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Ionization Rate vs. Voltage

1 Purpose

Calculating the ion production rate vs. gun voltage for the different gas species found in the "After 2 Days" RGA spectrum below:

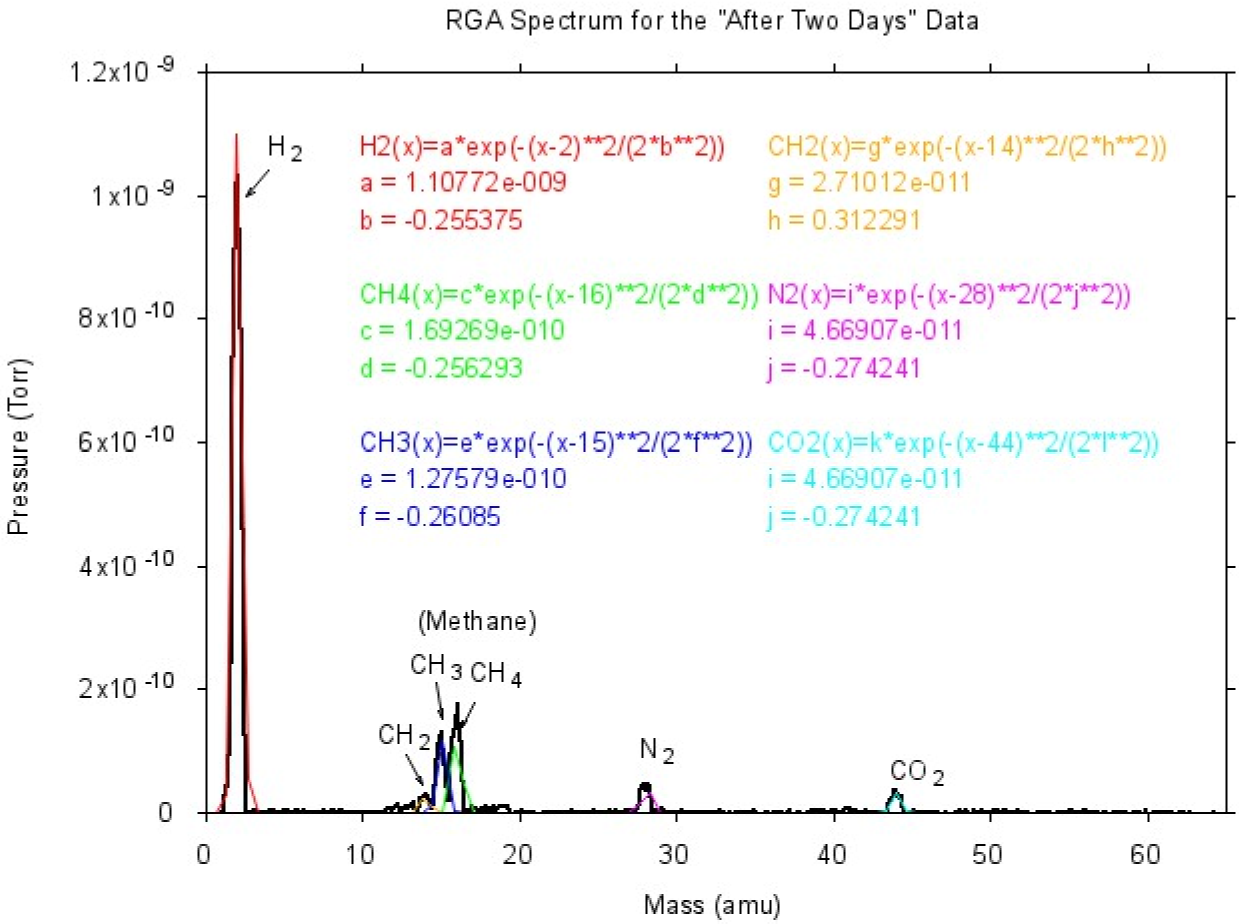

Figure 1: Analysis of the RGA spectrum for the "After 2 Days" data (before correction factor)

2 Ion Production Rate vs. Gun Voltage

The ion production rate (i.e. change in number density over time) is given by Reiser[1]

$$\frac{dn}{dt} = n_b n_g \sigma_i v = n_b n_g \sigma_i \beta_e c \tag{1}$$

We would like to rewrite the ion production rate as a function of voltage (or really in terms of beam energy T_e). We can start by rewriting β_e in terms of T_e:

$$
\begin{aligned}
T_e &= (\gamma - 1)\, m_e c^2 \\
\gamma &= 1 + \frac{T_e}{m_e c^2} \\
\frac{1}{\sqrt{1-\beta_e^2}} &= 1 + \frac{T_e}{m_e c^2} \\
1 - \beta_e^2 &= \left(\frac{1}{1+\frac{T_e}{m_e c^2}}\right)^2 = \left(\frac{m_e c^2}{m_e c^2 + T_e}\right)^2 \\
\beta_e^2 &= 1 - \left(\frac{m_e c^2}{m_e c^2 + T_e}\right)^2 \qquad (2)
\end{aligned}
$$

The numerical equation for the ionization cross section given by Reiser [1]

$$
\begin{aligned}
\sigma_{i[\mathrm{m}^2]} &= \frac{1.872 \times 10^{-24} A_1}{\beta_e^2} f(T_e) \left[\ln\left(7.515 \times 10^4 A_2 \beta_e^2 \gamma^2\right) - \beta_e^2\right] \qquad (3) \\
f(T_e) &= \frac{I_i}{T_e}\left(\frac{T_e}{I_i} - 1\right) \\
A_1 &= M^2 \\
A_2 &= \frac{e^{\frac{C}{M^2}}}{7.515 \times 10^4}
\end{aligned}
$$

Plugging in equation (2) into (3) yields:

$$
\begin{aligned}
\sigma_i(T_e) = &\frac{1.872 \times 10^{-24} A_1}{1 - \left(\frac{m_e c^2}{m_e c^2 + T_e}\right)^2} \frac{I_i}{T_e}\left(\frac{T_e}{I_i} - 1\right) \times \\
&\ln\left[7.515 \times 10^4 A_2 \left(1 - \left(\frac{m_e c^2}{m_e c^2 + T_e}\right)^2\right)\left(1 + \frac{T_e}{m_e c^2}\right)^2\right] - \left(1 - \left(\frac{m_e c^2}{m_e c^2 + T_e}\right)^2\right) \qquad (4)
\end{aligned}
$$

We can then plug (2) and (4) into (1) to yield a relationship between $\frac{dn}{dt}$ and T_e. Using Mathematica, this relationship can be shown graphically below for each gas species. We see that for each gas species, the graph has a zero at the ionization energy and is dominated by the logarithm in σ_i for high beam energies. A table of ion production rates (IPR) for selected beam energies is also shown below.

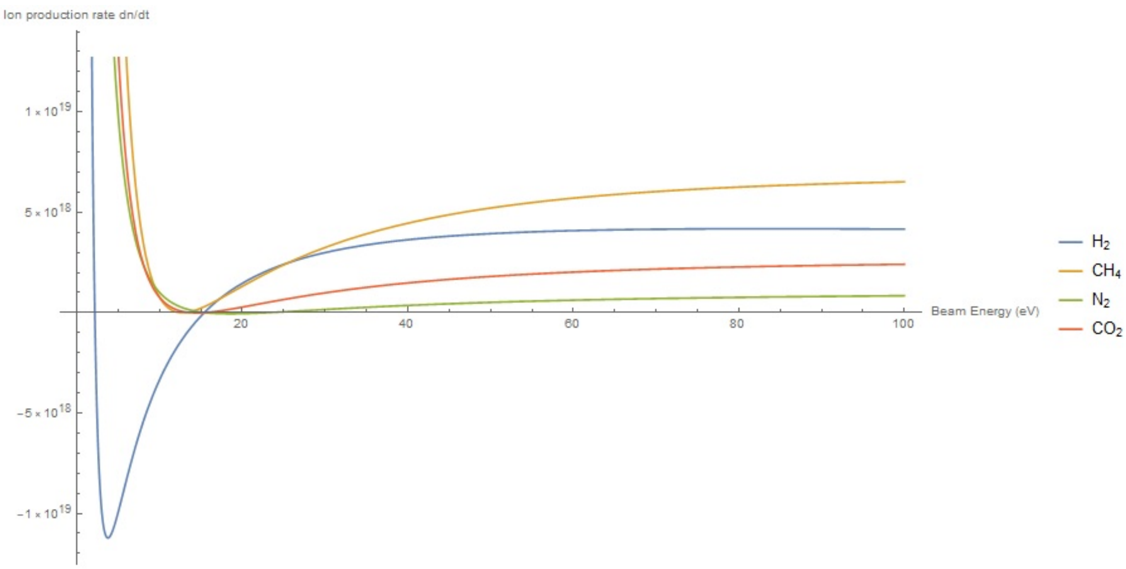

Figure 2: Ion production rate as a function of low beam energies for the gas species found in the RGA spectrum excluding CH_3 and CH_2.

Gas Species	IPR at 50eV $(\mathrm{m^{-3}\,s^{-1}})$	IPR at 100eV $(\mathrm{m^{-3}\,s^{-1}})$	IPR at 500eV $(\mathrm{m^{-3}\,s^{-1}})$	IPR at 1keV $(\mathrm{m^{-3}\,s^{-1}})$
H_2	3.93×10^{18}	4.15×10^{18}	3.03×10^{18}	2.45×10^{18}
CH_4	5.22×10^{18}	6.52×10^{18}	5.81×10^{18}	4.95×10^{18}
N_2	5.22×10^{17}	8.43×10^{17}	9.07×10^{17}	7.99×10^{17}
CO_2	1.80×10^{18}	2.41×10^{18}	2.26×10^{18}	1.94×10^{18}
Gas Species	IPR at 100keV $(\mathrm{m^{-3}\,s^{-1}})$	IPR at 130keV $(\mathrm{m^{-3}\,s^{-1}})$	IPR at 180keV $(\mathrm{m^{-3}\,s^{-1}})$	IPR at 1MeV $(\mathrm{m^{-3}\,s^{-1}})$
H_2	4.88×10^{17}	4.52×10^{17}	4.16×10^{17}	3.46×10^{17}
CH_4	1.15×10^{18}	‘1.07×10^{18}	9.91×10^{17}	8.46×10^{17}
N_2	2.00×10^{17}	1.87×10^{17}	1.73×10^{17}	1.50×10^{17}
CO_2	4.60×10^{17}	4.29×10^{17}	3.97×10^{17}	3.40×10^{17}

Table 1: Ion Production Rates (IPR) of each gas species for selected beam energies.

References

[1] Martin Reiser. *Theory and Design of Charged Particle Beams.* Wiley VCH Verlag GmbH, 2008.