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# Rectangular dipole matrix

- The rectangular dipole matrix is defined as :

$$M_{dipole}(\rho\theta) = \begin{bmatrix} \cos \theta & \rho \sin \theta & 0 & \rho(1 - \cos \theta) \\ -\frac{1}{\rho} & \cos \theta & 0 & \sin \theta \\ -\sin \theta & -\rho(1 - \cos \theta) & 1 & (\frac{\rho\theta}{\gamma^2}) - \rho(\theta - \sin \theta) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- $L_{dipole} = \rho\theta$
- $\rho$  is the bend radius.
- $\theta$  is the bend angle.
- $[y, y']$  and  $[x, x', z, \delta]$  elements are decoupled.

# Sector dipole matrix

- The sector dipole matrix is defined as :

$$M_{dipole}(\rho\theta) = \begin{bmatrix} \cos \theta & \rho \sin \theta & 0 & 0 \\ -\frac{1}{\rho} & \cos \theta & 0 & 0 \\ ? & ? & 1 & (\frac{\rho\theta}{\gamma^2}) - \rho(\theta - \sin \theta) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- $L_{dipole} = \rho\theta$
- $\rho$  is the bend radius.
- $\theta$  is the bend angle.
- $[y, y']$  and  $[x, x', z, \delta]$  elements are decoupled.

# Achromaticity condition

To simplify the mathematics we apply:

$$\text{Achromaticity criterion : } D = \begin{bmatrix} \eta_{x \text{ exit}} \\ \eta'_{x \text{ exit}} \end{bmatrix} = \begin{bmatrix} \eta_{x \text{ entrance}} \\ \eta'_{x \text{ entrance}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$D_{\text{exit}} = \begin{bmatrix} M_x & 0 \end{bmatrix} \times D_{\text{entrance}} + \begin{bmatrix} R_{16} \\ R_{26} \end{bmatrix}$$

$$R_{16} = R_{26} = 0$$

# Achromaticity condition

$$M_{chicane} = \begin{bmatrix} 1 & R_{12} & R_{13} & R_{14} & R_{15} & 0 \\ R_{21} & 1 & R_{23} & R_{24} & R_{25} & 0 \\ R_{31} & R_{32} & 1 & R_{24} & R_{25} & 0 \\ R_{41} & R_{42} & R_{43} & 1 & R_{25} & 0 \\ 0 & 0 & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z_{exit\ chicane} = R_{55}z_0 + R_{56}\delta_0$$

$$\Delta z = R_{56}\delta_0$$

# Longitudinal beam chirp

- Using  $z$  &  $\frac{\Delta P}{P}$  space, we have:

$$\kappa = \frac{d\delta_p}{dz} = \frac{-keV_0}{E_0 + eV_0 \cos \phi} \sin \phi$$

- $k = 2\pi \frac{f}{c}$  [ $m^{-1}$ ]
- $f$  is the cavity frequency
- $eV_0$  Cavity acceleration [MeV]
- $E_0$  Central energy [MeV]
- $\phi$  Cavity phase advance.

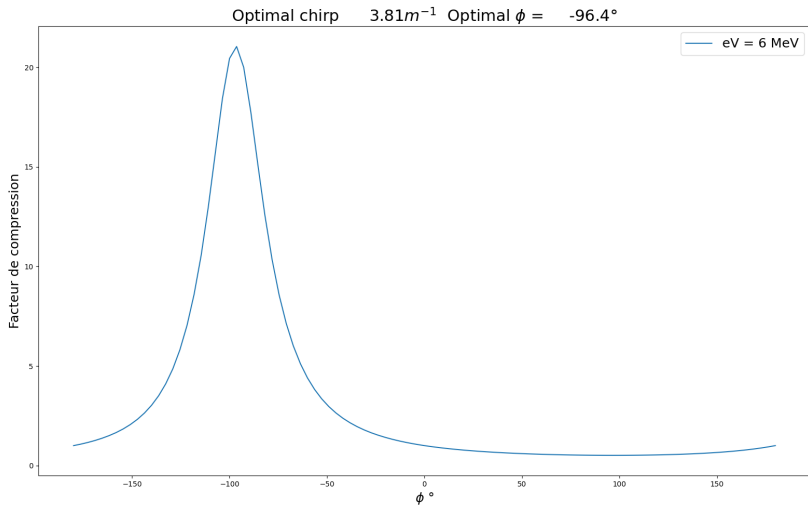
- Compression factor

$$C = \frac{1}{1 + [R_{56} \times \kappa]}$$
$$C = \frac{1}{1 + \left[ R_{56} \times \frac{-keV_0}{E_0 + eV_0 \cos \phi} \sin \phi \right]}$$



# Compression factor

- $R_{56} = -0.25 \text{ m}$



# Beam size along the chicane

- How to reduce the beam size along the chicane?

- **Answer : FODO**

- **Motivation:**  $\frac{\Delta P}{P_0} = \pm 10\%$

- **Focusing quadrupole =**

$$\begin{bmatrix} \cos \sqrt{K}L_q & \frac{1}{\sqrt{K}} \sin \sqrt{K}L_q \\ -\sqrt{K} \sin \sqrt{K}L_q & \cos \sqrt{K}L_q \end{bmatrix}$$

- **Defocusing quadrupole =**

$$\begin{bmatrix} \cosh \sqrt{K}L_q & \frac{1}{\sqrt{K}} \sinh \sqrt{K}L_q \\ -\sqrt{K} \sinh \sqrt{K}L_q & \cosh \sqrt{K}L_q \end{bmatrix}$$

- **FODO**

$$M_{FODO} =$$

$$M_{HALF\ QF} M_{DRIFT} M_{QD} M_{DRIFT} M_{HALF\ QF}$$

- **Initial FODO parameters**

- Focusing Quadrupole strength  $K_{QF} = 0.6 \text{ m}^{-2}$
- Quadrupole length  $L_Q = 0.2 \text{ m}$
- Defocusing quadrupole strength  $K_{QDF} = ?$

- **Drift parameter:**

- Drift length  $L_{drift} = 5.6 \text{ m}$

- **Motivation** Apply the periodicity condition on the FODO lattice to

get : 
$$\begin{bmatrix} \beta_{exit} \\ \alpha_{exit} \\ \gamma_{exit} \end{bmatrix} = \begin{bmatrix} \beta_{entrance} \\ \alpha_{entrance} \\ \gamma_{entrance} \end{bmatrix}$$

- $\beta$   $\alpha$  and  $\gamma$  are the twiss parameters of the beam which describes the behaviour of the optics along the lattice.
- In periodic system, for stability of the equation of the motion we have :

$$|\text{trace}(M)| < 2$$

- If the FODO matrix is given by :

$$M(s_1 s_2) = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix}$$

- The transformation matrix from point  $s_1$  to  $s_2$  in the lattice is given by :

$$\begin{bmatrix} \beta_{s2} \\ \alpha_{s2} \\ \gamma_{s2} \end{bmatrix} = \begin{bmatrix} C^2 & -2SC & S^2 \\ -CC' & SC' + S'C & -SS' \\ C'^2 & -2S'C' & S'^2 \end{bmatrix} \begin{bmatrix} \beta_{s1} \\ \alpha_{s1} \\ \gamma_{s1} \end{bmatrix}$$

- From the stability condition:

$$|\text{trace } M(s_1 s_2)| = C + S' < 2$$

We get :

$$K_{QDF} = -1.096 \text{ m}^{-2}$$

at the beginning of the FODO :  $\alpha = 0$ , then we have  $\beta = \beta_{\text{max}}$  at the center of the focusing quadrupole.

- The FODO matrix become :

$$M_{FODO} = \begin{bmatrix} 0.95 & 6.59 \\ -0.014 & 0.95 \end{bmatrix}$$

- With  $\alpha = 0$  then we have  $\beta = \beta_0$  and  $\gamma = \frac{1}{\beta_0}$ , then Using the transformation matrix:

$$\beta_0 = 11.6 \text{ m}$$

- We define the phase advance matrix per cell:

$$\begin{bmatrix} \cos \phi + \alpha \sin \phi & \beta \sin \phi \\ -\gamma \sin \phi & \cos \phi - \alpha \sin \phi \end{bmatrix}$$

- We can immediately get the phase advance :

$$\cos \phi = 0.95 \quad (1)$$

$$\phi = \arccos 0.95$$

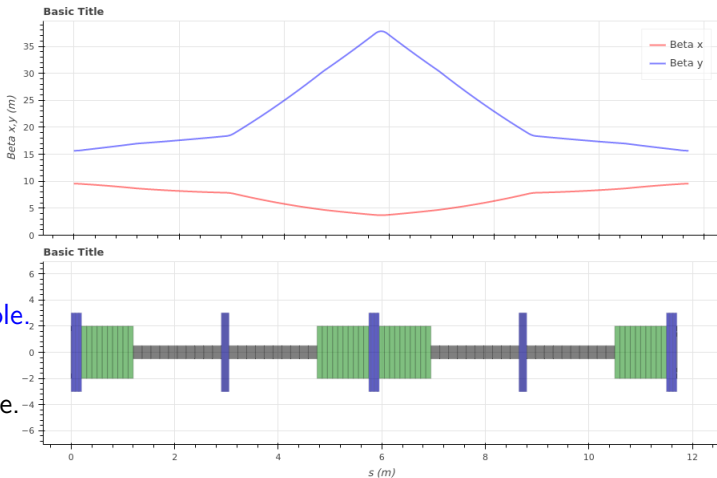
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# ELEGANT Results

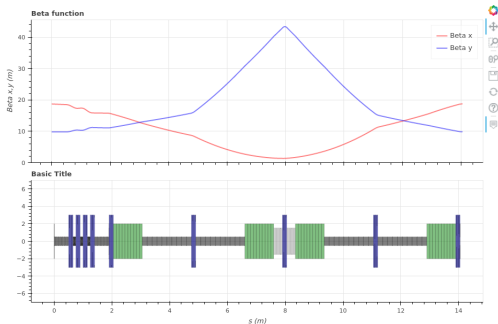
- Layout :



- Quadrupole.
- Dipole.
- Beam pipe.

# ELEGANT Results with matching section

- Layout :

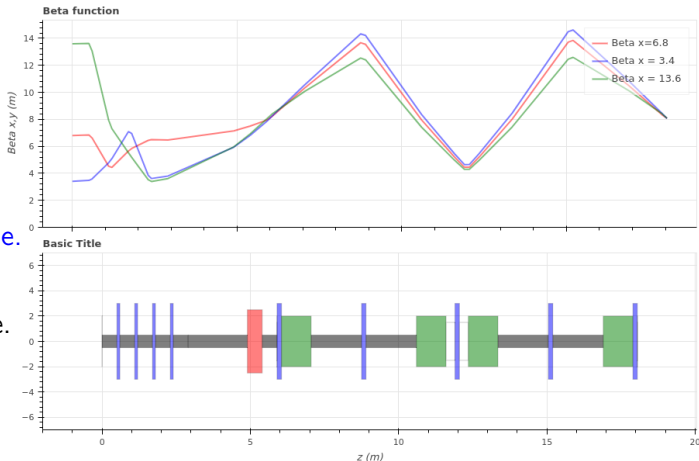


- Quadrupole.
- Dipole.
- Beam pipe.



# ELEGANT Results with matching section

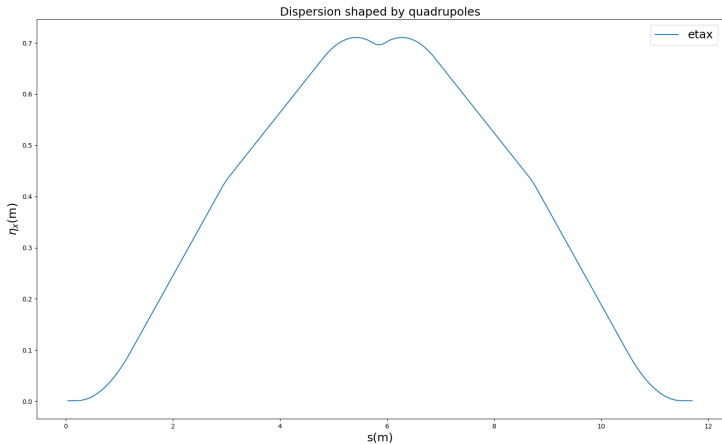
- Layout :



- Quadrupole.
- Dipole.
- Beam pipe.
- Cavity.

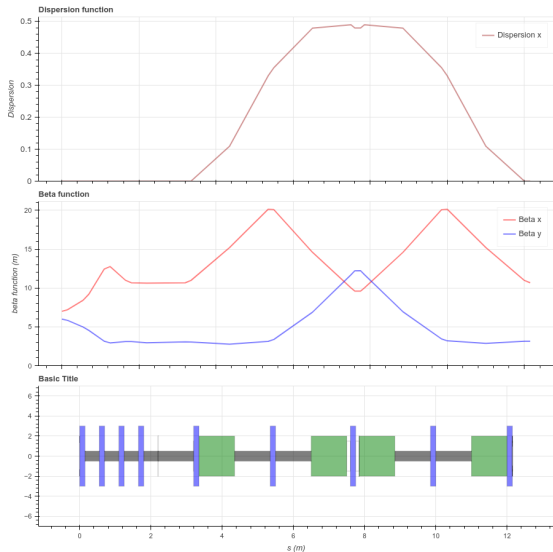
# Dispersion

- Dispersion peak at the middle of the chicane.

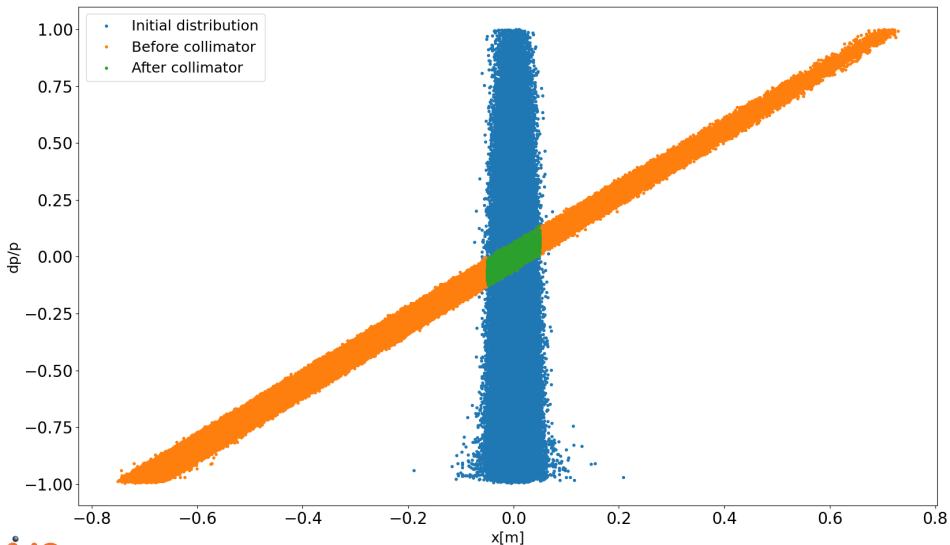


# ELEGANT Results with matching section

- Layout :

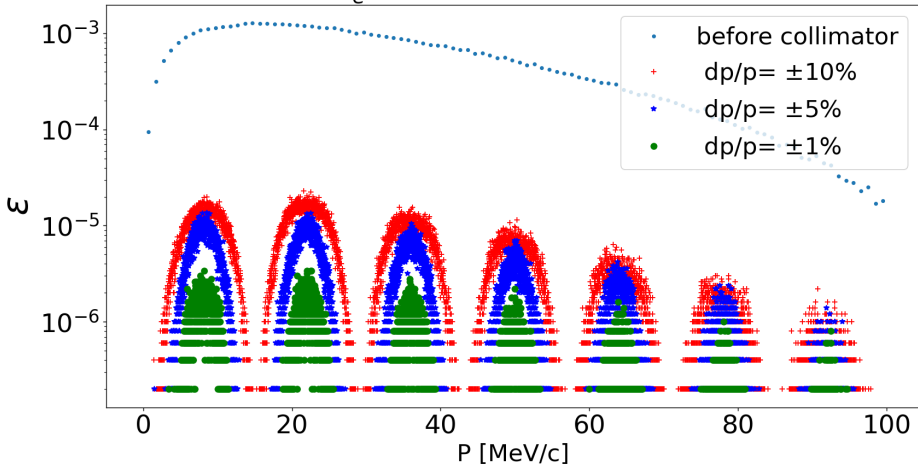


- Quadrupole.
- Dipole.
- Beam pipe.



# Moving the collimator

$T_e^- = 120 \text{ MeV}$   $L_W = 4 \text{ mm}$



# Beam size [ $x, x'$ ] plane

- The beam size at the middle of the chicane is given by :

$$\sigma = \sqrt{\epsilon \times \beta_{min}}$$

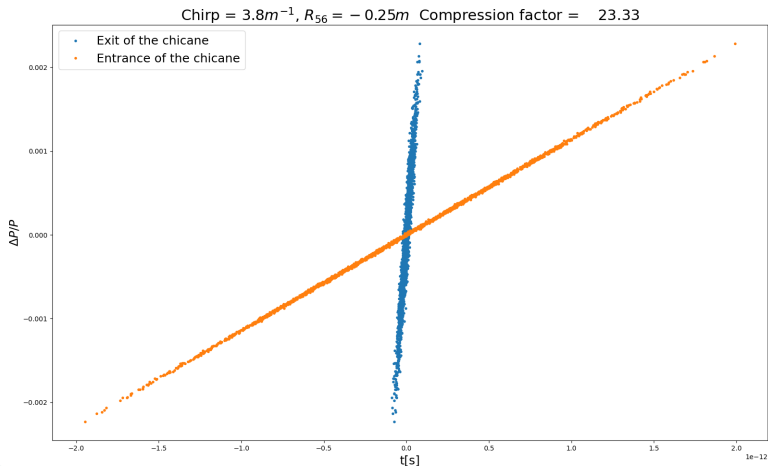
- From the positron distribution  $\epsilon = 0.039 \text{ mm rad}$ , and from the  $\beta$  function, we get  $\beta_{min}$  at the middle of the chicane:

$$\beta_{min} = 3.7 \text{ m}$$

- The beam size at the middle of the chicane is:

$$\sigma = 0.012 \text{ m}$$

# Chicane exit



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# Conclusion

- Fodo lattice allow us to control the beam size along the chicane.
- Optimized cavity to chirp the beam.
- Need to increase the dispersion at the middle of the chicane.
- Need an optimized matching section (quadrupoles) before the FODO lattice to match the twiss parameters.
- Mathematic calculations helps a lot for the software optimization.
- To be continued ...