GEANT4 Simulation of the Jlab MeV Mott Polarimeter

Martin McHugh
mjmchugh@jlab.org

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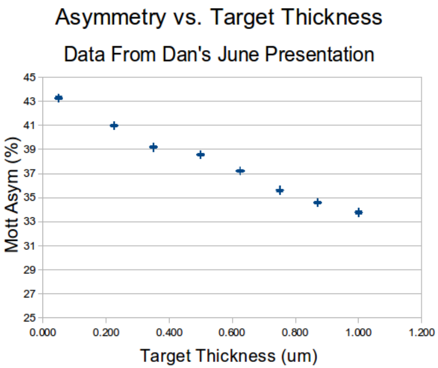
The Problem

We don't know the form of the effective Sherman function for targets of finite thickness, $S(d)$.



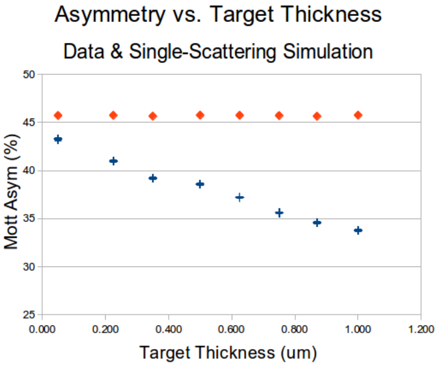
How Can Simulation Help Us?

- Allows us to examine contributions to detector signal individually.
- Beam is treated as 100% polarized in y direction.
- Gaussian, circular beam profile with width of 1 mm.

Why Brute Force doesn't work: 1 μA is 6.24×10^{12} e^-/s and we need $\approx$ 1000 μAs of data for a decent measurement. Can only simulate 100 million events per day...

How to Generate Single-Scattering Events

1. Pick point $\vec{x}_1$ in the beam profile on the target.
2. Calculate energy loss to $\vec{x}_1$. Get new energy E_1.
3. Pick point $\vec{x}_2$ in acceptance to throw at.
4. Calculate $\sigma(\theta_1, \phi_1, E_1)$ based on $\vec{x}_1$, $\vec{x}_2$.
5. Throw random number, x. If $x < \sigma$ throw electron. Else, repeat from 1.



How to Generate Double-Scattering Events

1. Pick point $\vec{x}_1$ in the beam profile on the target.
2. Calculate energy loss to $\vec{x}_1$. Get new energy E_1.
3. Pick point $\vec{x}_2$ in target with $|\vec{x}_2 - \vec{x}_1| < r_E$.
4. Calculate $\sigma_1(\theta_1, \phi_1, E_1)$ based on $\vec{x}_1$, $\vec{x}_2$.
5. Calculate energy loss to $\vec{x}_2$. Get new energy E_2.
6. Pick point, $\vec{x}_3$, in acceptance to throw at.
7. Calculate $\sigma_2(\theta_2, \phi_1, E_2)$.
8. Throw random number, x. If $x < \sigma_1\sigma_2$ throw electron. Else, repeat from 1.

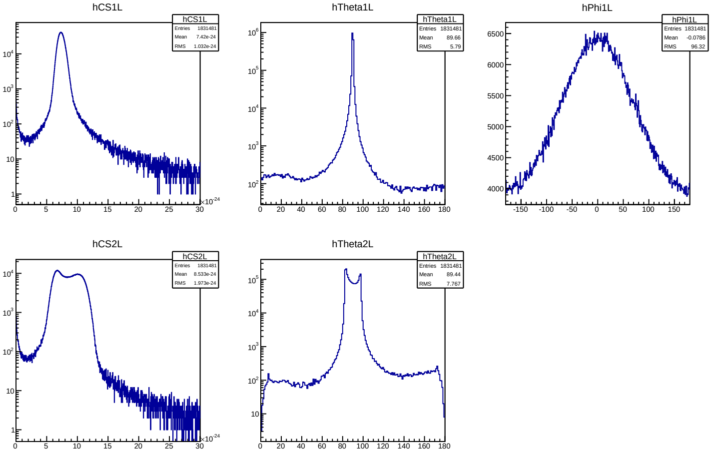
Double Scattering Asymmetry

The Asymmetry is calculated to be:

$$A_{d.s.} = \frac{L - R}{L + R} = -01.05\% \pm 0.06\%$$

for all target thicknesses. The problem now becomes one of determining how much of a dilution this is at each target thickness.

Scattering into Left Detector



Double Scattering Generation: Method 2

1. Pick a scattering position, $\vec{x}_1$, within the intersection of the beam and our target.
2. Pick a direction direction, (θ_1, ϕ_1), from the uniform unit sphere.
3. Pick a point, $\vec{x}_2$, uniformly between $\vec{x}_1$ and the edge of the foil (or 0.16 mm as in the previous example).
4. Pick a point, $\vec{x}_3$, in the acceptance the primary collimator.
5. Throw from $\vec{x}_2$ towards $\vec{x}_3$.

Not weighting by cross section allows for an easier integral in the rate calculation.

Calculating Rates

In order to compare both types of simulation and to compare simulation to data, we must be able to calculate rates. The rate is given as

$$\mathcal{R} = \mathcal{L} \int_x \sigma(x)\epsilon(x)$$

where ϵ is the effective acceptance of the detectors and x are the degrees of freedom over which the integral is performed.

Calculating Rates (Single Scattering)

In the case of single scattering the integral can be simplified to:

$$\mathcal{R} \approx \mathcal{L}\langle\sigma\rangle \frac{N_{hit}}{N_{thrown}} \Delta\cos\theta\Delta\phi$$

d (μm)	$\mathcal{R}_L$ (Hz/μA)	$\mathcal{R}_R$ (Hz/μA)	$\mathcal{R}_{\mathrm{avg}}$	$\mathcal{R}_{\mathrm{data}}$
0.05	5.3	11.6	8.5	9.3
1.00	104.3	241.9	173.1	214.3

This is a good sanity check.

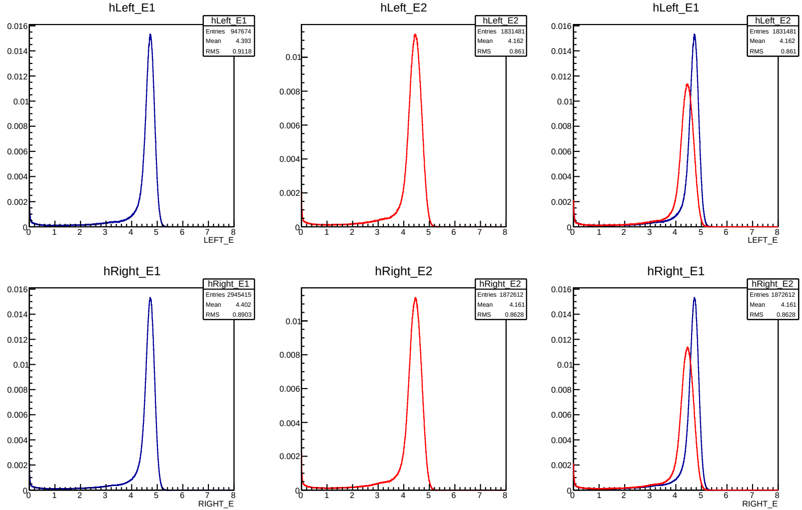
Calculating Rates (Double Scattering)

- Attempting the same simplification doesn't work in the double scattering case. Rates are $\approx 10^{-12}$ smaller than the single scattering case.
- More thought needs to go into performing this integral in order to make the simulation work.
- Once this works, we should be able to calculate

$$A(d) = \frac{[\mathcal{R}_{L_1}(d) - \mathcal{R}_{R_1}(d)] + [\mathcal{R}_{L_2}(d) - \mathcal{R}_{R_2}(d)]}{[\mathcal{R}_{L_1}(d) + \mathcal{R}_{R_1}(d)] + [\mathcal{R}_{L_2}(d) + \mathcal{R}_{R_2}(d)]}$$

directly from simulation.

Spectra



hLeft_E1
hLeft_E1
Entries 947674
Mean 4.393
RMS 0.9118
LEFT_E
hLeft_E2
hLeft_E2
Entries 1831481
Mean 4.162
RMS 0.861
hLeft_E1
hLeft_E2
Entries 1831481
Mean 4.162
RMS 0.861
LEFT_E
hRight_E1
hRight_E1
Entries 2945415
Mean 4.402
RMS 0.8903
RIGHT_E
hRight_E2
hRight_E2
Entries 1872612
Mean 4.161
RMS 0.8628
hRight_E1
hRight_E2
Entries 1872612
Mean 4.161
RMS 0.8628
RIGHT_E

Summary

- Single scattering simulation gives good results but no *d*-dependence in Asymmetry.
- Can't calculate rate yet for Double-scattering. Asymmetry is small. Need to determine proper dilution.
- Spectra look decent, can't tell how much but it looks like the double scattering will influence the low energy shoulder to some degree.

Energy Loss in the Gold Foils

Using the table at right, we determine the linear fit

$$\frac{dE}{dx}(E) = \frac{0.272}{\mathrm{mm}} \times E + 1.888\frac{\mathrm{MeV}}{\mathrm{mm}}.$$

Numerical integration of the above gives us energy loss within the target. Note: A particle with initial energy of 5 MeV will only lose 3 keV/μm and will lose 500 keV in $\approx$ 200μm.

Energy [MeV]	dE/dx [MeV/mm]
1.0	2.179
2.0	2.422
3.0	2.702
4.0	2.980
5.0	3.254
6.0	3.526
7.0	3.796
8.0	4.065

Data from NIST estar database.

What Steigerwald Did

His method of calculating multiple scattering's influence was direct integration of some form:

$$N = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \int_{x_1=0}^{D} \int_{\psi=\theta_2}^{\theta_2+\Omega_\theta} \sigma_1(x_1, \theta, \phi)\sigma_2(x_2, \theta_2)E(x_1, x_2) d\psi dl d\phi d\theta.$$

The problem is that his source for this integral is in German and his code is poorly documented and also in partial German.