National Institute of
Advanced Industrial Science
and Technology
AIST

JLab seminar 8/17/2016

X-ray vortices from nonlinear inverse Thomson scattering

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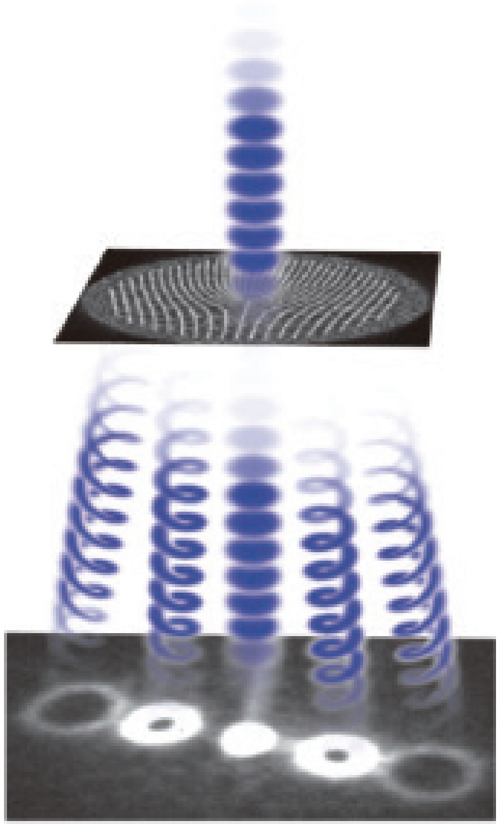
Optical vortex



B. J. McMorran et al., Science 331 (2011) 192.

Characteristics

- Forming a helical wave front.
- Carrying an orbital angular momentum (OAM).
- Total angular momentum
  = OAM $m\hbar$ + spin angular momentum $\hbar$
  on the paraxial approximation.

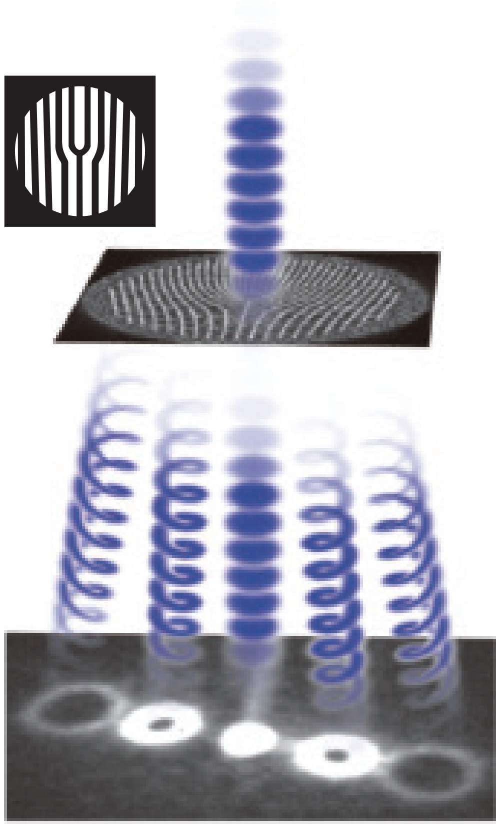
Beams

- Laser, 300 keV electron, 10 keV X-ray,
  Terahertz radiation, Cold neutron.

Application

- Optical tweezer, quantum entanglement, etc.

Optical vortex



B. J. McMorran et al., Science 331 (2011) 192.

Generation

- Cylidrical lens,
- Spiral phase plate,
- Fork grating,
- Electromagnetic radiation from an electron.

Laguerre Gaussian beam

$$E = \frac{C}{\sqrt{1+\left(z/z_R\right)^2}}\left(\frac{r\sqrt{2}}{\omega(z)}\right)^{|m|} L_p^m\left(\frac{2r^2}{\omega^2(z)}\right)\exp\left(-\frac{r^2}{\omega^2(z)}\right)$$

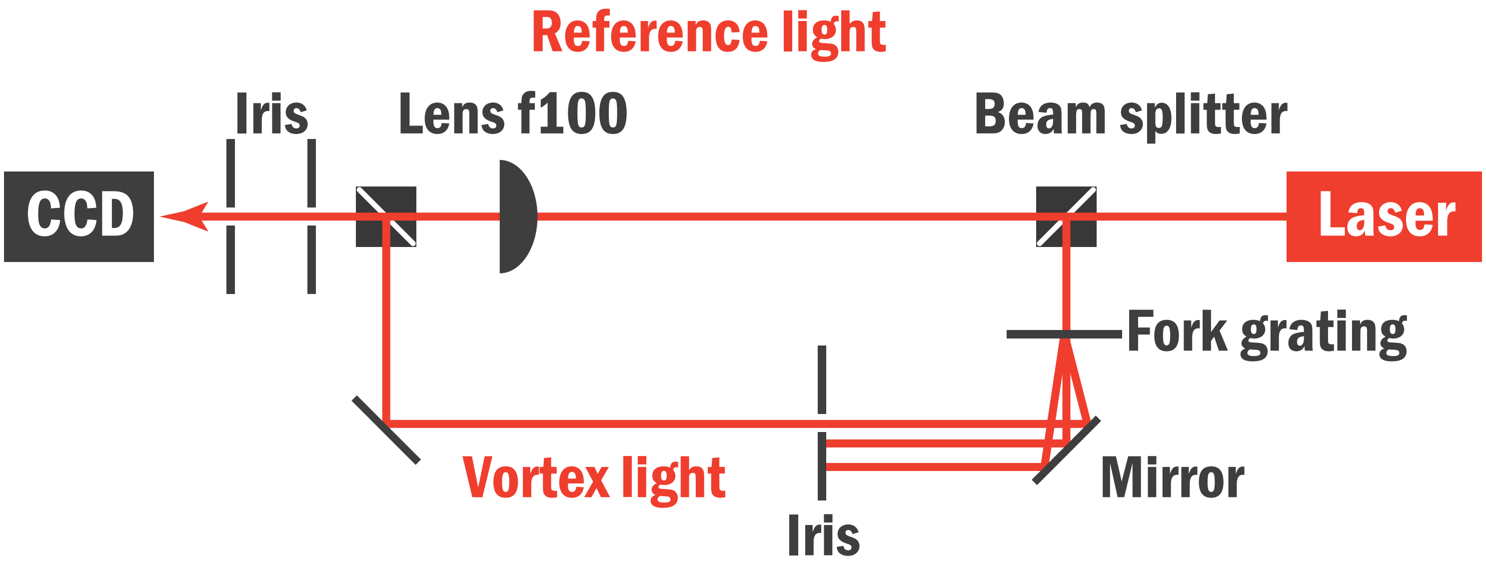
$$\times \exp\left\{-i\frac{kr^2 z}{2\left(z^2+z_R^2\right)}\right\}\exp\left(-im\phi\right)\exp\left\{i\left(2p+m+1\right)\tan^{-1}\frac{z}{z_R}\right\}$$

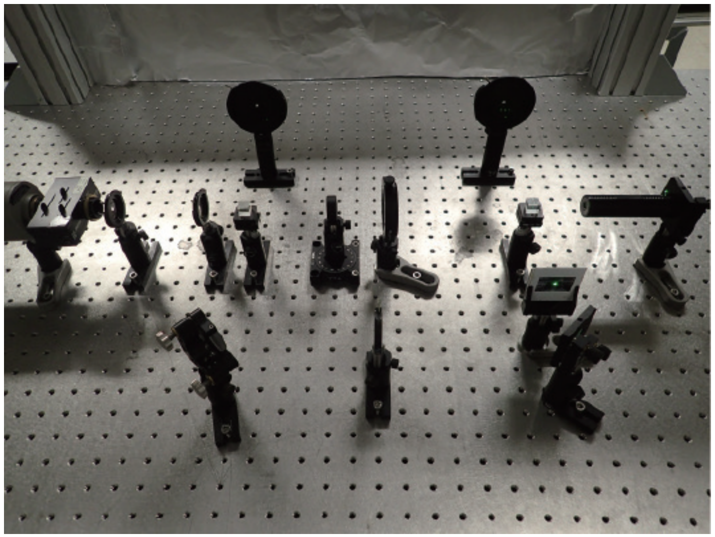
This term represents the helical wave front.

OAM operator:

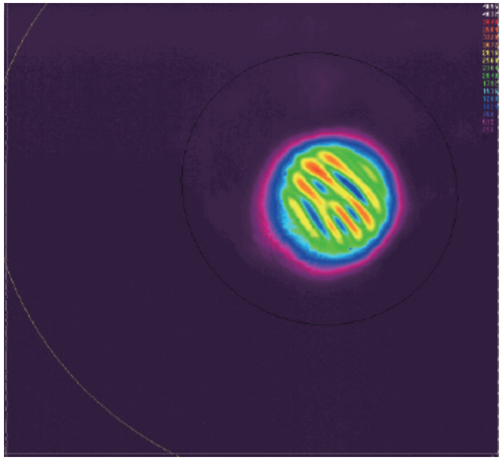
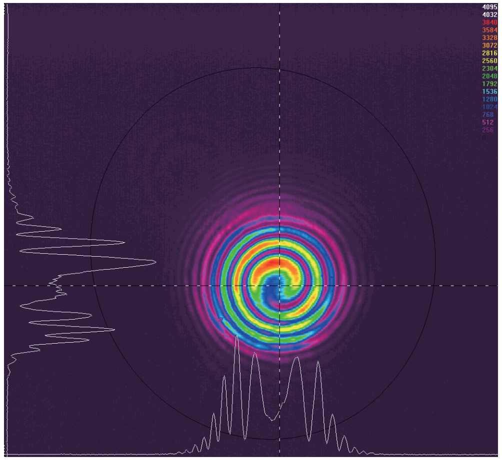
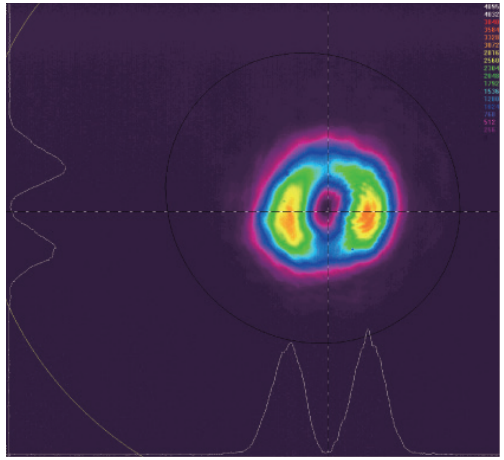
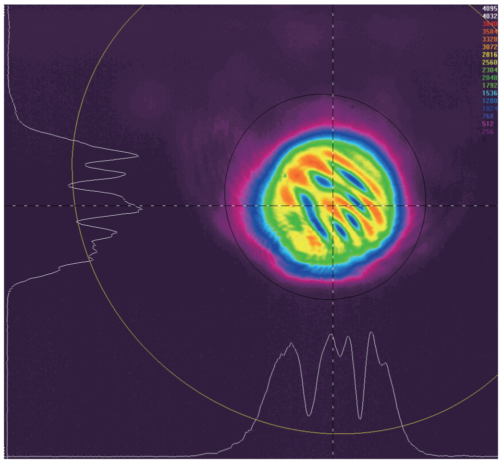
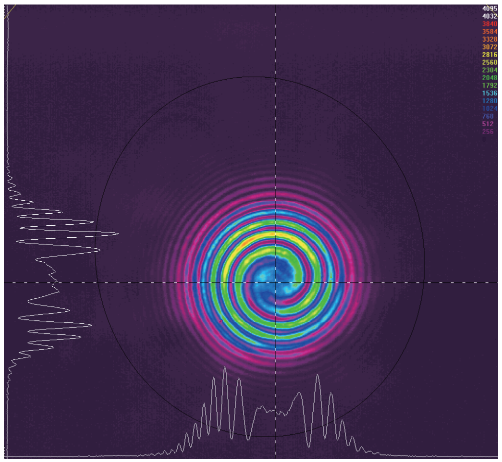
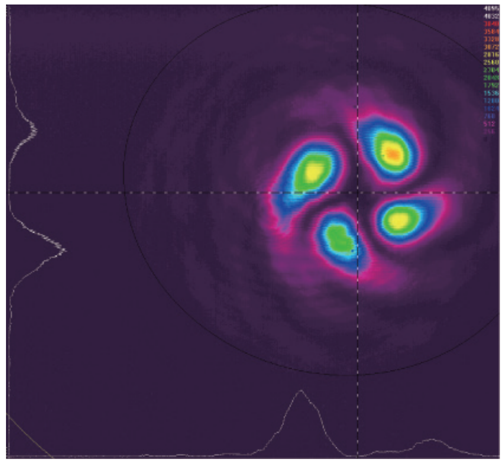
$$L_z = i\hbar\frac{\partial}{\partial\phi} = m\hbar$$

Interferometer





Interference pattern

OAM value	Plane wave + vortex	Spherical wave + vortex	Vortex + vortex
1			
2			

Purpose

Generate high energy X-ray vortex (more than MeV) and develop its application.

Application possibility

X-ray dichroism, Nuclear physics, Solid state physics,
Generation of positron vortex via pair production.

How to generate X-ray/gamma-ray vortex

- **Helical undulator**

  High energy electron, more than 100 GeV, is required if MeV gamma ray vortex generats.

- **Inverse Thomson/Compton scattering between plane wave electron and optical vortex laser**

  Already proposed by U. D. Jentchura and V. G. Serbo.
  PRL 106 013001 (2011).

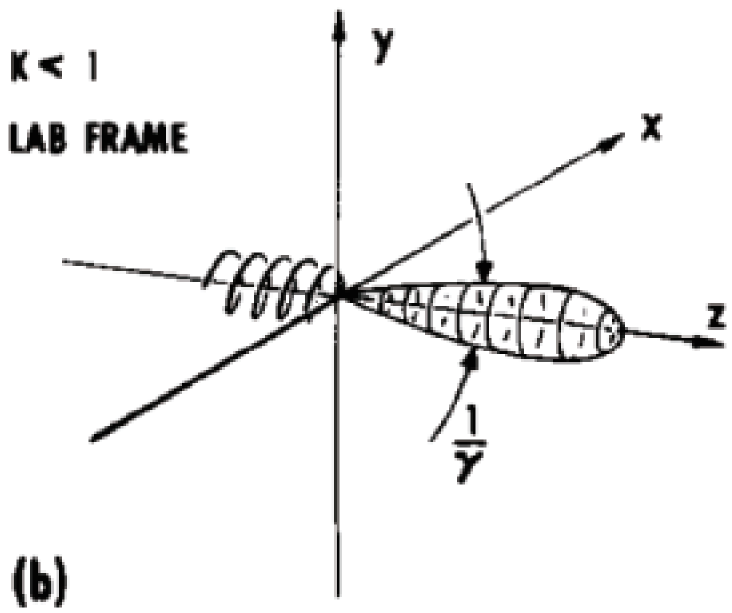
- **Nonlinear inverse Thomson/Compton scattering between plane wave electron and intense circularly polarized laser (not vortex)**

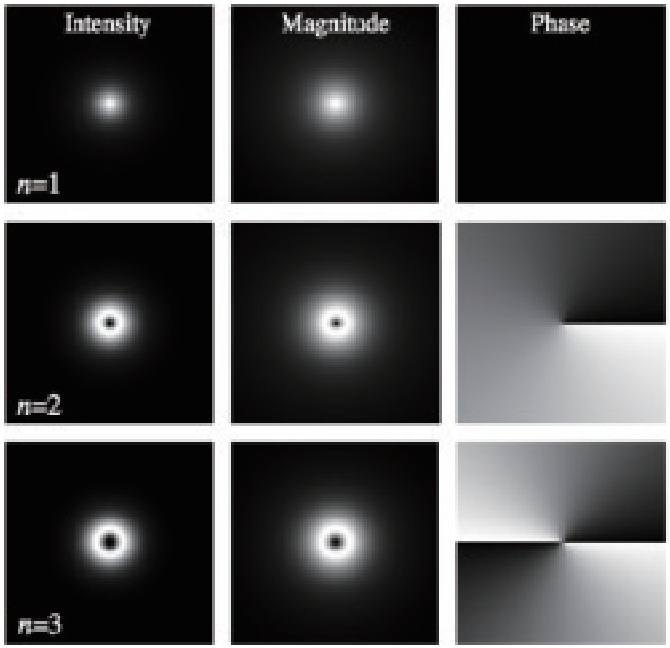
  Newly proposed by Y. Taira and M.Katoh.

What is important to generate X-ray vortex?

Helical motion of the relativistic electron.

X-ray vortex generation by a helical undulator was proposed in 2008.

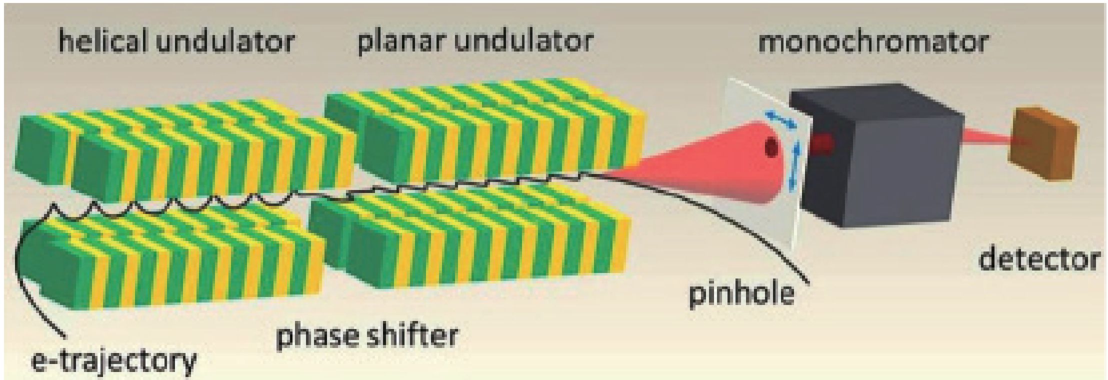


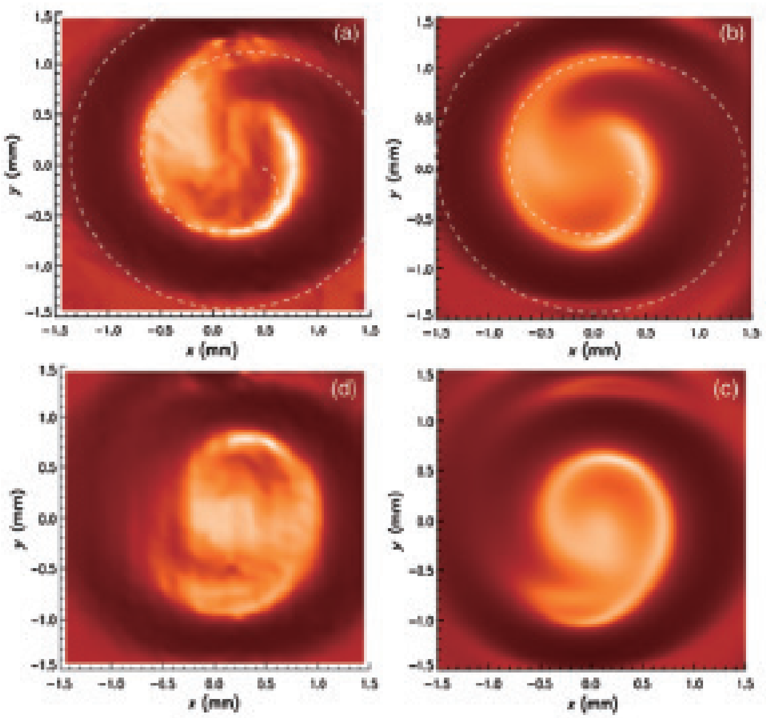


Higher harmonic carry ±(n-1)ħ OAM.

B. M. Kincaid, J Appl. Phys. 48 2684 (1977).
S. Sasaki et al., PRL. 100 124801 (2008).

POP experiment at BESSY-II





Interference pattern between second harmonics from helical undulator (vortex beam) and fundamental from planar undulator (non vortex beam). Photon energy was 99 eV.

J. Bahrdt et al., PRL 111 034801 (2013).

Helical motion of electron

Also it is induced by the intense circularly polarized laser field.

Electron orbits r = (x, y, z)

$$x(\eta) = x_0 + \left(r_1/\sqrt{2}\right)\sin k_0\eta$$
$$y(\eta) = y_0 \mp \left(r_1/\sqrt{2}\right)\cos k_0\eta$$
$$z(\eta) = z_0 + \beta_1\eta$$

sigh - : positive helicity
sigh +: negative helicity

$$\eta = z + ct$$
$$k_0 = 2\pi/\lambda_0$$
$$r_1 = a_0/h_0k_0$$
$$a_0 = 0.85\times10^{-9}\lambda_0(\mu\mathrm{m})\sqrt{I_0 \quad (\mathrm{W/cm^2})}$$

λ_0 : Wavelength of circularly polarized laser

a_0 : Laser strength parameter

I_0 : Intensity of the laser

High energy X-ray vortex will be produced by nonlinear inverse Thomson scattering !

E. Esarey et al., PRE. 48 3003 (1993).

Electric field

Emitted by a single electron in an arbitrary orbit

$$\vec{E}(\omega) = -i\sqrt{\frac{e^2k^2}{32\pi^3\varepsilon_0^2R^2}}\exp(ikR)\int_{-\infty}^{\infty}\mathrm{d}t\left\{\vec{n}\times\left(\vec{n}\times\vec{\beta}\right)\right\}\exp\left\{i\omega\left(t-\frac{\vec{n}\cdot\vec{r}(t)}{c}\right)\right\}$$

ω : Angular freq. of emitted radiation

R : Distance from origin of r to the observation point

$$E_\theta(\omega) = \sum_{n=1}^{\infty} i\sqrt{\frac{e^2k^2\lambda_0^2N_0^2}{32\pi^3\varepsilon_0^2c^2R^2}}\exp i(\psi_0 + kR \pm n\phi)\left(\frac{\sin \bar{k}\eta_0}{\bar{k}\eta_0}\right)\left(\frac{nk_0\cos\theta}{k\sin\theta} - \beta_1\sin\theta\right)J_n(p)$$
$$= \sum_{n=1}^{\infty} iC_\theta \exp i(\psi_0 + kR \pm n\phi)$$

$$E_\phi(\omega) = \sum_{n=1}^{\infty} \mp\sqrt{\frac{e^2k^2\lambda_0^2N_0^2}{32\pi^3\varepsilon_0^2c^2R^2}}\exp i(\psi_0 + kR \pm n\phi)\left(\frac{\sin \bar{k}\eta_0}{\bar{k}\eta_0}\right)\frac{a_0}{\sqrt{2}h_0}J_n'(p)$$
$$= \sum_{n=1}^{\infty} \mp C_\phi \exp i(\psi_0 + kR \pm n\phi)$$

$\eta_0 = N_0\lambda_0/2$

N_0 : Number of the period interacting with the elecctrons

Polarization and OAM characteristics

Electric field in the x-y plane

$$E = \frac{i}{\sqrt{2}}\left(C_\theta \cos\theta + C_\phi\right)\exp i\left\{\psi_0 + kR + \underline{(n-1)\phi}\right\}e_+ \qquad e_\pm = \frac{e_x \pm i e_y}{\sqrt{2}}$$

$$+ \frac{i}{\sqrt{2}}\left(C_\theta \cos\theta - C_\phi\right)\exp i\left\{\psi_0 + kR + \underline{(n+1)\phi}\right\}e_-$$

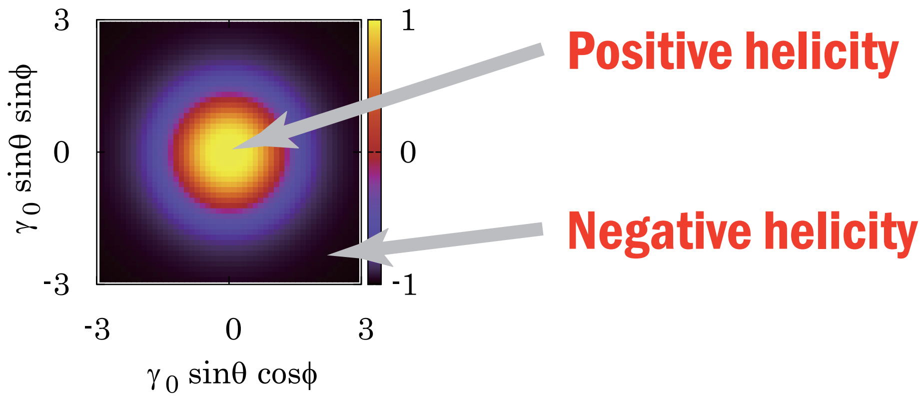
n: Harmonic number

Positive helicity carry (n-1)ħ OAM

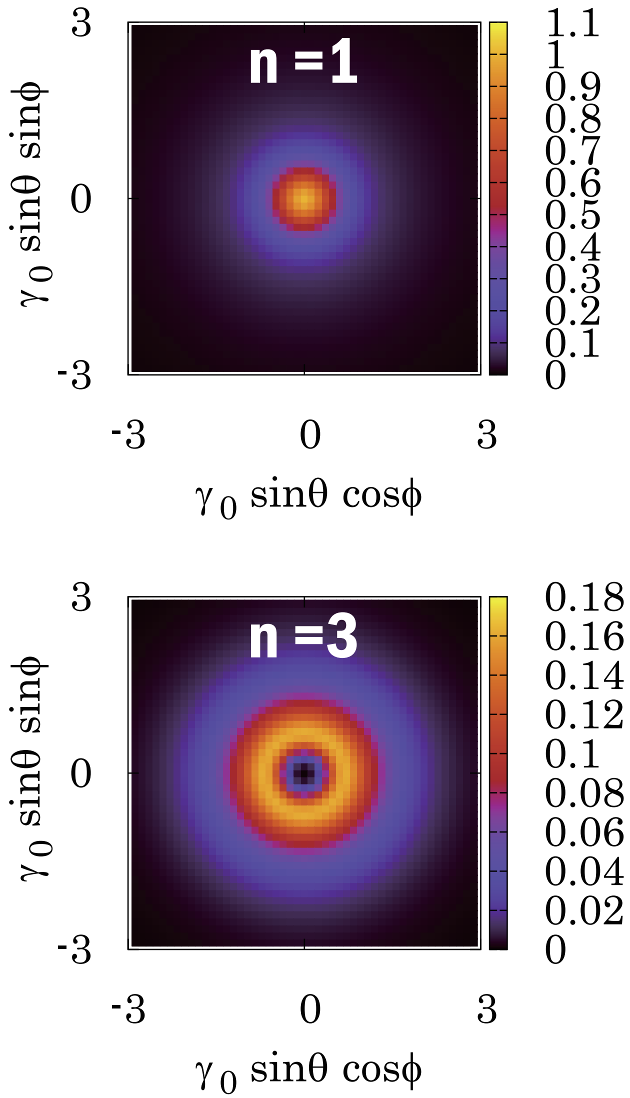
Negative helicity carry (n+1)ħ OAM.

Stokes parameter

$$\frac{S_3}{S_0} = \frac{\left(C_\theta \cos\theta + C_\phi\right)^2 - \left(C_\theta \cos\theta - C_\phi\right)^2}{\left(C_\theta \cos\theta + C_\phi\right)^2 + \left(C_\theta \cos\theta - C_\phi\right)^2} = \frac{2C_\theta C_\phi \cos\theta}{C_\theta^2 \cos^2\theta + C_\phi^2}$$



Spatial distribution



Annular profile of higher harmonic is due to the phase singularlity.

Characteristics of X-rays

Angle	$\theta < 0.6/\gamma_0$	$2.4/\gamma_0 < \theta < 2.47/\gamma_0$
Helicity	Positive helicity	Negative helicity
Fundamental (n = 1)		
N	6×10^{11} photons/sec	2×10^{10} photons/sec
E	11-13 MeV	2.6-2.7 MeV
OAM	0	$2\hbar$
Second (n = 2)		
N	2×10^{11} photons/sec	2×10^{10} photons/sec
E	21-26 MeV	5.2-5.5 MeV
OAM	$\hbar$	$3\hbar$

$a_0 = 1.0 \quad \gamma_0 = 2000 \quad \lambda_0 = 1.0\ \mu\text{m} \quad N_e = 10^9$ electrons/sec

$N_0 = 500$ **Pulse width of the laser = 1.7 ps**

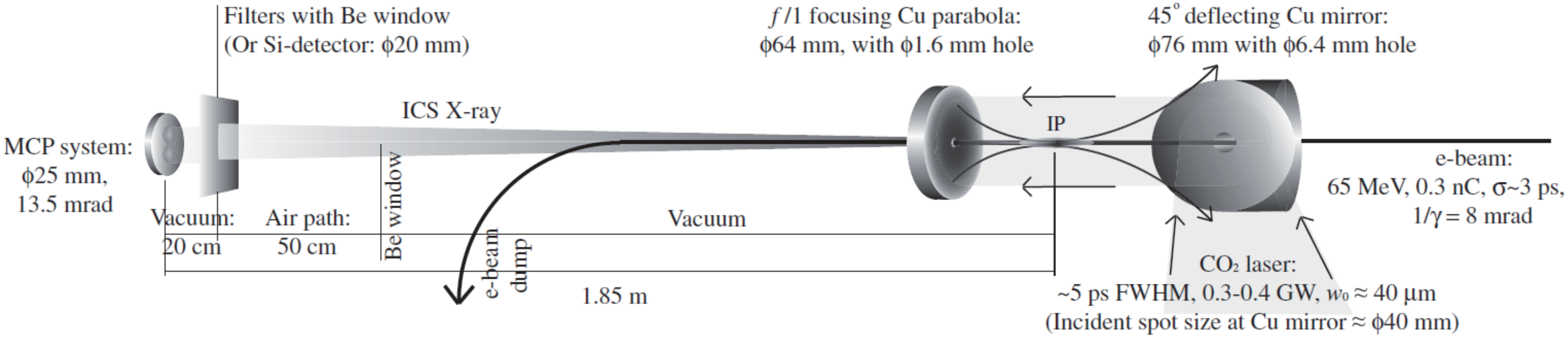
Where can we do experiment?

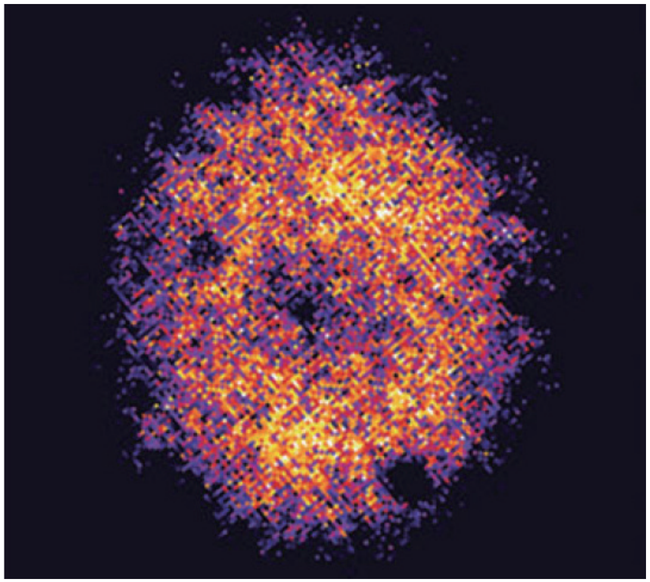
JLab ?

BNL ?

Second harmonic from nonlinear inverse Thomson scattering was observed.
But, they probably don’t realize it is X-ray vortex.

Nonlinear inverse Thomson X-rays at BNL





E = 13 keV (λ = 0.095 nm)

Annular shape of the second harmonics was measured. This will be X-ray vortex.

Electron beam

γ = 128

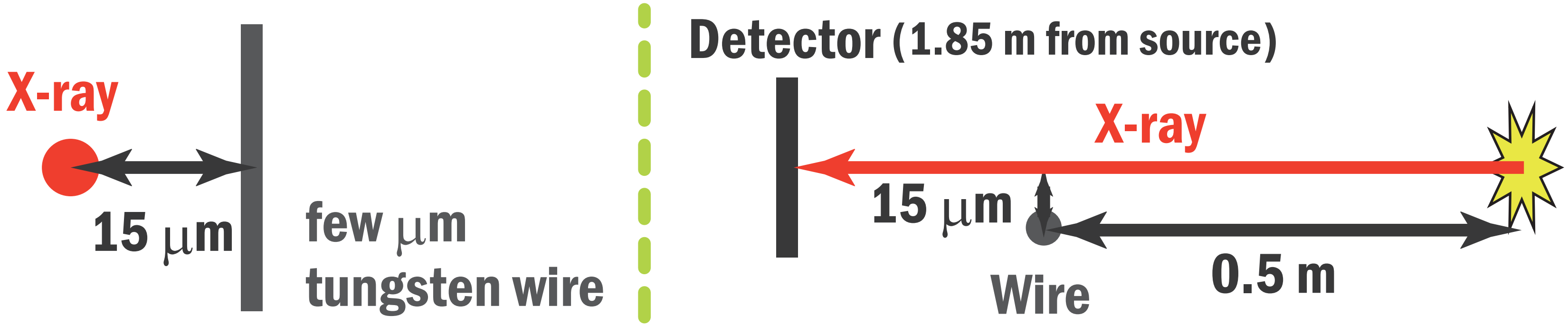
Laser

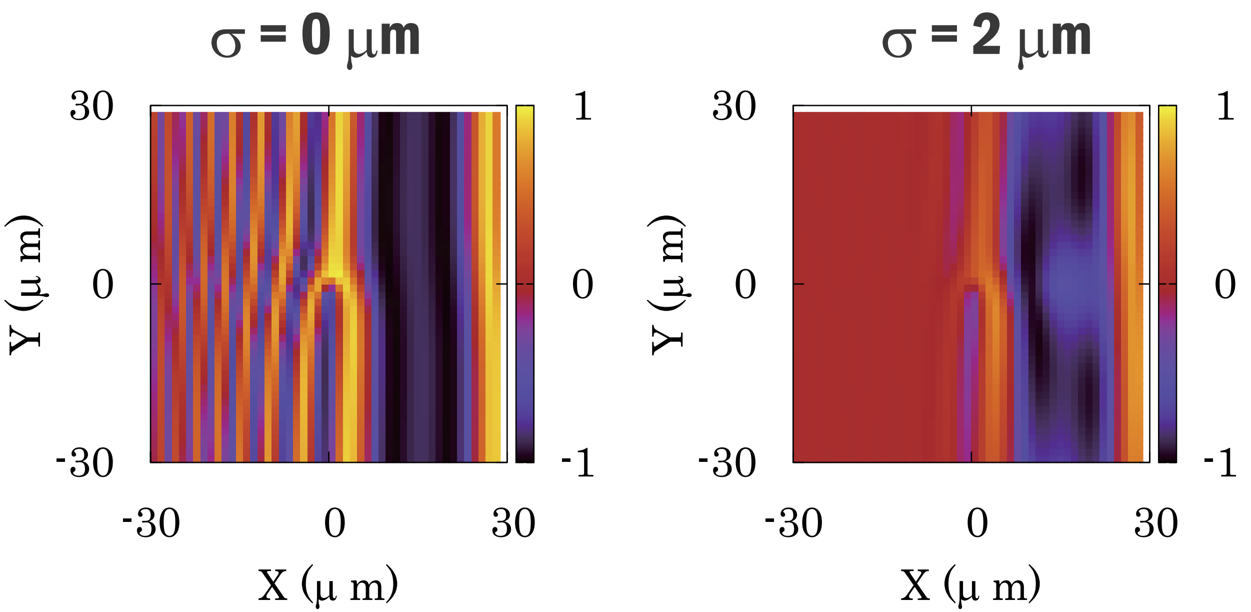
λ_0 = 10.6 μm, pulse energy = 2 J,
$2\omega_0$ = 100 μm, pulse width = 5 ps (FWHM),
I = 0.4 x 10^{16} W/cm^2, a_0 = 0.6

M. Babzien et al., PRL. 96 054802 (2006).
Y. Sakai et al., PRSTAB 18 060702 (2015).

Experimental demonstration

Interference pattern measurement between the incident X-ray vortex and diffracted X-ray from the wire.





σ: rms beam size of the electrons

Are X-ray vortices generated at universe?

Crab Nebula, Wikipedia

X-ray vortex can be generated if relativistic electron and intense circularly plarized electromagnetic wave coexist. For example, neutron star, quasar, and supernova.

Conclusion

- X-ray vortex can be generated by nonlinear inverse Thomson scattering of the intense circularly polarized laser.
- Second harmonics of the nonlinear inverse Thomson scattering was observed at BNL.
- A candidate of the experimental demonstration is an interference measurement of the X-ray vortex using a metal wire.