β Calculation

Sajini Wijethunga

➢ Focal length

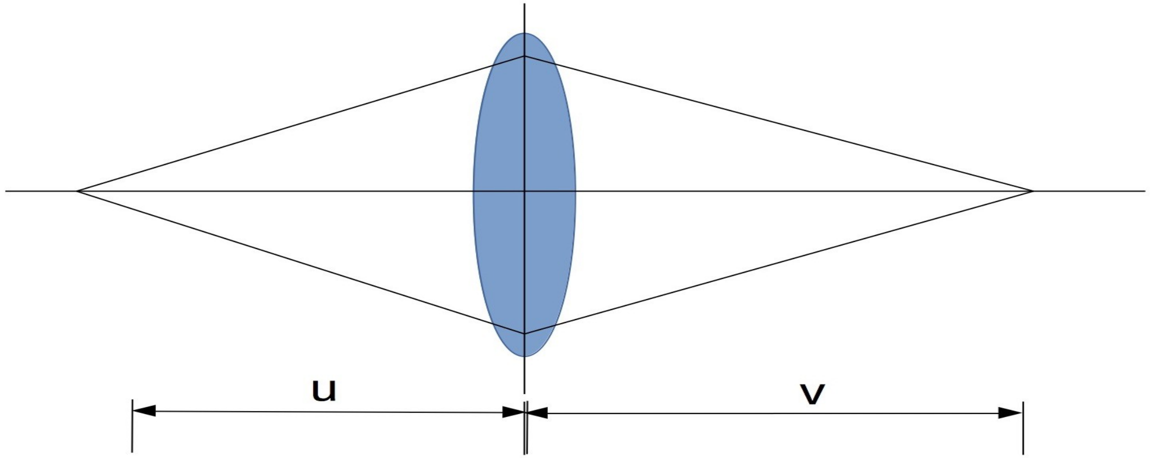
- For a solenoid

$$\frac{1}{f} = \frac{e^2 B_z^2 dz}{4\beta_z^2 \gamma^2 m^2 c^2}$$

- From optics, lens equations

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

where, u-distance to the object, v-distance to the image

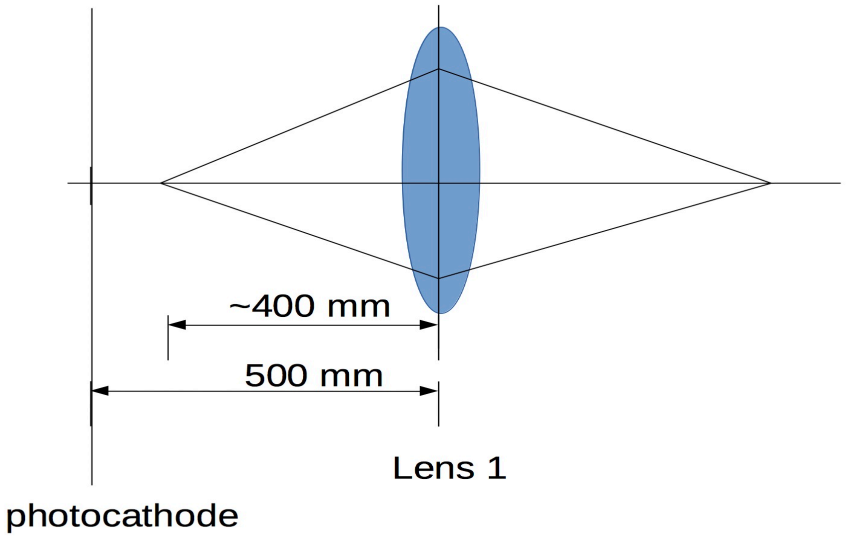


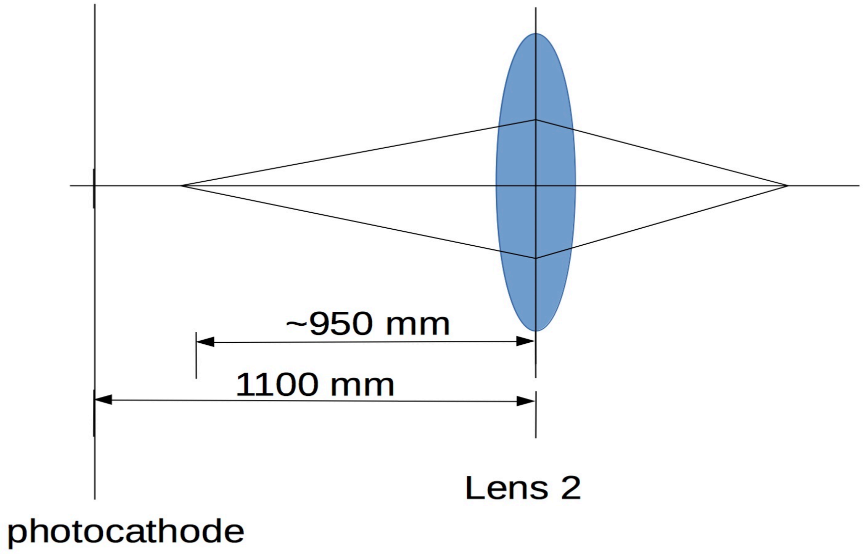
- Calculated focal length values from lens equation and from current

Lens	Viewer	$f = \frac{uv}{u+v}$ (mm)	f from the current (mm)
1	1	333.33	271.515
1	2	375.00	306.64
2	1	293.33	284.42
2	2	495.00	461.276

➢ Calculated distance to the photocathode using the focal length from the current

Lens	Viewer	Distance to object (mm)
1	1	372.7118
1	2	385.4327
2	1	984.3225
2	2	946.2632



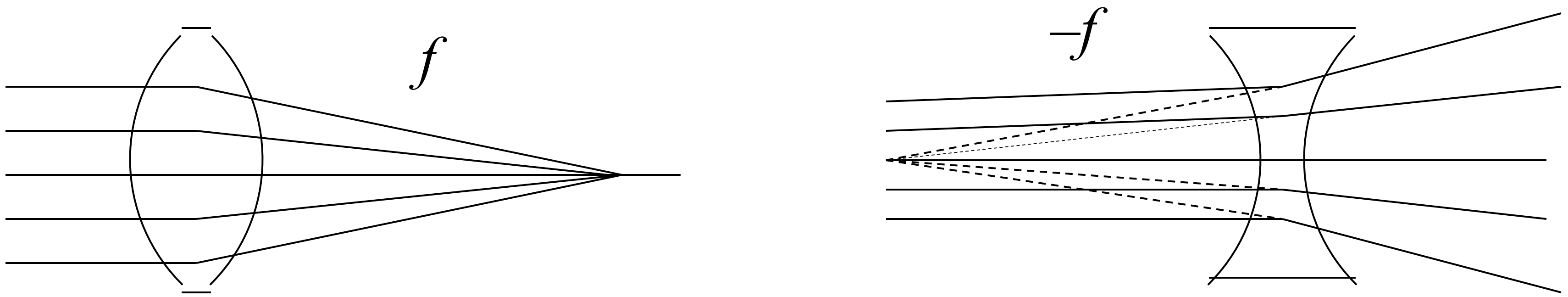


Accelerator Physics

G. A. Krafft
Jefferson Lab
Old Dominion University

Lecture 5

Thin Lenses



Thin Focusing Lens (limiting case when argument goes to zero!)

$$\begin{pmatrix} x(s_{lens}+\varepsilon) \\ \dfrac{dx}{ds}(s_{lens}+\varepsilon) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x(s_{lens}-\varepsilon) \\ \dfrac{dx}{ds}(s_{lens}-\varepsilon) \end{pmatrix}$$

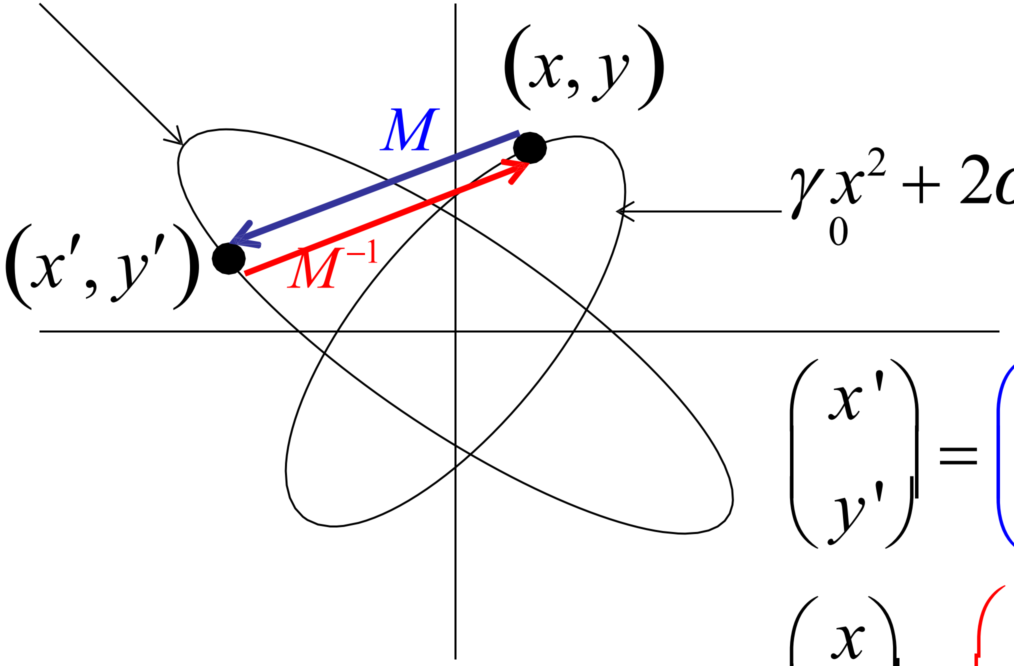
Thin Defocusing Lens: change sign of f

Effect of Transformation

Let the final ellipse be $\gamma x^2 + 2\alpha xy + \beta y^2 = \varepsilon$

The transformed coordinates must solve this equation.

$$\gamma x'^2 + 2\alpha x'y' + \beta y'^2 = \varepsilon$$



$$\gamma_0 x^2 + 2\alpha_0 xy + \beta_0 y^2 = \varepsilon_0$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} M^{-1}_{11} & M^{-1}_{12} \\ M^{-1}_{21} & M^{-1}_{22} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

The transformed coordinates must also solve the initial equation transformed.

Because

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (M^{-1})_{11} & (M^{-1})_{12} \\ (M^{-1})_{21} & (M^{-1})_{22} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

The transformed initial ellipse is

$$\gamma x'^2 + 2\alpha x' y' + \beta y'^2 = \varepsilon_0$$

$$\gamma = (M^{-1})_{11}^2 \gamma_0 + 2(M^{-1})_{11}(M^{-1})_{21}\alpha_0 + (M^{-1})_{21}^2 \beta_0$$

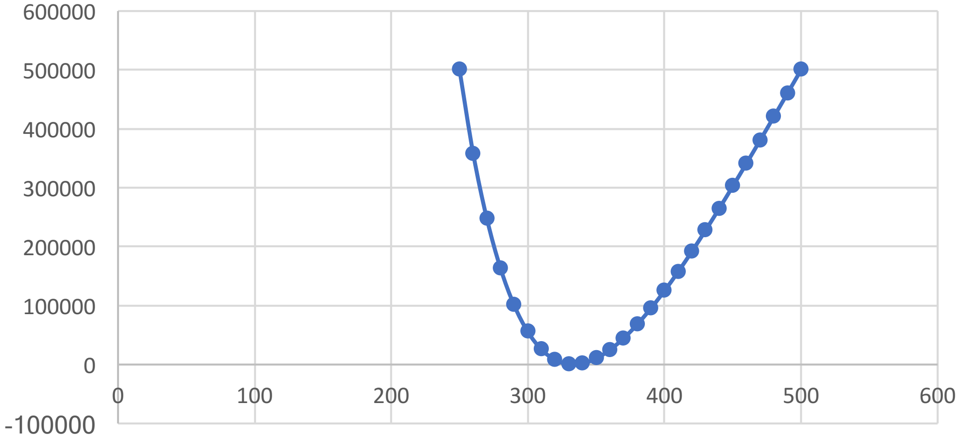
$$\alpha = (M^{-1})_{11}(M^{-1})_{12}\gamma_0 + \left((M^{-1})_{11}(M^{-1})_{22} + (M^{-1})_{12}(M^{-1})_{21}\right)\alpha_0 + (M^{-1})_{21}(M^{-1})_{22}\beta_0$$

$$\beta = (M^{-1})_{12}^2 \gamma_0 + 2(M^{-1})_{12}(M^{-1})_{22}\alpha_0 + (M^{-1})_{22}^2 \beta_0$$

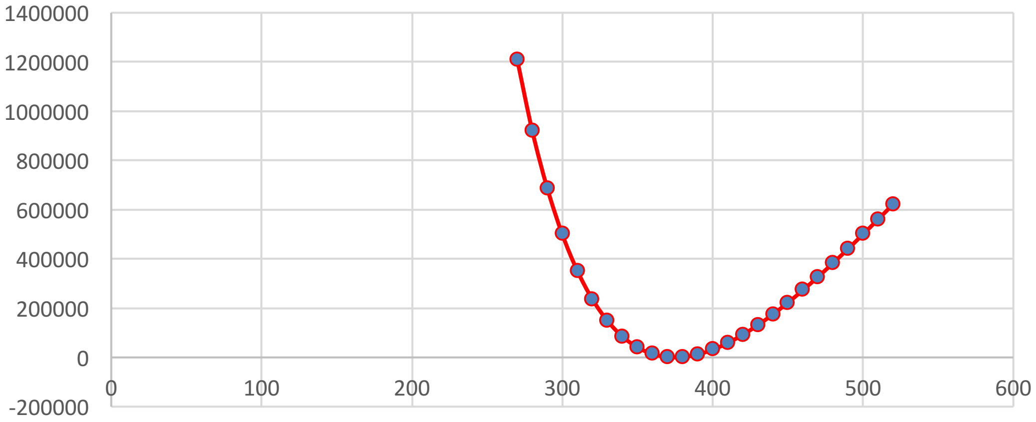
$$\beta = \frac{1}{0.5}\left(-u - v + \frac{uv}{f}\right)^2 + 0.5\left(1 - \frac{v}{f}\right)^2$$

$$\gamma_0 = \frac{1}{0.5}\, mm^{-1}\ ,\ \beta_0 = 0.5\, mm\ ,\ \alpha_0 = 0$$

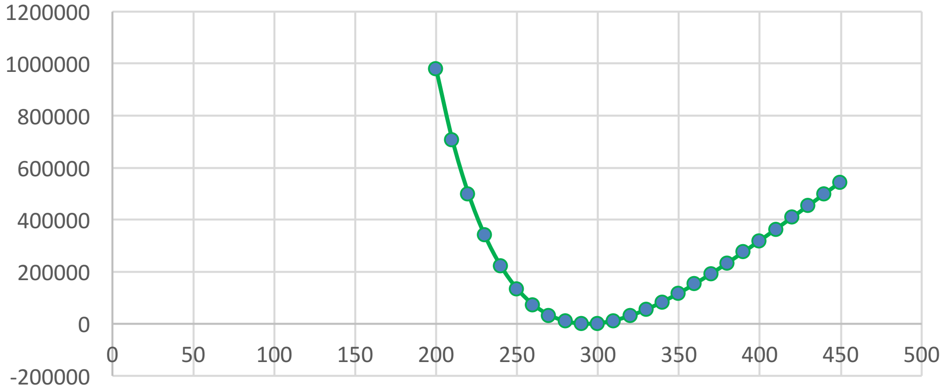
Lens 1 -viewer 1 (f=333.33 mm)



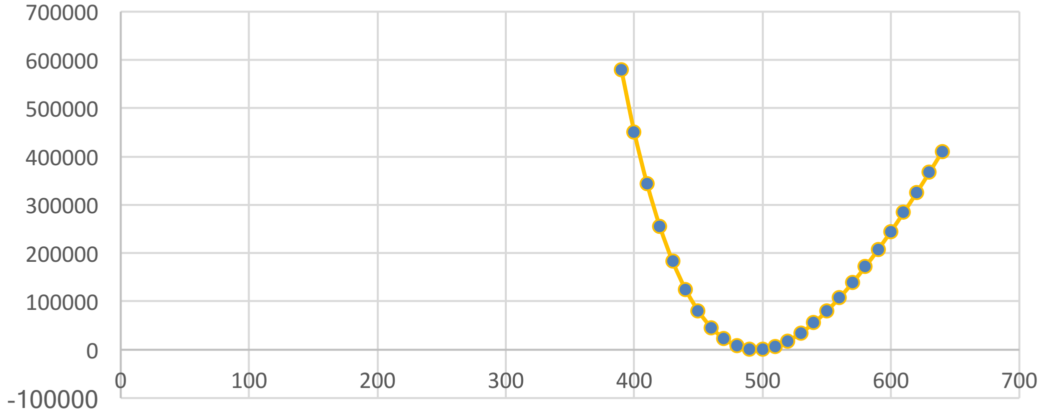
Lens 1-viewer 2 (f=375 mm)



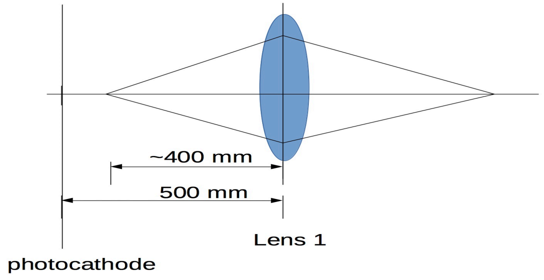
Lens 2-viewer 1 (f=293.33 mm)



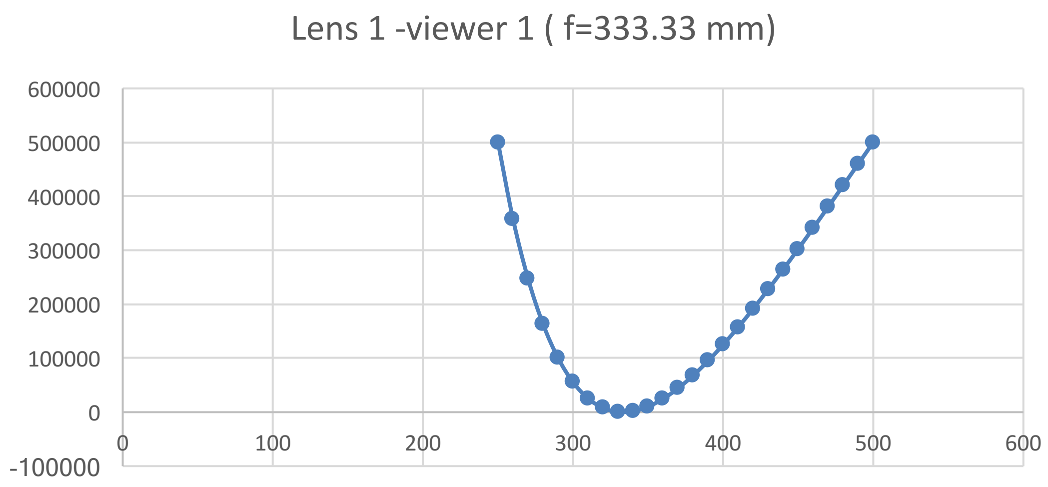
Lens 2-viewer 2 (f=495 mm)

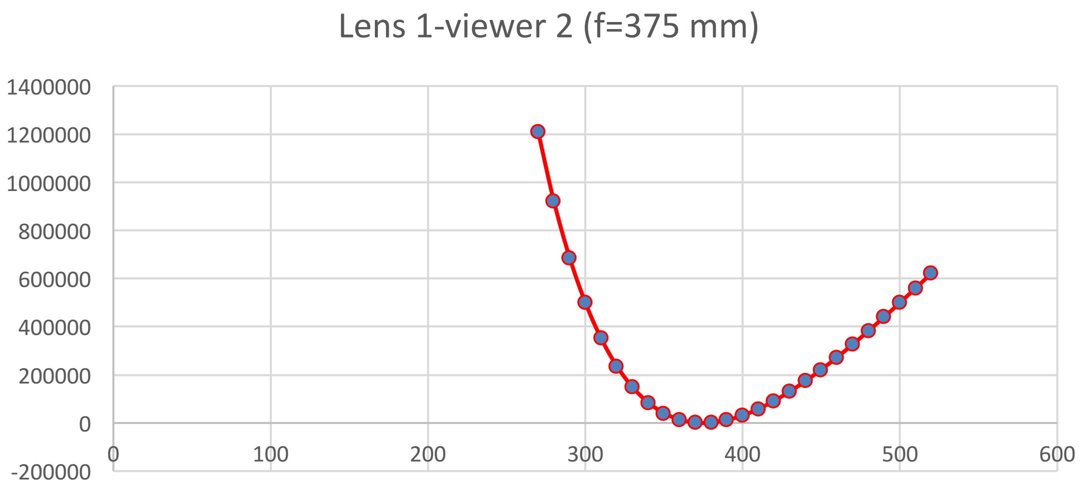


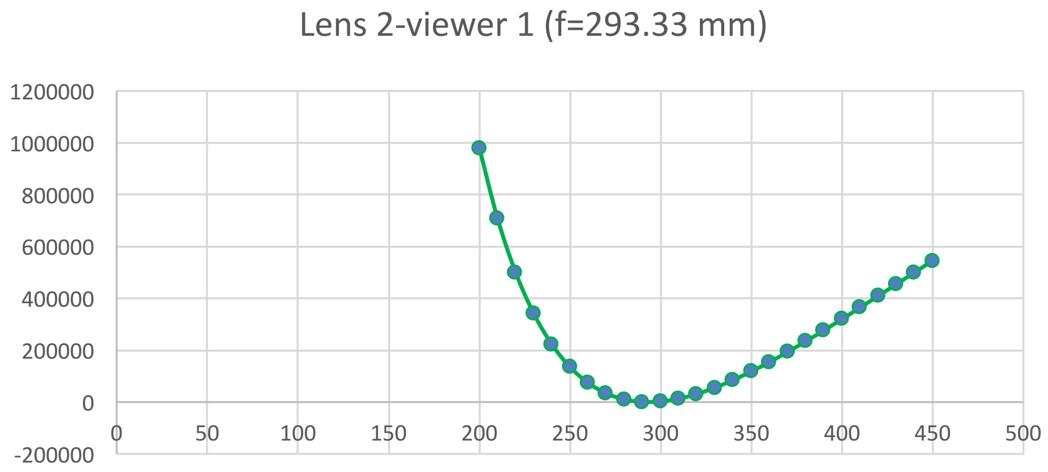
$$\beta = \frac{1}{0.5}\left(-u - v + \frac{uv}{f}\right)^2 + 2\times 2.5\times 10^{-3}\left(-u - v + \frac{uv}{f}\right)\left(1 - \frac{v}{f}\right) + 0.5\left(1 - \frac{v}{f}\right)^2$$

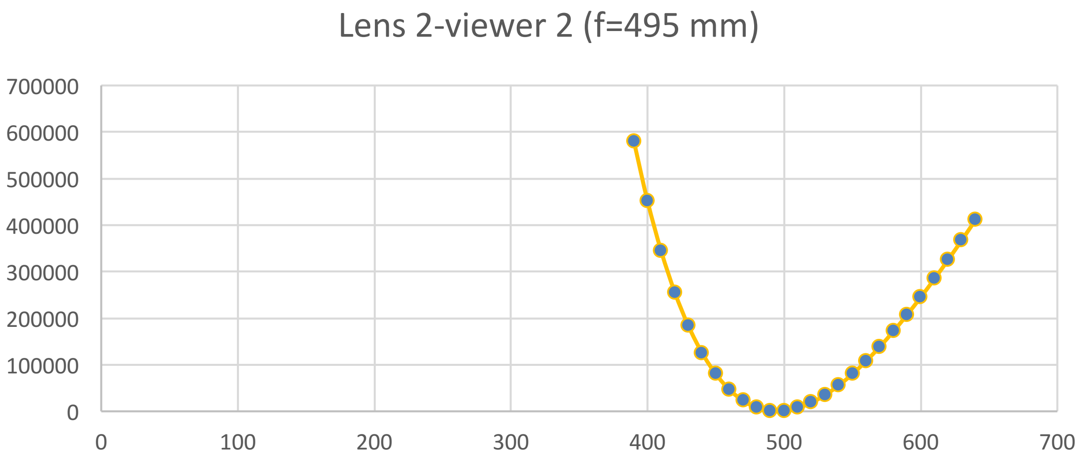


$$\gamma_0 = \frac{1}{0.5}\, mm^{-1},\ \beta_0 = 0.5\ \text{mm}\ ,\ \alpha_0 = \frac{\beta}{2f} = 2.5\times 10^{-3}$$









$$\beta = \frac{1}{0.5}\left(-u - v + \frac{uv}{f}\right)^2 - 2\times 2.5\times 10^{-3}\left(-u - v + \frac{uv}{f}\right)\left(1 - \frac{v}{f}\right) + 0.5\left(1 - \frac{v}{f}\right)^2$$

$$\gamma_0 = \frac{1}{0.5}\, mm^{-1},\ \beta_0 = 0.5\, mm\,,\ \alpha_0 = -2.5\times 10^{-3}$$