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To my knowledge, the focus of theoretical errors in the calculations may come from:

1) Numerical accuracy. The **convergence of the partial wave series** may not always reach a very high accuracy. Regarding convergence, I expect a much better accuracy in the theoretical calculations than the one required for the calibration of the polarimeter. **I can estimate this error,** in any case. *(Please remind me, the goal accuracy for the calibration of the polarimeter required a 1% or smaller theoretical error for the analyzing power?)*

2) Nuclear Recoil: there exist a **kinematic and dynamic corrections due to the recoil of the nucleus.**

The kinematic corrections due to the finite mass of the target nucleus M can be taken into account just by a kinematic factor in the cross section measured in the Lab [see for example F. Gross, "Relativistic Quantum Mechanics and Field Theory, (Wiley 99)" or *Review of Modern Physics* **36** (1964) 881]. I expect the relative change in the **analyzing power** due to the correction to be at most of the same order as the relative change in the differential cross section ($\Delta\sigma/\sigma$). Specifically, if the **electron beam energy is much smaller than the mass of the target divided by the mass of the projectile** $E_e \ll M/m$ the kinematic recoil is negligible. Considering $M \sim 2000\,Am$, where A is the nuclear mass number, $E_e \ll 2000\,A \sim 10^4$ MeV for medium mass nuclei. Therefore, $E_e \leq 10$ MeV may be enough for ensuring a negligible recoil. *(In addition, one may expect that GEANT4 takes care of this effect but this I do not know).*

The so called **dynamic correction** is due to the **change in the phase shift of the outcome electron wave function produced by the nuclear recoil** [see *Physical Review* **113** (1959) 1147]. This effect has been seen to produce a **relative change in the cross section** proportional to the fine structure constant times the charge of the nucleus [$\alpha Z \sim 50/(2\times137) \sim 0.2$ assuming $Z \sim A/2$] multiplied by $m/E_e \approx 0.5/10 = 0.05$ and $m/M \approx 1/(2\times10^3\times50) = 10^{-5}$, that is approximately, a relative change of the order of $0.2\times0.05\times10^{-5} \sim 10^{-7}$. Note that the **dynamical correction depends on the inverse of the initial electron beam energy and would be negligible for** $m \ll E_e$ **even though** m **is not much smaller than** M.

On the other side (see point **1**), one may expect some inaccuracies in the theoretical calculations where big cancellations in summing the series to obtain the scattering amplitudes have to be done. Even though these inaccuracies may deteriorate the relative accuracy, I would not expect the nuclear dynamical recoil correction to be crucial for the calibration of the polarimeter.

3) Radiative effects. Bremsstrahlung. GEANT4 simulations take into account this effect *(isn't it?)*.[See also *Review of Modern Physics* **36** (1964) 881].

In the experiment, all scattered electrons by a Coulomb field emit photons with a probability of unity! That is, elastic electron scattering (with no photon emission) is an idealized process that requires radiative corrections when compared to experimental data.

At most, the **energy radiated** should **not exceed** the electron energy $E_e = 10$ MeV in the Lab frame. Specifically, large electron beam energy and heavy nuclei increase the importance of Bremsstrahlung radiation.

4) Radiative corrections to atomic parity violation: this corrections are thought to be not relevant for parity conserving analyzing power.

5) Finite nucleus charge distribution and point nucleus: calculations considering both situations can be done. For **few MeV electrons, the fine details of the nuclear charge distribution are not relevant.** One may use existent experimental charge densities parametrized with simple forms (such as Fermi distribution) for the calculations.

6) Screening of atomic electrons: the screening can be neglected for electrons of few MeV or more (may be except for very small scattering angles $\theta \sim 1^o$.).

7) Interesting (recent) literature on electron scattering from spin-1/2 and spin-0 nuclei. May be you know already this works. I give you here the references and do not summarize what it was done since I have not carefully read them yet. I wanted to bring into your attention such references with enough time before the meeting on January 30th in order to let you have a look to them: *Physical Review* C **87**, 064609 (2013) / *J. Phys. G: Nucl. Part. Phys.* **39** (2012) 025102 / *Nuclear Physics* A **896** (2012) 59-73

Just as an example, note that if spin-1/2 nuclei are used, magnetic transition amplitudes may have some influence in the cross section and analyzing power depending on the kinematics of the experiments.

8) Sum rule $S+P+L=1$ [see for example *Review of Modern Physics* **36** (1964) 881]. In terms of the direct $f(\theta)$ and spin-flip $g(\theta)$

amplitudes,

$\frac{d\sigma}{d\Omega}=|f|^2+|g|^2$; $S=i\frac{fImg(g)-Img(f)g}{|f|^2+|g|^2}$; $T=\frac{|f|^2-|g|^2}{|f|^2+|g|^2}$; $U=\frac{fImg(g)+Img(f)g}{|f|^2+|g|^2}$

and

$P=U\cos\theta-T\sin\theta$; $L=U\sin\theta+T\cos\theta$

Therefore, one can rewrite the sum rule as,

$S+(\cos\theta+\sin\theta)U+(\cos\theta-\sin\theta)T=1$

Can S, U and T be measured independently? If so, you have more than probably thought about measuring S on one side and U and T on the other side since it would allow for two independent measurements of S thanks to the sum rule, isn't it?. Would be useful to take this into consideration for improving the statistical error? In addition, it also provides a cross-check for the theoretical calculations.