Talman SPIN2016 comments

The reference is to Talman’s talk at SPIN 2016, available here [https://indico.cern.ch/event/570680/contributions/2310168/attachments/1341808/2026008/Talman_Spin2016-talk.pdf](https://indico.cern.ch/event/570680/contributions/2310168/attachments/1341808/2026008/Talman_Spin2016-talk.pdf).

- Talman denotes the lab frame by $K$ and the electron rest frame by $K'$. But $K'$ (as Talman employs it) is not the rest frame. To clarify concepts, I define two terms: “longitudinal” means “parallel to the particle momentum (velocity)” and “axial” means “parallel to the beamline reference axis.” I employ a coordinate system $(x,y,s)$. The boost from $K$ to $K'$ is an axial boost, such that $p_x(K') = 0$ (or $v_x(K') = 0$). However the particle still has a nonzero momentum $p_{x,y}(K') = p_{x,y}(K)$. What this means is that there is still a $v \times E$ term for the spin-orbit interaction in the frame $K'$.

- More generally, for relativistic particles the Stern-Gerlach force is not proportional to the magnetic dipole moment. The coupling to the spin is given by the same $\Omega \cdot s$ Hamiltonian which yields the BMT equation. Writing $a = (g - 2)/2$, the transverse magnetic fields in $\Omega \cdot s$ are multiplied by the coefficient $(\gamma a + 1)$ and longitudinal magnetic fields are multiplied by the coefficient $(a + 1)$. These are well known facts. They apply equally to the relativistic Stern-Gerlach force. Expressing matters in the lab frame $K$, the equations of motion for the momenta $p_{x,y}$, for the coupling to the spin, are

$$\frac{dp_x}{dt} = -\left(\frac{\partial \Omega}{\partial x}\right) \cdot s, \quad \frac{dp_y}{dt} = -\left(\frac{\partial \Omega}{\partial y}\right) \cdot s. \quad (1)$$

- Consider motion through a quadrupole and treat only $p_x$ below. Then to a sufficient approximation for this note, with $-eB/(p_sc) = K_1(xe_y + ye_x)$,

$$\Omega = -\frac{e}{mc} \frac{\gamma a + 1}{\gamma} B = (\gamma a + 1)v_sK_1(xe_y + ye_x). \quad (2)$$

Then

$$\frac{dp_x}{dt} = -\left(\frac{\partial \Omega}{\partial x}\right) \cdot s = -(\gamma a + 1)v_sK_1 s_y. \quad (3)$$

1 Or Thomas-BMT or Thomas-Frenkel-(anyone else you like)-BMT equation.
Say the quadrupole length is $L_q$. The time to transit the quadrupole is $\Delta t = L_q/v_s$. Then, approximately, the Stern-Gerlach momentum kick upon traversing the quadrupole is

$$\Delta p_{x}^{SG} \simeq \frac{dp_x}{dt} \Delta t \simeq -(\gamma a + 1)L_q K_1 s_y. \quad (4)$$

The angular deflection is

$$\Delta \theta_x^{SG} = \frac{\Delta p_{x}^{SG}}{p_s} \simeq -\gamma a + 1 \frac{L_q K_1 s_y}{m \gamma v_s}. \quad (5)$$

- Talman’s expression is

$$\left(\Delta \theta_x^{SG}\right)_{\text{Talman}} = -\frac{\mu_x^*}{e\beta_s} q_x. \quad (6)$$

Here $q_x = 1/f$ is the inverse focal length. (I have changed Talman’s “$v$” to $\beta_s$, see below.)

- Let us compare the two expressions. There are some issues about normal and skew quadrupoles, but that is not essential here. Talman states that “$\mu_x^*$ and $\mu_y^*$ differ from the Bohr magneton (not magnetron) $\mu_B$ only by $\sin \theta$ and $\cos \theta$ factors respectively.” Here $\theta = \pi/4$ and is not important. Then, approximately, $L_q K_1 \simeq 1/f = q_x$. Also $\mu_x^* \simeq (e/mc)s_x$, up to factors of $1/\sqrt{2}$. Then I obtain

$$\Delta p_{x}^{SG} \simeq -\gamma a + 1 \frac{q_x s_y}{\gamma m v_s}. \quad (7)$$

Talman’s expression is

$$\left(\Delta \theta_x^{SG}\right)_{\text{Talman}} = -\frac{1}{mc\beta_s} q_x s_x = -\frac{q_x s_x}{mv_s}. \quad (8)$$

See my comment above about $\beta_s$. I ignore global minus signs and the fact that my expression has $s_y$ and Talman’s contains $s_x$; this may be because I treated a normal quadrupole and Talman treated a skew quadrupole. Both “$s_x$” and “$s_y$” are simply spin components, of $O(\hbar/2)$. For the CEBAF injection line, $E \simeq 123$ MeV so $\gamma \simeq 240$ (and $\gamma a \simeq 0.25$, which can be ignored). Hence overall

$$\Delta p_{x}^{SG} \simeq \frac{1}{\gamma m v_s} \frac{q_x}{2}, \quad \left(\Delta \theta_x^{SG}\right)_{\text{Talman}} \simeq \frac{q_x}{mv_s} \frac{h}{2}. \quad (9)$$

- The angular deflection due to the Stern-Gerlach force is a factor $1/\gamma$ relative to that derived by Talman. For a 123 MeV beam line this is about a factor of 240.

- The effects of the Lorentz boosts have not been calculated correctly. For relativistic particles, the Stern-Gerlach force is not proportional to the magnetic dipole moment. There may be other errors.