Twisted Electrons Meeting

March 4, 2015

J. Grames

- Goal is to make connection of TEM to GaAs source in terms of…
  - Brightness
  - Spatial coherence
  - Temporal coherence

- First thing I "gave up" is idea of multiple electrons interacting at same time…

  I=1 nA
  T=100 keV ($\beta \sim 0.55$)

  $n_e \sim (\beta * c)/(I/e) = (0.55 * 2.99 \times 10^8$ m/s) / (1×10^{-9} C/s * e-/1.6×10^{-19} C)
  $n_e \sim$ 26 millimeters/electron

- What follows are some preliminary investigations and thoughts…

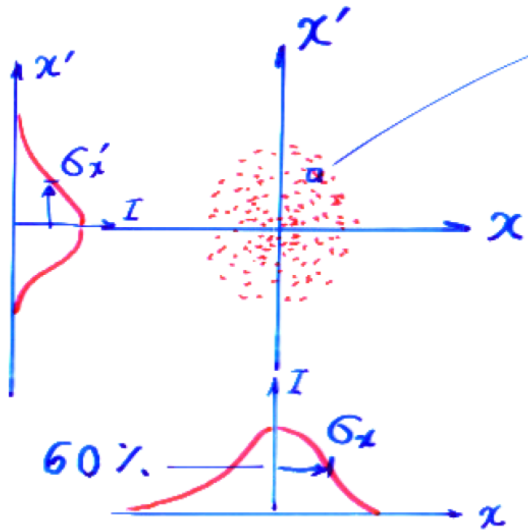
Emittance Definition

We use Normalized r.m.s. emittane

$$\varepsilon_n = \gamma\beta\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

$$= \frac{1}{m_0 c}\sqrt{\langle x^2\rangle\langle P_x^2\rangle - \langle x P_x\rangle^2}$$

Gaussian Beam



$$dN = \frac{Ne}{2\pi\sigma_x\sigma_x'} e^{-\left(\frac{x^2}{2\sigma_x^2} + \frac{x'^2}{2\sigma_x'^2}\right)} \cdot dx\,dx'$$

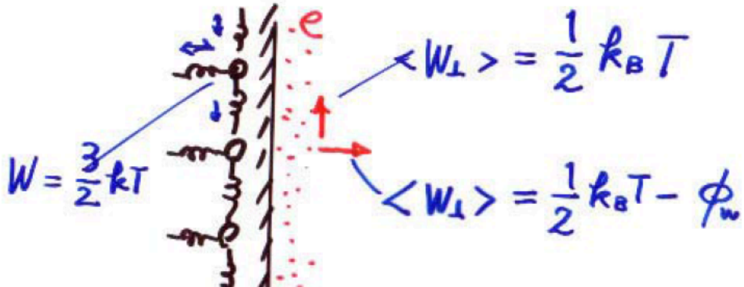
$$\langle xx'\rangle^2 = 0$$

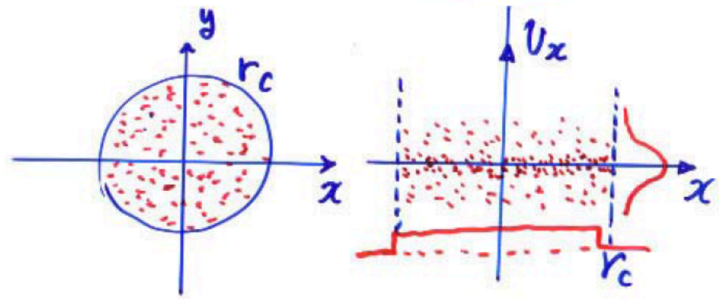
$$\langle x^2\rangle = \frac{\iint x^2\,dN}{\iint dN} = \cdots = \sigma_x$$

$$\langle x'^2\rangle = \frac{\iint x'^2\,dN}{\iint} = \cdots = \sigma_x',$$

$$\varepsilon_n = \gamma\beta\,\sigma_x\sigma_x' \iff \text{Area}: A = \pi\,\sigma_x\sigma_x'$$

Thermal Emittance (lower limit)





Transverse Energy

$$\frac{1}{2} m_0 \langle v_x^2 \rangle = \frac{1}{2} k_B T$$

Transverse Momentum

$$\sqrt{\langle p_x^2 \rangle} = \gamma m_0 \sqrt{\langle v_x^2 \rangle}$$
$$= \gamma \sqrt{m_0 k_B T}$$

r.m.s Cathode Size

$$\langle x^2 \rangle = \frac{\int x^2 dS}{\int dS}$$
$$= \frac{r_c^2}{4}$$
$$\sqrt{\langle x^2 \rangle} = r_c/2$$

Thermal Emittance

$$\varepsilon_{xN} = \frac{1}{m_0 c} \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle - \langle x p_x \rangle^2}$$

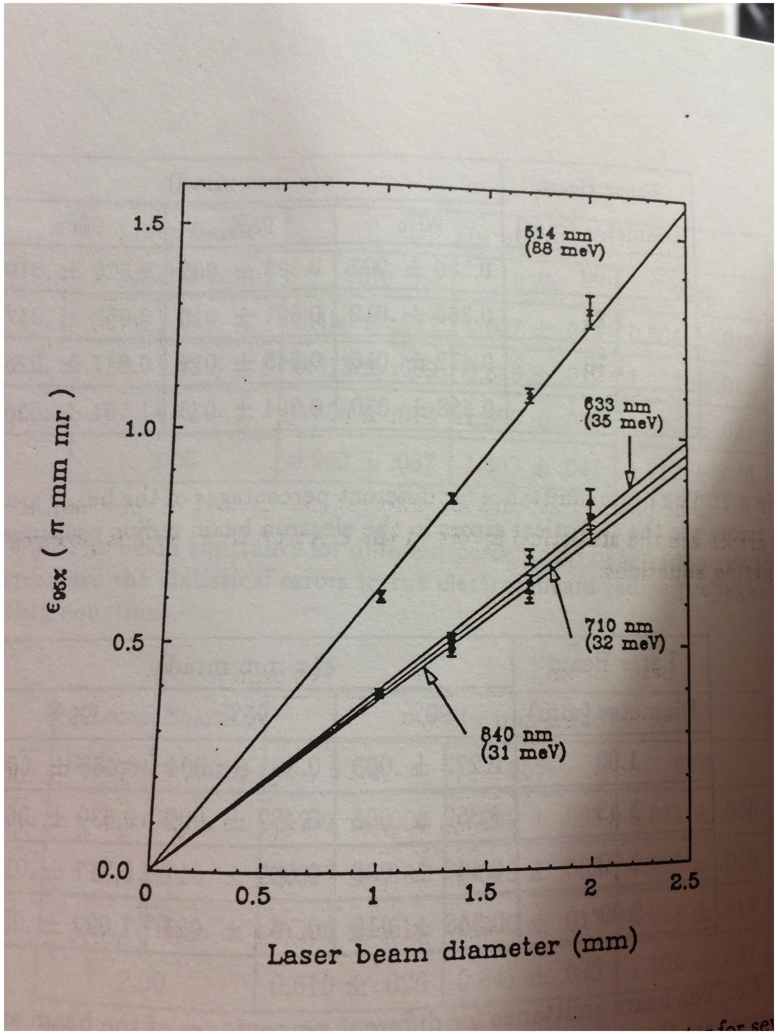
$$= \frac{\gamma r_c}{2} \sqrt{\frac{k_B T}{m_0 c^2}}$$

Boltzmann constant

$$k_B = 1.38 \times 10^{-23} \ (J/\deg)$$

Tsumoru Shintake

Thermal emittance of Bulk GaAs at room temperature



$E_T \sim (\varepsilon_n/r_c)^2$

λ (nm)	E_T (meV) 90%	95%	99%
840	16 ± .2(± 2)	31± .5(± 3)	53 ±.8(± 4)
710	17 ± .6(± 2)	32± .6(± 3)	62 ±1(± 4)
633	19 ± .2(± 2)	35± .5(± 3)	63 ±9(± 4)
514	48 ± 1(± 4)	88± 2(± 6)	153 ±3(± 10)

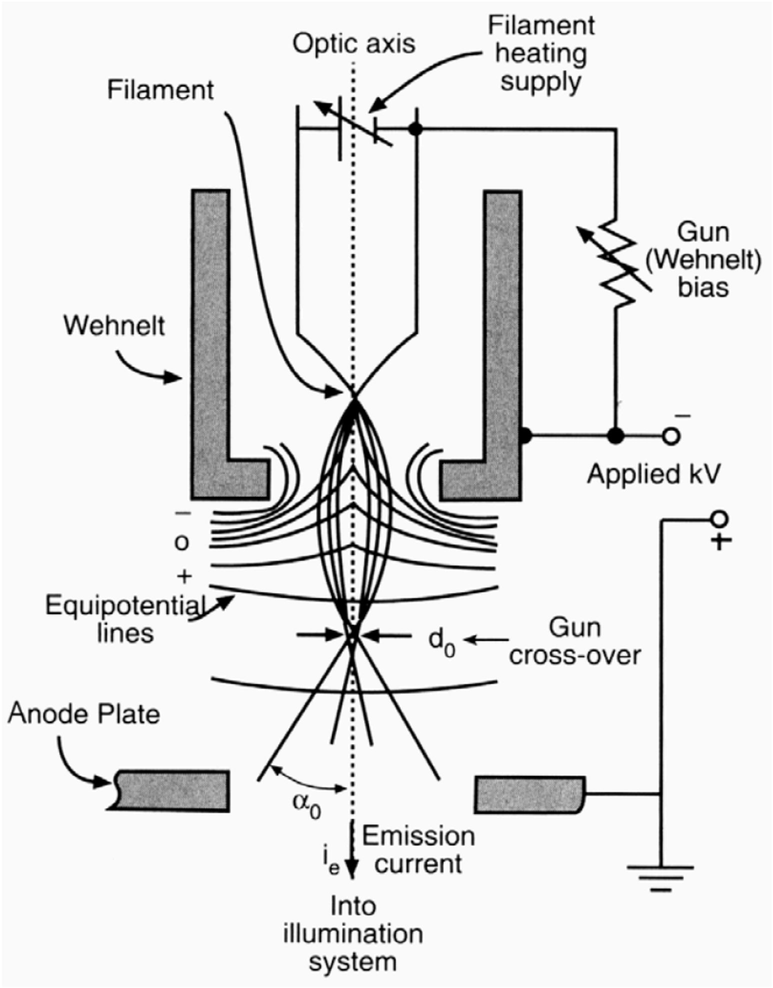
Bruce Dunham, PhD thesis (1993)

Brightness = Current density per unit solid angle = [A / (m^2-rad^2)]

$$\beta = \frac{i_e}{(\pi d_o \alpha_o)^2}$$

Brightness ($A/m^2 \cdot sr$)

Thermionic	**10^9**
Schottky	**$5 \cdot 10^{10}$**
Cold field emission	**10^{13}**



Connection to accelerator beam brightness

Brightness = Current density per unit solid angle = [A / (m^2-rad^2)]

Similar definition: $B = J/d\Omega = dI/(dS\, d\Omega)$

Average brightness: $\langle B \rangle = I/V_4$ is intensity of (x,x',y,y') phase sub-space

In terms of emittance: $\langle B \rangle = 2Ie / (\pi^2 \varepsilon_x \varepsilon_y)$ where at source often $\varepsilon_x = \varepsilon_y$

Normalizing to momenta in an accelerator is useful…

- Normalized emittance: $\varepsilon_n = \beta\gamma\, \varepsilon$
- Normalized brightness: $B_n = 2I_e / \pi^2 \varepsilon_n^{\,2}$

One can then write the normalized brightness for emission in terms of a current density specified as $J_c = I/(\pi r_c^{\,2})$ to determine a normalized brightness:

$B_n = (J_c/2\pi) * (mc^2/k_B T)$

- Using upper bound thermal emittance: $k_B T \sim 0.1$eV (GaAs)
- Using 100 nA in a $r_c \sim 10\mu$m => $J_c \sim 0.03$ A/cm^2

$B_n \sim 2 \times 10^8$ A/(m-rad)2

Emission physics
summary

	β (A/m^2sr)	ΔE (eV)	d	Vacuum (Pa)
W	10^9	1.5 - 3	20 - 50 μm	10^{-3}
LaB_6	$5 \cdot 10^9$	1 - 2	10 - 20 μm	10^{-4}
Schottky FEG	$5 \cdot 10^{10}$	0.7	15 nm	10^{-6}
Cold FEG	10^{13}	0.3	2.5 nm	10^{-8}

GaAs 2×10^8 0.1 20 um

x10: Try to reduce energy spread from 0.10 to 0.01 eV by choosing wavelength ?
x100: Increase beam intensity but choose a 1% duty factor to remain 100 nA ?

Spatial (Transverse) Coherence

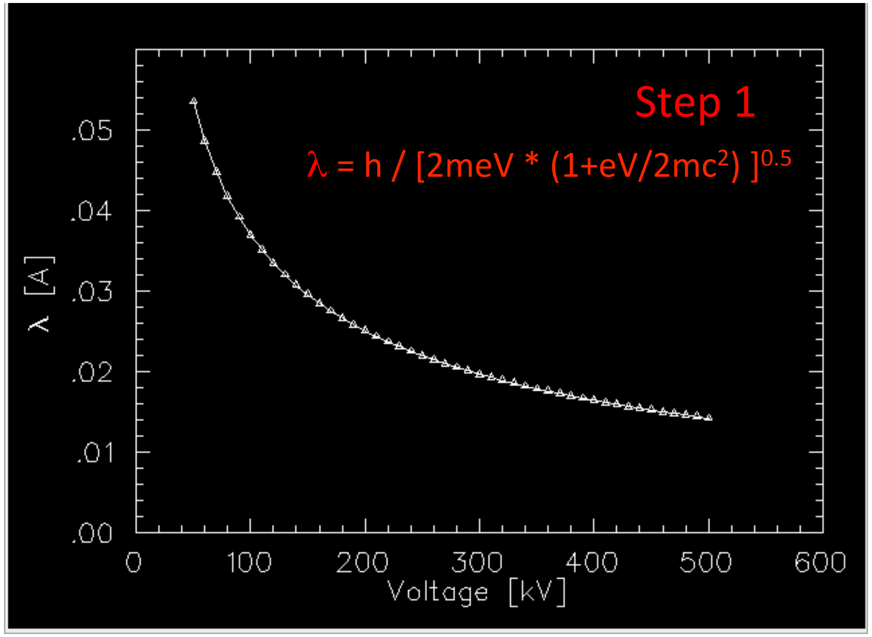
Spatial coherence is related to source size.

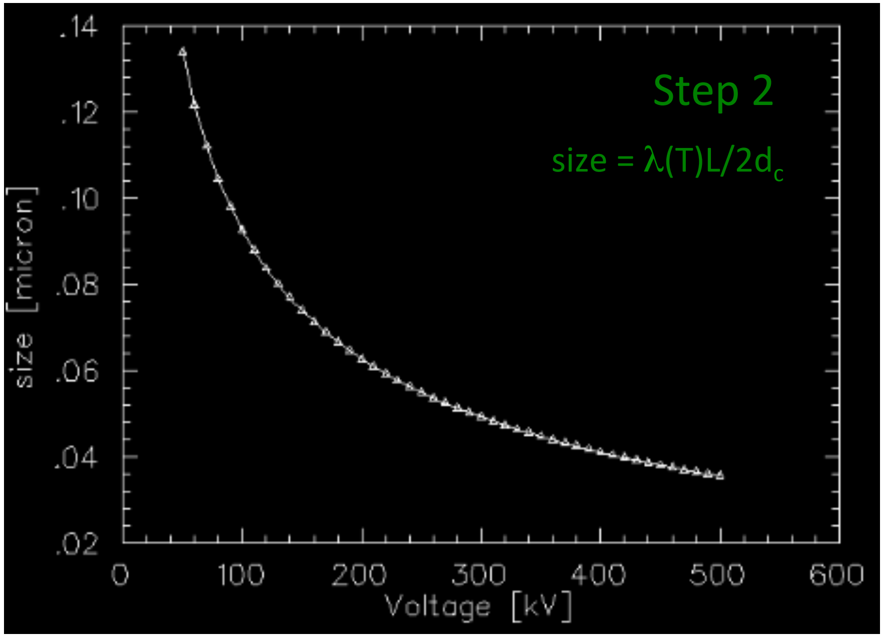
$d_c = \lambda(T)/2\alpha \quad \Rightarrow \quad \alpha = \lambda(T)/(2d_c)$

- d_c source size (spot/tip)
- α subtends specimen = grating/distance ?

Step 1. $pc = (T^2 + 2Tmc^2)^{0.5} = hc/\lambda$

Step 2. Assuming d_c = 20 μm laser what is grating at 1 meter ?





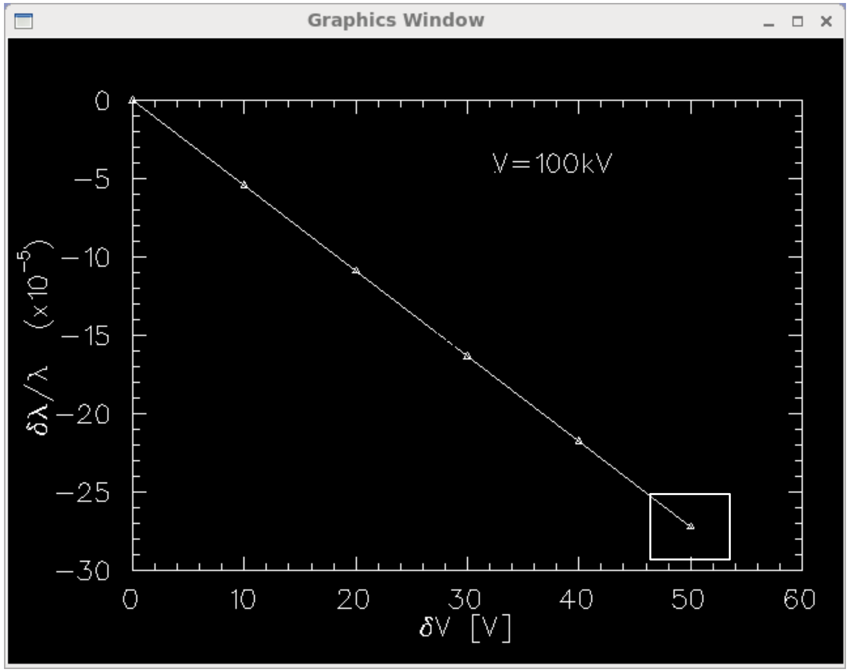
Temporal Coherence Length

My interpretation thinking in terms of $\delta\lambda/\lambda$:

T = 100,000 V

λ ~ 0.037 A

ΔE = 50 eV => $\delta\lambda/\lambda \sim -2.7\text{x}10^{-4}$



TEM defines it as: $\lambda_c = vh/\Delta E$

T = 100,000 V (β=0.548)

ΔE = 50 eV

λ_c = (0.548 * 2.99x10^8 * 6.626x10^{-34} J-s) / 50 eV = 128 A

So, I've made some progress but still need to read more to appreciation spatial and temporal coherence.