gas species.

Ionization Cross Section vs Beam Energy cont'd

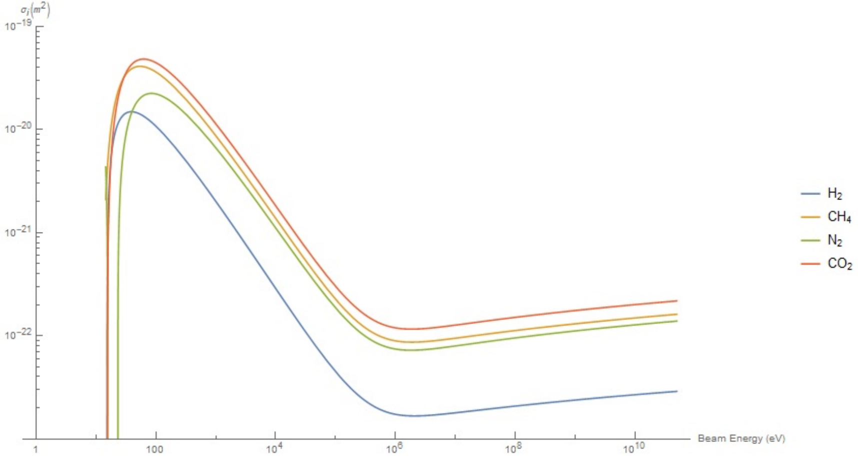

Figure: Plot of the ionization cross section σ_i vs. beam energy T_e

Example RGA Spectrum for Calculating Ion Production Rates

- For given values of n_g, n_b and v, we can calculate σ_i and the ion production rate $\frac{dn}{dt}$. As an example, we can use the RGA spectrum below to get the densities of the gas species n_g in the accelerator vacuum:

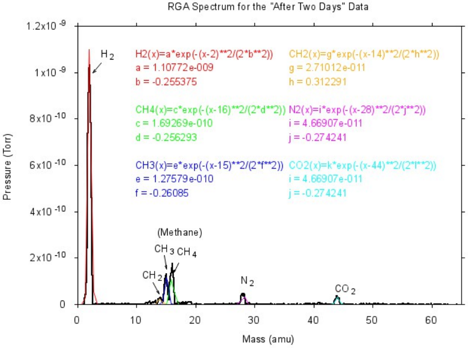

Figure: Analysis of the RGA spectrum for the "After 2 Days" data (before correction factor)

Gas Densities

We can assume that the residual gas behaves ideally (obeys Newton's laws, volume of gas molecules is much smaller than the gas volume, no external forces on the molecules, molecules in random motion). At standard temperature ($T_0 = 273.15\text{K}$) and pressure ($p_0 = 760\text{torr} = 1\text{atm}$) the density of an ideal gas in a given volume is given by Loschmidt's number:

$$n_0 = \frac{p_0}{k_B T_0} \approx 2.687 \times 10^{25} \text{m}^{-3}$$

Thus, for a given gas, its density is

$$n_g \left[\text{m}^{-3}\right] = \left(3.54 \times 10^{22}\right) p \left(\text{torr}\right)$$

The partial pressures are calculated from the Gaussian fit functions (given in the RGA spectrum) using the Gaussian integral:

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} A e^{-\frac{(x+b)^2}{2\sigma^2}} dx = A\sqrt{2\pi\sigma^2}$$

Gas Densities cont'd

- These partial pressures then need to be corrected using correction factors that adjust the pressures of the gas species relative to nitrogen N_2 (from MKS website).
- The correction factor for each parent ion is assumed to be the same for each ion in its class (as in the case of CH_4, CH_3, and CH_2). Assuming an extractor gauge pressure of 2×10^{-12}torr, we can normalize these partial pressures by a normalization factor α that is equal to the sum of the corrected partial pressures divided by the extractor gauge pressure. Each partial pressure is then multiplied by α so that the sum of the partial pressures is 2×10^{-12}torr. In this case, $\alpha \approx 2.87 \times 10^{-3}$.
- From the normalized partial pressures, the number densities and ion production rates can be calculated.

Table of Values for n_g, σ_i and $\frac{dn}{dt}$

Gas species	Uncorrected Pressure (torr)	Correction factor	Corrected Pressure (torr)	Normalized Pressure (torr)
H_2	7.09085×10^{-10}	0.46	3.26×10^{-10}	9.28×10^{-13}
CH_4	1.08744×10^{-10}	1.40	1.52×10^{-10}	4.33×10^{-13}
CH_3	8.34180×10^{-11}	1.40	1.17×10^{-10}	3.33×10^{-13}
CH_2	2.12148×10^{-11}	1.40	2.97×10^{-11}	8.45×10^{-14}
N_2	3.20961×10^{-11}	1.00	3.21×10^{-11}	9.14×10^{-14}
CO_2	3.20961×10^{-11}	1.42	4.56×10^{-11}	1.30×10^{-13}

Gas species	Gas Density n_g $\left(\text{molecules}/\text{m}^3\right)$	Ionization Cross Section σ_i $\left(\text{m}^2\right)$	Ion Production Rate $\left(\text{ions}/\text{m}^3\text{s}\right)$
H_2	3.29×10^{10}	2.99×10^{-23}	4.06×10^{17}
CH_4	1.53×10^{10}	1.53×10^{-22}	9.66×10^{17}
CH_3	1.18×10^{10}	8.00×10^{-23} *	3.89×10^{17}
CH_2	2.99×10^{9}	9.00×10^{-23}*	1.11×10^{17}
N_2	3.24×10^{9}	1.27×10^{-22}	1.70×10^{17}
CO_2	4.60×10^{9}	2.04×10^{-22}	3.87×10^{17}

*Denotes values from NIST here
https://physics.nist.gov/PhysRefData/Ionization/intro.html

Ion Production Rate vs Beam Energy

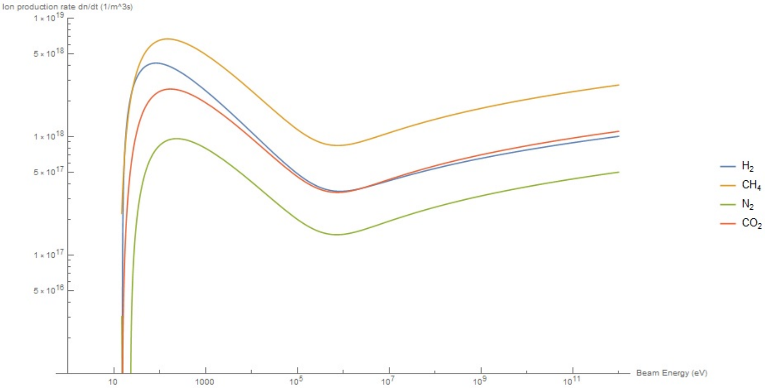

Figure: Log-log plot of ion production rate vs. beam energy for each gas species in the RGA spectrum assuming a 1mm^2, 1mA uniform cylindrical electron beam.

Normalizing IPR to Beam Current

- We can rewrite the equation for IPR and normalize to beam current

$$\frac{dn}{dt} = n_g \sigma_i \left(n_b v \right)$$

- The quantity in parentheses can be thought of as the volume *number* density of electrons moving at velocity v. We can relate this to conventional current I via the definition of the volume *current* density J for a volume *charge* density ρ:

$$J = \rho v \equiv e n_b v = \frac{dI}{da_\perp}$$

where e is the elementary charge and $a_\perp$ is the geometric transverse area of the electron beam.

Normalizing IPR to Beam Current Cont'd

- We can integrate through the entire electron beam and rewrite this as:

$$J = \frac{I}{a_\perp} = en_b v$$
$$n_b v = \frac{I}{ea_\perp}$$

Thus,

$$\frac{dn}{dt} = n_g \sigma_i \frac{I}{ea_\perp}$$
$$\frac{dn/dt}{I} = \frac{n_g \sigma_i}{ea_\perp}$$

- The above equation yields the number of ions produced per second per ampere per cubic meter. Note that, in most cases, $a_\perp$ and n_g are *not* constant and have distance dependence.

Normalized IPR vs. Beam Energy

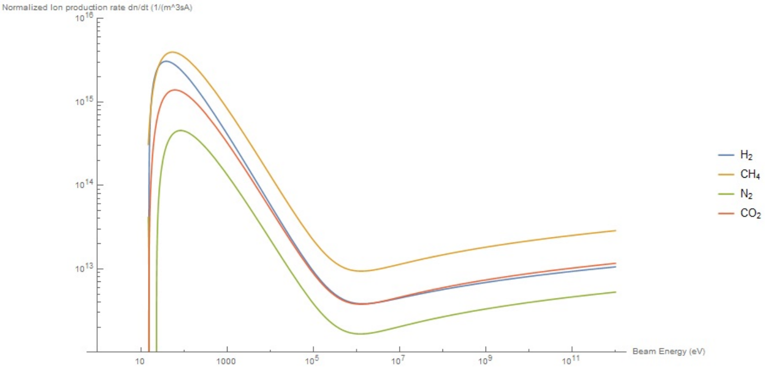

Figure: Log-Log plot of normalized ion production rate vs. beam energy for each gas species in the RGA spectrum assuming a 1mm^2, 1mA uniform cylindrical electron beam.

Fast Ion Instability

- Fast ion instability refers to when an ion that only stays in the beam for one pass causes an instability in the electron beam[2]. Applicable for clearing gap method.

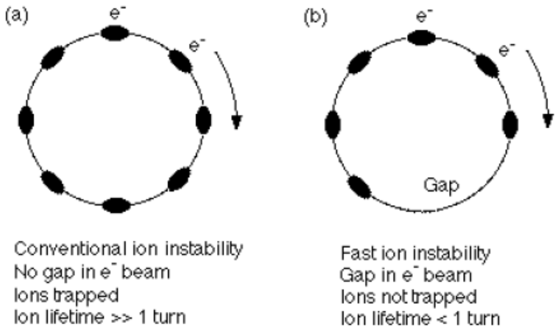

Figure: Comparison between conventional ion instability (left) and fast ion instability (right)[2].

Ionization

- Assume we have an electron beam in a long linac. Let the beam have N electrons and have a transverse distribution of a uniform disk of radius a and a longitudinal distribution of length l. Assume that the accelerator vacuum has residual gas of volume density n. Then:
  - The number of ions per unit length is

$$\lambda = \Sigma n N$$

where Σ is the ionization cross-section. Each electron ionizes Σn ions per unit length.

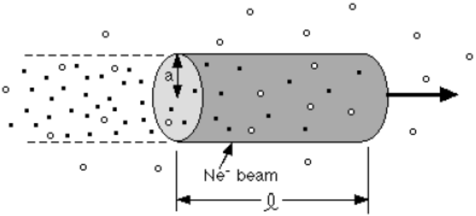

Figure: The electron beam ionizes residual gas in the accelerator vacuum. The circles represent neutral gas molecules and the black dots represent ions[2].

Ion Motion with Unperturbed Electrons

- Assume that each ion is produced at rest and the electron beam is unperturbed by the ions. Each ion will have charge $+e$.
- The ion motion is assumed to be non-relativistic and the electron beam is assumed to be relativistic (or at least have sufficient energy to ionize the residual gas and have its motion remain unaffected to first order. Note that last time we showed this to always be true: $v_e \gg v_i$).
- Let y be the transverse displacement of the ion relative to the center of the electron beam. Then the electric field at the location of the ion due to the electron beam can be derived using Gauss' Law:

$$\int \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$$

Electric field

- The volume charge density of the entire electron beam is $\rho_{tot} = \frac{-Ne}{\pi a^2 l}$. The volume charge density enclosed within a cylinder of radius y is $\rho_{enc} = \frac{q_{enc}}{V} = \frac{q_{enc}}{\pi y^2 l}$. Because the beam is uniform,

$$\rho_{enc} = \rho_{tot}$$

$$\frac{q_{enc}}{\pi y^2 l} = \frac{-Ne}{\pi a^2 l}$$

$$q_{enc} = -Ne\frac{y^2}{a^2}$$

- Thus,

$$E\left(2\pi y l\right) = \frac{-Ney^2}{a^2\varepsilon_0} = \frac{-Ney}{2\pi\varepsilon_0 a^2 l}$$

In atomic units, $\frac{1}{4\pi\varepsilon_0} = 1 \rightarrow \frac{1}{2} = 2\pi\varepsilon_0$, thus $E = -\left(\frac{2Ne}{la^2}\right) y$ and is purely radial.

Equation of motion for unperturbed electrons

- The electric force on the ion is $F_e = eE = -\left(\frac{2Ne^2}{la^2}\right)y$. Thus, the equation of motion of the ions is

$$M\ddot{y} + \frac{2Ne^2}{la^2}y = 0$$

where $M = Am_p$ is the mass of the ion, which is A times the mass of a proton. Using $\frac{e^2}{m_p c^2} = r_p = 1.54 \times 10^{-16}\text{cm}$ as the proton classical radius, the frequency of oscillation of the ions due to the electrons is

$$\omega_I = \sqrt{\frac{2Nr_p c^2}{la^2 A}}$$

For $N = 10^{11}$, $l = 1\text{cm}$, $a = 1\text{mm}$, $A = 14$ (nitrogen or carbon monoxide), then $\omega_I/2\pi = 70\text{MHz}$. Note that ω_I is independent of the energy of both the ions and electrons.

Ion distribution

- The ions will undergo oscillations about the center of the electron beam. Let z be the longitudinal location between the ion and the head of the elecron beam ($l > z > 0$). Then the ion distribution $\rho(r|z)$ is

$$\rho(r|z) = \frac{\Sigma n N}{\pi l a^2} \int_0^z \frac{dz'}{\cos^2\left(\frac{\omega_I (z - z')}{c}\right)} \quad \frac{r}{a} < \left|\cos\left(\frac{\omega_I (z - z')}{c}\right)\right|$$

$$\approx \frac{2\Sigma n N z}{\pi^2 l a^2} \frac{\sqrt{a^2 - r^2}}{r} \text{ for } \omega_I z / c \gg \pi$$

where z' refers to a location where ions can be produced.

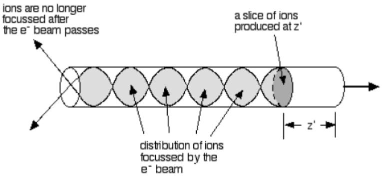

Figure: Diagram of the ion distribution after being produced by the electron beam. A slice of the ions produced at z' will have radius $a\left|\cos\left(\omega_I (z - z') / c\right)\right|$.[2]

Electron motion with unperturbed ions

- Assume we have an ion distribution as a rigid unmoving uniform cylinder with radius $a/\sqrt{2}$ unperturbed by the electrons.

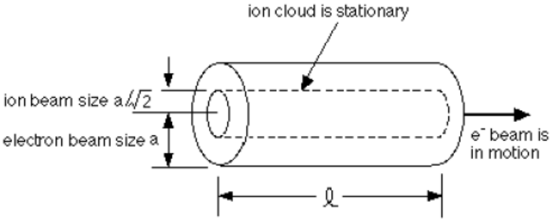

Figure: Diagram of the ion and electron beams[2].

Equation of motion

- An electron sees an electric field due the ion beam of $E = 4\left(e\Sigma nN/a^2\right)(z/l)\,y$ that is purely radial (opposite in direction).
- Thus, the equation of motion for the electrons is

$$\ddot{y} + K\frac{z}{l}y = 0$$

$$K = 4\frac{\Sigma nNc^2 r_e}{\gamma a^2}$$

For $\lambda = 6.4/\text{cm}$, $\gamma = 10^4$, and $a = 1\text{mm}$, then $K = 6.5 \times 10^7\text{s}^{-2}$ and the frequency of oscillation of an electron at $z = l$ (at the end of the beam) due to the ions is $\omega_\beta = \frac{\sqrt{K}}{2\pi} = 1.3\text{kHz}$

Coupled ion-electron equations of motion

- We can consider the case when the two beams are coupled and mutually perturbing. Assume that the electron and ion beam distributions (transverse) are uniform discs with radii a and $a/\sqrt{2}$ respectively and there are no self direct space charge effects (ion-ion or electron-electron).
- Let $y_e(s|z)$ denote the motion of the centroid of the electron beam slice. Let s be the distance along the accelerator. If the head of the beam passes $s = 0$ at $t = 0$, then for the electron beam slice, $s = ct - z$.
- Let $y_I(s, t|z)$ denote the motion of an ion produced at z'. Note that y_I is only defined after the ion is born $(s < ct - z')$.

Coupled ion-electron equation of motion for ions

- Recall the unperturbed equation of motion for the ions:

$$M\ddot{y} + \frac{2Ne^2}{la^2} y = 0$$
$$\ddot{y} + \omega_I^2 y = 0$$

- The coupled equation of motion for the ions is

$$\frac{\partial^2}{\partial t^2} y_I \left(s, t|z'\right) + \omega_I^2 \left[y_I \left(s, t|z'\right) - y_e \left(s|ct - s\right)\right] = 0$$

$y_e \left(s|ct - s\right)$ appears since it is the electrons that are interacting with the ions with $z = ct - s$

- This coupled equation requires the initial conditions:

$$y_I \left(s, \frac{s + z'}{c}|z'\right) = y_e \left(s|z'\right)$$
$$\left[\frac{\partial}{\partial t} y_I \left(s, t|z'\right)\right]_{t = \frac{s+z'}{c}} = 0$$

i.e. ions and electrons are in the same place when the ions are produced at time $t = \dfrac{s + z'}{c}$. Thus, the ions must have the same distribution as the electrons at that moment. The second condition implies that the ions initially have zero velocity.

Coupled ion-electron equation of motion for electrons

- Recall the unperturbed equation for the electrons:

$$\ddot{y} + K\frac{z}{l}y = 0$$

$$K = 4\frac{\Sigma nNc^2 r_e}{\gamma a^2}$$

- The coupled equation motion for the electrons is:

$$c^2\frac{\partial^2}{\partial s^2}y_e\left(s|z\right) + \omega_\beta^2 y_e\left(s|z\right) + \frac{K}{l}\left[zy_e\left(s|z\right) - \int\limits_0^z dz' y_I\left(s, \frac{s+z}{c}|z'\right)\right] = 0$$

The second term describes external betatron focusing. The fourth term describes the center of charge of all ion slices integrated from $z' = 0$ to $z' = z$. That is, it is the contribution to electron motion due to the production of all ions prior to z.

Resonance

- Any asymptotic behavior will be dominated by the resonance between the ion and electron motion. To see this:
  - Consider electron motion described by

$$y_e\left(s|z\right) \sim e^{i\omega_\beta s/c+ikz}$$

    for some k.
  - The ions are in simple harmonic motion

$$y_I\left(s,t|z'\right) \sim y_I\left(s,t_0|z'\right)e^{\pm i\omega_I(t-t_0)}$$

    where we'll keep the $+$ sign for instability and t_0 is the time when the ions are produced.
  - Using the first initial condition at $t=t_0$,

$$\begin{aligned} y_I\left(s,t|z'\right) &\sim y_e\left(s|z'\right)e^{i\omega_I(t-t_0)} \\ &\sim e^{-i\omega_\beta s/c+ikz'}e^{i\omega_I t-i\omega_I\left(s+z'\right)/c} \end{aligned}$$

Resonance cont'd

- Resonance occurs when y_e and y_I have the same time dependence when observed at fixed s. For $z = ct - s$,

$$y_e\left(s|z\right) \sim e^{-i\omega_\beta s/c + ikct - iks}$$

- Recalling that $y_I \sim e^{-i\omega_\beta s/c + ikz'} e^{i\omega_I t - i\omega_I\left(s+z'\right)/c}$, we see that resonance occurs when $kc = \omega_I$. We have that,

$$y_e\left(s|z\right) \sim e^{-i\omega_\beta s/c + i\omega_I z/c}$$
$$y_I\left(s,t|z'\right) \sim e^{-i(\omega_\beta + \omega_I)s/c + i\omega_I t}$$

Resonance cont'd

- At resonance, y_I is independent of z' - all ions at s oscillate in unison regardless of where/when they were produced. When observed at fixed s, both electrons and ions have the same time dependence of $e^{i\omega_I t}$. At fixed t, their behavior is $e^{-i(\omega_\beta+\omega_I)s/c}$.
- At resonance, y_e contains information on ω_I. Thus, we could use a beam position monitor to observe the electron beam. If fast ion instability exists, then we should be able to observe a frequency peak at the ion frequency ω_I. Recall that ω_I contains information on the identity of the ion species, so this may be another way to determine what kinds of ions are present in the accelerator. The article has a subsection on this...I won't include it here for brevity.

Asymptotic behavior

- Assume that y_e and y_i have the form

$$y_e\left(s|z\right) = \tilde{y}_e\left(s|z\right)e^{-i\omega_\beta s/c + i\omega_l z/c}$$
$$y_l\left(s,t|z'\right) = \tilde{y}_l\left(s,t|z'\right)e^{-i(\omega_\beta+\omega_l)s/c + i\omega_l t}$$

where $\tilde{y}_e\left(s|z\right)$ is slowly varying in s and $\tilde{y}_l\left(s,t|z\right)$ is slowly varying in t.

- We can plug y_e and y_l into the coupled equations of motion and drop small terms $\frac{\partial^2 \tilde{y}_l}{\partial t^2}$ and $\frac{\partial^2 \tilde{y}_e}{\partial t^2}$ to get equations of motion at resonance

$$\frac{\partial}{\partial t}\tilde{y}_l\left(s,t|z'\right) + \frac{i\omega_l}{2}\tilde{y}_e\left(s|ct-s\right) = 0$$
$$\frac{\partial}{\partial s}\tilde{y}_e\left(s|z\right) + \frac{iK}{2\omega_\beta cl}z\tilde{y}_e\left(s|z\right) - \frac{iK}{2\omega_\beta cl}\int_0^z dz'\tilde{y}_l\left(s,\frac{s+z}{c}|z'\right) = 0$$

with the (first) initial condition $\tilde{y}_l\left(s,\frac{s+z'}{c}|z'\right) = \tilde{y}_e\left(s|z'\right)$ (Recall $s = ct - z' \to t = \frac{s+z'}{c}$)

Asymptotic behavior cont'd

- From the first equation of motion and the initial condition, we have

$$\begin{aligned}\tilde{y}_I\left(s,t|z'\right) &= \tilde{y}_e\left(s|z'\right) - \frac{i\omega_I}{2}\int\limits_{(s+z')/c}^{t} dt'\,\tilde{y}_e\left(s|ct'-s\right) \\ &= \tilde{y}_e\left(s|z'\right) - \frac{i\omega_I}{2c}\int\limits_{z'}^{ct-s} dz''\,\tilde{y}_e\left(s|z''\right)\end{aligned}$$

- Substituting this result into the second equation of motion yields

$$\frac{\partial}{\partial s}\tilde{y}_e\left(s|z\right) + \frac{iK}{2\omega_\beta cl}z\int\limits_0^z dz'\,z'\left[\frac{\partial}{\partial z'}\tilde{y}_e\left(s|z'\right) + \frac{i\omega_I}{2c}\tilde{y}_e\left(s|z'\right)\right] = 0$$

- Taking $\dfrac{\partial}{\partial z}$ of both sides yields

$$\frac{\partial^2}{\partial s\partial z}\tilde{y}_e\left(s|z\right) + \frac{iK}{2\omega_\beta cl}z\left[\frac{\partial}{\partial z}\tilde{y}_e\left(s|z\right) + \frac{i\omega_I}{2c}\tilde{y}_e\left(s|z\right)\right] = 0$$

Asymptotic behavior cont'd

- The first term in the square bracket is much smaller than the second, since $\tilde{y}_e$ varies slowly in z compared to ω_I (I'll provide the corresponding validity condition in the next slide). Dropping the first term...

$$\frac{\partial^2}{\partial s \partial z}\tilde{y}_e\left(s|z\right) - \frac{K\omega_I}{4\omega_\beta c^2 I} z\tilde{y}_e\left(s|z\right) \approx 0$$

- Defining $\eta \equiv \frac{z}{c}\sqrt{\frac{K\omega_I s}{2\omega_\beta I}}$, we can rewrite this equation as

$$\eta\tilde{y}_e'' + \tilde{y}_e' - \eta\tilde{y}_e = 0$$
$$\eta^2\tilde{y}_e'' + \eta\tilde{y}_e' - \eta^2\tilde{y}_e = 0$$

where primes indicate $\frac{\partial}{\partial \eta}$. This is the modified bessel equation with solutions

$$\tilde{y}_e\left(s|z\right) \approx y_0 I_0\left(\eta\right)$$
$$\approx y_0 \frac{e^\eta}{\sqrt{2\pi\eta}} \;\; \eta \gg 1$$

Asymptotic behavior cont'd

- We see that the electron beam oscillation goes as e^{η} for $\eta \gg 1$, i.e. it is exponential in z but behaves like $e^{\sqrt{s}}$ in s. Since $\eta \propto z$, we see that in order to counter fast ion instability, we can introduce more gaps in the beam.
- The approximation from before was $\left|\frac{\partial}{\partial z}\tilde{y}_e\right| \ll \frac{\omega_I}{c}\left|\tilde{y}_e\right|$. With the approximation $\eta \gg 1$, the validity criterion is

$$\frac{\omega_I l}{c} \gg \eta \gg 1 \text{ or } \frac{\omega_\beta \omega_I l}{K} \gg s \gg s_0$$

where $s_0 = \frac{2\omega_\beta c^2}{K\omega_I l}$, the distance along the accelerator at which the electron amplitude $\tilde{y}_e$ grows by a factor of e. This criterion implies that $\frac{\omega_I l}{c} \gg 1$, i.e. there must be many ion oscillations within the length of the beam and s must be within the above range.

- Experimental verification? See Meseck article: https://www.researchgate.net/publication/276884133_Numerical_studies_of_the

Charge Neutralization by Poncet

- When ions are formed by colliding electrons and become trapped in the beam, they can accumulate over time. The total (net) charge of the beam thus decreases until the number of ions equals the number of electrons, at which case the beam is fully neutralized[3]
- We can define a neutralization factor η:

$$\eta = \frac{n_i}{n_e}$$

  where n_i is the neutralizing charge by the ions and n_e is the electron beam charge.
- Since η is unlikely to be constant along the beam, we can define a local neutralization factor:

$$\eta\,(s) = \frac{2\pi R}{n_e}\frac{dn_i}{ds}$$

  where $2\pi R$ is the circumference of the accelerator and $\frac{dn_i}{ds}$ is the local ion density (in charge/meter).

Charge neutralization time

- We can simply integrate the ion production rate equation to yield the charge neutralization time:

$$\begin{aligned} \frac{dn}{dt} &= n_b n_g \sigma_i v \\ \rightarrow n(t) &= n_b t / \tau_N \\ \tau_N &= \frac{1}{n_g \sigma_i v} \end{aligned}$$

which is valid at low ion density and short times.

GPT

- I created a simulation of protons, electrons and neutrons passing through a series of alternating focusing and defocusing quadrupoles to see whether or not we can distinguish between the particles (and to see if GPT can do something useful). The quadrupoles steadily gain strength down the accelerator. The particles are color coded by charge so that we can distinguish between them: blue particles are electrons; red particles are ions, and green particles are neutral atoms (in this case neutrons). No space charge effects are included.
- See email

Other

- List of references uploaded to wiki in bibliography format
- Next USPAS meeting in Jan-Feb 2019
  http://uspas.fnal.gov/programs/2019/knoxville/index.shtml
- Specifications of the accelerator for calculations & GPT, i.e. distance, strength, length etc. info for each element that affects the motion of the beam.

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