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GPT-Spin

Introduction

The GPT-Spin extension is able to track particle spin coordinates in GPT using the generalized Thomas-BMT equation. For performance reasons this functionality has to be specifically requested on the command-line using the new `-s` option of GPT.

When spin-tracking is requested on the command-line, every particle has an additional spin coordinate **s**. This spin coordinate **s** is similar to the particle position and momentum, in the sense that it is part of the initial particle distribution, it is being tracked in the laboratory frame, and it being written to the outputfile.

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1. GPT-Spin

1.1 Differential equations

When spin-tracking is enabled, the following additional differential equation is solved for each particle *i*:

$$\frac{d\mathbf{s}_i}{dt} = \mathbf{\Omega}_i \times \mathbf{s}_i \qquad [1.1]$$

where

$$\mathbf{\Omega}_i = -\frac{q_i}{m_i}\left[\left(a_i + \frac{1}{\gamma_i}\right)\mathbf{B}_i - \frac{a_i\,(\gamma_i \boldsymbol{\beta}_i \cdot \mathbf{B}_i)}{\gamma_i + 1}\boldsymbol{\beta}_i - \left(a_i + \frac{1}{\gamma_i + 1}\right)\frac{\boldsymbol{\beta}_i \times \mathbf{E}_i}{c}\right] \qquad [1.2]$$

With $\mathbf{s}$ the particle spin coordinate, and $a_i = g/2 - 1$ where g is the particle's magnetic moment. The current implementation assumes that all particles are electrons with a hard coded $g = 2.00231930436256$. This is planned to be generalized in a later version of GPT-Spin.

1.2 Accuracy

The accuracy statement in GPT-Spin has an optional third parameter that can be used to set the spin-accuracy. The new command is as follows:

`accuracy(GBacc,[xacc]) ;`
Specifies the calculation accuracy.

`GBacc`	Negative base 10 logarithm of the simulation accuracy for γβ. If not specified, 4 is assumed.
`xacc`	Negative base 10 logarithm for the simulation accuracy for position. If not specified, 6 is assumed.
`spinacc`	Negative base 10 logarithm for the simulation accuracy for spin tracking. If not specified, 3 is assumed.

The simulation accuracy is calculated as follows:

$$\begin{aligned} \Delta\gamma\beta_{\text{max}} &= 10^{-\mathtt{GBacc}} \\ \Delta x_{\text{max}} &= 10^{-\mathtt{xacc}} \\ \Delta\text{spin}_{\text{max}} &= 10^{-\mathtt{spinacc}} \end{aligned} \qquad [1.3]$$

The requested accuracy is the minimal accuracy per step. The final accuracy should be checked by increasing the accuracy and verifying that the result does not change significantly.

1.3 New (and modified) GPT elements

1.3.1 Setfile (modified)

The GPT-Spin version of GPT reads the initial spin coordinates from the columns **`spinx`**, **`spiny`** and **`spinz`**.

1.3.2 Tout, snapshot and screen (modified)

All output routines in GPT-Spin write the spin coordinates of all particles as **`spinx`**, **`spiny`** and **`spinz`** to the GPT outputfile

1.3.3 Setspinxdist, setspinydist and setspinzdist

Set the spin *x*, *y* and *z*-coordinates analogously to how **`setxdist`**, **`setydist`** and **`setzdist`** set the initial *x*, *y* and *z*-coordinates.

1.3.4 Setspinphidist

`setspinphidist(set,distribution);`
Set azimuthal angle (φ) of the particle spin coordinates.

Modifies the *xy*-coordinates of the particle spin as:

$$\mathrm{s_x} = |\mathrm{s_{xy}}| \cos(\mathbf{distribution})$$
$$\mathrm{s_y} = |\mathrm{s_{xy}}| \sin(\mathbf{distribution}) \quad [1.4]$$

where $|\mathrm{s_{xy}}| = \sqrt{1 - s_z^2}$.

The spin-angle in the *xy*-plane of all particles in the **`set`** is modified according to the specified distribution. The distribution is interpreted as a 1D distribution. The polar angle of the spin coordinates will not be affected.

1.3.5 SetGBthetadist (2D spherical)

`setGBthetadist(set,distribution);`

Set polar angle (θ) of the particle spin coordinates.

Modifies the *xyz*-coordinates of the particle spin such that:

$$\mathrm{s_x} = \sin(\mathbf{distribution})\, \mathrm{s}_x / |\mathrm{s_{xy}}|$$
$$\mathrm{s_y} = \sin(\mathbf{distribution})\, \mathrm{s}_y / |\mathrm{s_{xy}}|$$
$$\mathrm{s_z} = \cos(\mathbf{distribution}) \quad [1.5]$$

where $|\mathrm{s_{xy}}| = \sqrt{s_x^2 + s_y^2}$.

The particle spin coordinates of the particles in the **`set`** are modified to create the specified distribution. The distribution is interpreted as a 2D spherical distribution. The azimuthal angle of the spin coordinates will not be affected. When both s_x and s_y are initially zero, an angle of zero in the xy-plane is assumed.

1.4 New GDFA programs

The GDFA programs listed in Table 1-A can be used to extract macroscopic spin information.

Table 1-A: GDFA programs related to GPT-Spin

Name	Equation	Description
`avgspinx`	$\overline{s_x} = \frac{\sum n_i s_{x_i}}{\sum n_i}$	Average s_x-coordinate [m].
`avgspiny`	$\overline{s_y}$	Average s_y-coordinate [m].
`avgspinz`	$\overline{s_z}$	Average s_z-coordinate [m].
`stdspinx`	$\mathrm{std}(s_x) = \sqrt{\frac{\sum n_i \left(s_{x_i} - \overline{s_x}\right)^2}{\sum n_i}}$	Standard s_x-deviation.
`stdspiny`	$\mathrm{std}(s_y)$	Standard s_y-deviation.
`stdspinz`	$\mathrm{std}(s_y)$	Standard s_z-deviation.
`stdspin`	$\sqrt{\mathrm{std}(s_x)^2 + \mathrm{std}(s_y)^2 + \mathrm{std}(s_z)^2}$	Standard spin-deviation.

2. References