

Adapting Optics for High Energy Electron Cooling

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Abstract

A large value of beam horizontal emittance in ultrarelativistic electron synchrotrons is an obstacle to efficient introduction of electron cooling in hadron colliders. In this work, an adapting optics is proposed for the electron beam, in order to cancel the horizontal emittance's contribution to electron beam temperature in the cooling section. It involves a solenoid along the cooling section and special skew quadrupole blocks to match the solenoid and rest of electron ring, which has conventional optics. As result, the horizontal emittance becomes responsible only for the electron beam cross-section in the solenoid, while the transverse temperature in solenoid is determined by the outside vertical emittance, which is very small. This adaptation essentially raises the prospective efficiency of the high energy electron cooling.

1 Introduction

Electron cooling [1] with an electron beam in a synchrotron was proposed and treated earlier as a method to cool high energy proton and anti-proton colliding beams at energies ~ 200 GeV [2-7]. Recently, studies of physics and the capabilities of high energy electron cooling (HEEC) were revived [8-12] in connection with the quest to raise the luminosity of hadron colliders in the energy range 0.25-1 TeV.

In accordance with the basic principle of electron cooling [1], electron and ion beams circulating in two synchrotrons have equal average velocities ($\bar{\gamma}_e = \bar{\gamma}_i$), but different temperatures ($T_e \ll T_i$). An electron beam, being cooled by the synchrotron radiation serves the ion beam as a thermostat, by means of Rutherford's collisions between electrons and ions. The Rutherford's cross-section, hence, the cooling rate, is determined by the value of the relative particle velocities' spread, $\Delta(\vec{v}_i - \vec{v}_e)$, in the co-moving frame ($\vec{v}_\perp =$

$\gamma c \bar{\theta}$, $v_{\parallel} = c \Delta \gamma / \gamma$, where $\bar{\theta}$ is two-dimensional angle deviation of a particle relative to a fixed orbit in the cooling section). Conventionally, the electron horizontal spread, Δv_{ex} , appears to be the largest among all the spread components (either electron or hadron); thus, it becomes the most significant factor, reducing the cooling rate. This peculiarity is due to the particle closed orbit sensitivity to energy; energy jumps at radiation quanta emission, or intrabeam scattering, excite the horizontal particle oscillation in the focussing field. This mechanism is universal and unavoidable. It can be optimized, but not removed, therefore, it is considered usually as a ground deficiency of HEEC.

However, in this work we will show that there is a way to avoid the electron radial temperature transfer to the cooling section. Applying special skew quadrupole blocks, one can transform a plane electron beam (of a large emittance ϵ_{ex}) into a round (or elliptical) beam of a vortical state of motion [13-15] (Fig.1); then, the edge field of a solenoid can stop the vortex. The beam becomes almost parallel in the solenoid, with a low transverse temperature due to a small vertical emittance, ϵ_{ey} . In other words, the large beam emittance can be transformed into the beam cross-section in the solenoid, while the small emittance becomes responsible for the particles' Larmor oscillations. The effective emittance of the adapted electron beam in the solenoid becomes equal to the geometrical mean of two values, ϵ_{ex} and ϵ_{ey} . Thus, the vortex-solenoid optical trick can be considered as a way to make the intrinsic feature of electron synchrotrons— to form electron beams of very small vertical emittance— useful for cooling purposes.

2 Plane-vortex transformation and cancellation of large electron divergence

The above stated transformation of an electron beam can be performed with the optics shown in Fig.2. The key element is a quadrupole block, with matrices M and N in two normal planes, turned on some angle, α , around the beam direction. Let us introduce vector V of a particle position in the 4-dimensional transverse phase space $(x, x') \times (y, y')$:

$$V = \begin{pmatrix} X \\ Y \end{pmatrix}; \quad X = \begin{pmatrix} x \\ x' \end{pmatrix}; \quad Y = \begin{pmatrix} y \\ y' \end{pmatrix}; \quad (') \equiv \frac{d}{dz}$$

and rotation matrix

$$R = \frac{Ic}{-Is} \left| \frac{Is}{Ic} \right.,$$

where $c \equiv \cos \alpha$, $s \equiv \sin \alpha$, and I is the 2×2 unit matrix. Then, the 4×4 matrix of skew block, S , in the transition

$$V_2 = SV_1 \tag{1}$$

can be found as

$$S = R^{-1} \begin{pmatrix} M & O \\ O & N \end{pmatrix} R = \frac{Mc^2 + Ns^2}{(M - N)cs} \left| \frac{(M - N)cs}{Ms^2 + Nc^2} \right. \tag{2}$$

After an electron has passed the skew block, it encounters the solenoid fringe field, i.e. the edge transverse field. Let us describe its effect in the "thin lense" approximation. It is convenient to introduce a complex vector, u , as

$$u = x + iy, \quad u' = x' + iy';$$

then the edge effect can be found as

$$\Delta u'_2 = \frac{i}{2\beta_s} u_2, \quad (3)$$

with $\beta_s = pc/eB_s$.

Now, we consider the transformation of an initial (i.e. before the skew block) plane beam state, $Y_1 = 0$, with a condition that $x' = 0, y' = 0$ inside the solenoid.

Taking into account the equation in (3), we find a relation to satisfy:

$$u'_2 = -\frac{i}{2\beta_s} u_2, \quad (4)$$

or

$$Y_2 = FX_2, \quad (5)$$

with

$$F \equiv \begin{pmatrix} 0, & 2\beta_s \\ -1/2\beta_s, & 0 \end{pmatrix}. \quad (6)$$

Note that

$$F^2 = -1, \text{ i.e. } F^{-1} = -F. \quad (7)$$

The vortex state (5) has to be compared with the transformed beam state, according to (1) and (2), at $Y_1 = 0$:

$$X_2 = (Mc^2 + Ns^2)X_1,$$

and

$$Y_2 = cs(M - N)X_1.$$

Thus, we find a condition for matrices M and N as follows:

$$cs(M - N) = F(Mc^2 + Ns^2),$$

or

$$(csI - c^2F)M = (csI + s^2F)N. \quad (8)$$

Taking into account that $\det M = \det N = 1$, we find:

$$c^2 = s^2, \text{ i.e. } \alpha = \pm 45^\circ.$$

Using (8), we find a relation

$$(F \pm 1)N = -(F \mp 1)M,$$

or, taking into account (7),

$$N = \mp FM. \quad (9)$$

Thus, the required plane-vortex transformer is a quadrupole block possessing two normal matrices, M and N , with a relative phase advance of 90° (in terms of an effective focussing parameter, $2\beta_s$) and turned 45° around the beam direction to become a skew block.

The relationship in (9) puts 3 conditions on the elements of the quadrupole block. Three quadrupoles seem to be sufficient to satisfy (9). Finding a concrete model requires a separate investigation beyond the frame of this work. An example of a triplet adaptor for electron beam at $\gamma = 10^3$ has been simulated in the work [16]: skew block length 2.233 m, maximum field gradient 1.423 KGs/cm, solenoid field 1.03 T.

For further consideration, assume $\alpha = -45^\circ$, then

$$N = FM, \quad (10)$$

and the total matrix S in (2) takes a form as follows:

$$S = \frac{1}{2} \frac{(F+1)M}{(F-1)M} \left| \frac{(F-1)M}{(F+1)M} \right., \quad (11)$$

where M is a 2×2 matrix of a quadrupole transformation in a normal plane while matrix F is given in (6).

Now, using the above described conditioning, we introduce the drift position (x_d, y_d) of an electron in the solenoid, which is determined by the electron state (x_1, x'_1) in the horizontal plane before the vortical transformer (see (2)):

$$\begin{pmatrix} x_d \\ y_d \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2\beta_s M_{21} + M_{11} & M_{22} + \frac{M_{12}}{2\beta_s} \\ 2\beta_s M_{21} - M_{11} & M_{22} - \frac{M_{12}}{2\beta_s} \end{pmatrix} \begin{pmatrix} x_1 \\ 2x'_1\beta_s \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \bar{M} \begin{pmatrix} x_1 \\ 2x'_1\beta_s \end{pmatrix}, \quad (12)$$

$$x'_d = y'_d = 0.$$

Note that $\det \bar{M} = 1$.

Thus, the horizontal emittance of the electron beam is transformed into the beam cross-section in the solenoid while its contribution to the beam temperature is cancelled.

It should be noted that the cancellation of beam horizontal divergence in the solenoid does not contradict to the Liouville's theorem, since this theorem is formulated in terms of (\vec{r}, \hat{p}) with $\hat{p} = \vec{p} + \frac{e}{c} \vec{A}$, where $\vec{p} = \gamma m \vec{v}$ is the particle momentum, and \vec{A} is the vector potential of the electromagnetic field. With introduction of a solenoid, the transverse vector potential appears inside the solenoid as

$$\vec{A} = \frac{1}{2} \vec{B}_s \times \vec{\rho}, \quad \vec{\rho} = (x, y).$$

Obviously, the introduced 2-dimensional drift position in the solenoid can be returned to the horizontal plane as (x, x') , applying an out-transformer of a reverse type (for example, symmetrical to the in-transformer).

3 Transverse temperature and beam shape in the solenoid

Now, we can find a contribution of the vertical beam emittance to the beam state in the solenoid, applying matrix S in (11) to the vector V_1 component Y_1 :

$$\begin{aligned} X_2 &= \frac{1}{2}(F - 1)MY_1, \\ Y_2 &= \frac{i}{2}(F + 1)MY_1; \end{aligned} \quad (13)$$

then we find

$$Y_2 = -FX_2, \quad (14)$$

or, in complex form,

$$(x + iy)_2' = \frac{i}{2\beta_s}(x + iy)_2. \quad (15)$$

Thus, if the horizontal phase plane $(x, x')_1$ is transformed into a vortex, the vertical phase plane $(y, y')_1$ becomes transformed into the opposite vortex (see (4)).

Then, the solenoid will double the vortex (15), and inside the solenoid we obtain for this mode a relation $u' = i\beta_s u$, i.e., Larmor's, or cyclotron, motion:

$$u(z) = -i\beta_s(x'_o + iy'_o) \exp(iz/\beta_s),$$

with

$$\begin{pmatrix} -x'_o \\ y'_o \end{pmatrix} = \frac{1}{\sqrt{2}}\bar{M} \begin{pmatrix} y_1/\beta_s \\ y'_1 \end{pmatrix}. \quad (16)$$

Thus, the vertical emittance (y_o, y'_o) is transformed into the Larmor's heat motion in the solenoid.

The transition optics, i.e. skew block, can be optimized to obtain a minimum transverse temperature in the solenoid. Assume, for simplicity sake, $\beta'_x = \beta'_y = 0$ at $z = z_1$, $M_{21} = M_{11}/2\beta_s$, and $M_{12} = -M_{22} \times 2\beta_s$; taking into account that $\det M = 1$, we find $M_{22} = 1/2M_{11}$. Then, the Courant-Snyder ellipse

$$\frac{x_1^2}{\beta_x} + \beta_x x_1'^2 \equiv I_x/\gamma = Const$$

will be transformed inside the solenoid into ellipse

$$\frac{x_d^2}{M_{11}^2\beta_x} + \frac{\beta_x M_{11}^2}{\beta_s^2} y_d^2 = I_x/\gamma. \quad (17)$$

Respectively, we find the characteristic value of the cyclotron adiabatic invariant

$$I_s \equiv \gamma \beta_s [x_o'^2 + y_o'^2] = \left[\frac{M_{11}^2 y_1^2}{\beta_s} + \frac{\beta_s y_1'^2}{M_{11}^2} \right] \gamma ; \quad (18)$$

thus, the optimum condition is

$$M_{11} = \sqrt{\beta_s / \beta_y}, \quad (19)$$

with

$$I_s = I_y = \gamma \left(\frac{y_1^2}{\beta_y} + \beta_y y_1'^2 \right). \quad (20)$$

We define the emittance values as $\epsilon_{ex} = \langle I_x \rangle$, and $\epsilon_{ey} = \langle I_y \rangle$.

Taking into account the condition $\epsilon_{ey} \ll \epsilon_{ex}$, we can find the characteristic beam sizes in the solenoid:

$$\begin{aligned} \Delta x &\approx \Delta x_d = M_{11} \sqrt{\beta_x \epsilon_{ex} / \gamma}, \\ \Delta y &\approx \Delta y_d = \frac{\beta_s}{M_{11}} \sqrt{\epsilon_{ex} / \gamma \beta_x}, \end{aligned} \quad (21)$$

with

$$\Delta x_d \times \Delta y_d = \beta_s \times \epsilon_{ex} / \gamma, \quad (22)$$

$$\frac{\Delta y_d}{\Delta x_d} = \frac{\beta_s}{M_{11}^2 \beta_x}. \quad (23)$$

Note that the ratio in (23) is equal to β_y / β_x at the optimum condition in (19).

4 Shaping optimum electron beams in synchrotrons with adapting optics

The above described adaptation of the electron beam in the cooling section changes the approach to the electron beam forming in the storage rings. The electron storage rings possess an intrinsic property to shape electron beams of a small vertical emittance, ϵ_{ey} . In practice, this value is determined by the $x - y$ coupling; the lower limit of ϵ_{ey} , that is due to the quantum fluctuations of synchrotron radiation and intrabeam scattering, is smaller than a characteristic value of ϵ_{ex} in a few orders of value ($(T_y)_{min} \sim T_{||}$). However, this feature is not useful for electron cooling with conventional optics, since the spread of electron transverse velocities in the cooling section is determined by the ϵ_{ex} value ($\theta_e = \sqrt{\epsilon_{ex} / \gamma \beta}$). Oppositely, an increase of ϵ_{ey} (i.e. beam vertical size along the orbit) up to $\epsilon_{ey} \sim \epsilon_{ex}$ can be undertaken in order to reduce the heating effect of the intrabeam scattering on ϵ_{ex} and $\Delta\gamma_e$.

The situation becomes different with the vortex-solenoid adaption of the electron beam, since the contribution of ϵ_{ex} emittance to the electron temperature in the cooling section is cancelled there.

To compare the two optics, assume round cross-sections and equal sizes of the two beams, ion and electron, in cooling section. For typical conditions [10]

$$\Delta\gamma_i \ll \Delta\gamma_e ,$$

the cooling time dependence on the beams' parameters is found, in general, as follows [7,17]:

$$\begin{aligned} \tau_{\perp} &\propto \max(\theta_e^3, \theta_i^3)/n_e , \\ \tau_{\parallel} &\propto \max(\theta_e^2, \theta_i^2)\Delta\gamma_e/n_e , \end{aligned}$$

where n_e is the electron density.

Final reduction of these dependences is different for the two cases, conventional and adapted electron beams:

a) with conventional optics:

$$\begin{aligned} \tau_{\perp} &\propto [\max(\varepsilon_{ex}, \varepsilon_i)]^{5/2} , \\ \tau_{\parallel} &\propto \Delta\gamma_e [\max(\varepsilon_{ex}, \varepsilon_i)]^2 ; \end{aligned}$$

b) with vortex-solenoid adaptation:

$$\begin{aligned} \tau_{\perp} &\propto [\max(\varepsilon_{eff}, \varepsilon_i)]^{5/2} , \\ \tau_{\parallel} &\propto \Delta\gamma_e [\max(\varepsilon_{eff}, \varepsilon_i)]^2 , \end{aligned}$$

where

$$\varepsilon_{eff} \equiv \sqrt{\varepsilon_{ex}\varepsilon_{ey}} .$$

An extensive treatment of optimum beam shaping for HEEC goes beyond the frame of this work; we will limit our consideration of a gain estimation comparatively to the case of conventional electron optics in the cooling section. In accordance with the sense of this work, we have to undermine the situation

$$\varepsilon_o \gg \varepsilon_i ,$$

where ε_o is the electron transverse horizontal emittance optimized (i.e. minimized) to obtain the best cooling rate with conventional optics.

The quantum fluctuations contribution to ε_{ex} and $\Delta\gamma_e$ value is not a function of beam state; its minimization consists of optimum design of a magnetic lattice of arcs and wiggler sections. When this mechanism dominates, the gain of adapting optics at $\varepsilon_{ey} \ll \varepsilon_{ex}$ is obvious.

Similarly to quantum diffusion, the intrabeam scattering at high energies impacts the beam primarily in the radial direction, making the radial temperature high with respect

to the vertical and longitudinal ones. The heating rates dependence on beam sizes and velocity spreads can be found as [7,17]

$$(\dot{T}_{\parallel}, \dot{T}_x)_{ibs} \propto \frac{n_e}{\theta_{ex}} \propto \frac{1}{\epsilon_{ex} \sqrt{\epsilon_{ey}}}.$$

The vertical rate, in addition, is a function of $x - y$ coupling. Therefore, emittance ϵ_{ey} can be considered as a parameter to manipulate, while the equilibrium values ϵ_{ex} and $\Delta\gamma_e$ become functions of ϵ_{ey} :

$$\left. \begin{aligned} \epsilon_{ex} &= (\epsilon_o) \times (\epsilon_o/\epsilon_{ey})^{1/4}, \\ \Delta\gamma_e &= \Delta\gamma_e^o (\epsilon_o/\epsilon_{ey})^{1/8}, \end{aligned} \right\} \epsilon_{ey} \leq \epsilon_{ex}$$

where ϵ_o is a minimum equilibrium value of ϵ_{ex} , obtained at $\epsilon_{ey} = \epsilon_{ex}$.

Apparently, with conventional optics, beam vertical extension up to $\epsilon_{ey} = \epsilon_{ex}$ is an optimum measure to reduce the intrabeam scattering destructive effect on the cooling rate. Contrary to the conventional optics case, the cooling rates with adapting optics grow at a decrease of ϵ_{ey} (from value ϵ_o), i.e. $y - x$ coupling (remembering the condition $\epsilon_o \gg \epsilon_i$). For cooling rates dependences, we find

$$\epsilon_{eff} \propto \epsilon_y^{3/4},$$

$$\tau_{\perp} \propto \epsilon_{eff}^{5/2} \propto \epsilon_{ey}^{15/8},$$

and

$$\tau_{\parallel} \propto \epsilon_{eff}^2 \Delta\gamma_e \propto \epsilon_{ey}^{11/8}.$$

The increase stops at $\epsilon_{ey}\epsilon_{ex} = \epsilon_i^2$, i.e. at

$$\frac{\epsilon_{ey}}{\epsilon_{ex}} = \left(\frac{\epsilon_i}{\epsilon_o}\right)^{10/3},$$

but it can be extended in the longitudinal direction with an artificial increase of ϵ_{ex} at $\epsilon_{ex}\epsilon_{ey} = \epsilon_i^2$, as far as the $\Delta\gamma_e$ and ϵ_{ey} approach the quantum radiation limit. Note that the beam size in the cooling section grows with ϵ_{ex} as $\sqrt{\epsilon_{ex}}$; to compensate for this extension, if needed, the solenoid must be of suitable strength.

It should be emphasized again that the situation $\epsilon_{ex} \gg \epsilon_i$ appears to be a typical one in the design of electron cooling rings for hadron colliders. As an example, refer to calculations in the work [10]: horizontal beam emittance (in $\pi \times mm \times mrad$) of a value $\epsilon_{ex} = 12.36$ in the electron ring was calculated to cool colliding beams in Tevatron of emittance $\epsilon_i = 1.67$. This tendency is increased with energy and electron current. Therefore, the considered adaptation of the electron beam in the cooling section, accompanied by necessary reduction of $x - y$ coupling in the rest of electron ring, seems to be an important element of advanced electron cooling concepts.

5 Conclusions

We showed in this work that the efficiency of electron synchrotrons as coolers for high energy hadron beams can be dramatically increased with introduction of the adapting optics for the electron beam. These optics involve special skew quadrupole blocks with a solenoid along the cooling section. As a result, the vertical emittance becomes transformed into the Larmor heat motion, while the horizontal one is responsible only for the beam cross-section in the solenoid. The effective emittance is equal to the geometrical mean of two emittances, vertical and horizontal, shaped outside the cooling section under the effects of the synchrotron radiation and intrabeam scattering.

This adaptation changes the approach to the beam forming in the electron ring. It makes sense for the efforts to reduce the vertical-horizontal coupling due to the imperfections to a possibly low level in order to obtain minimum vertical emittance, and, finally, minimum or optimum value of the 6-dimensional phase space volume of the electron beam.

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List of Figures

Figure 1: Plane-vortex transformation of electron beam: a skew quadrupole block transforms the (x, x') plane into a vortex, while the (y, y') plane becomes transformed into the opposite vortex. General transformation is a superposition of two elementary ones.

Figure 2: Schematic of adapting optics for electron beam: 1) plane-vortex skew quadrupole transformer; 2) solenoid to stop the x -vortex; 3) vortex-plane transformer.

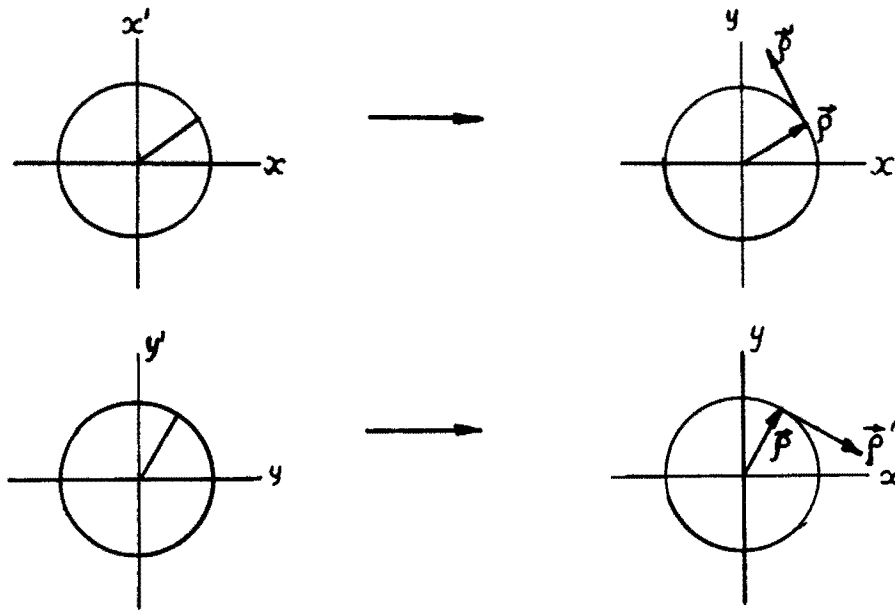


Figure 1.

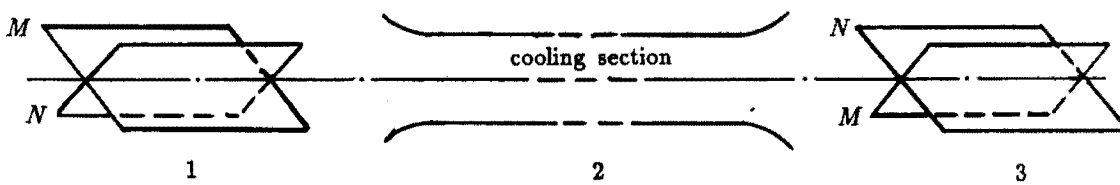


Figure 2.