

Error Analysis II

Relative Systematic Errors

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Penfold-Leiss Cross Section Unfolding

- Measure Yields at: $E = E_1, E_2, \dots, E_n$ where,
 $E_i - E_{i-1} = \Delta, i = 2, n$

$$Y(E_i) = \int_{th}^{E_i} n_\gamma(E_i, k) \sigma(k) dk \approx \sum_{j=1}^i N_\gamma(E_i, \Delta, E_j) \sigma(E_j)$$

- The solution can be written in two forms:

$$\sigma_i = \frac{1}{N_{ii}} \left[y_i - \sum_{j=1}^{i-1} (N_{ij} \sigma_j) \right]$$

- Or, Matrix form:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} N_{\gamma,11} & 0 & \cdots & 0 \\ N_{\gamma,21} & N_{\gamma,22} & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 \\ N_{\gamma,n1} & N_{\gamma,n2} & \cdots & N_{\gamma,nn} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_n \end{bmatrix}$$

$$[Y] = [N] \bullet [\sigma]$$

$$[\sigma] = [N]^{-1} \bullet [Y]$$

Systematic Error Propagation (1)

- For absolute beam energy uncertainty of δE ($= 0.1\%$) and zero relative beam energy uncertainty:

$$\frac{dy_i}{y_i} = \frac{y_i(E_i + \delta E) - y_i(E_i)}{y_i(E_i)}$$

$$\frac{dN_{ij}}{N_{ij}} = \frac{N_{ij}(E_i + \delta E) - N_{ij}(E_i)}{N_{ij}(E_i)}$$

$$E_0 = 7.8 + \delta E$$

$$E_i = E_0 + i\Delta$$

E_i (MeV)	dy_i/y_i (%)	$d\sigma_i/\sigma_i$ (%)
7.9	12.5	12.6
8.0	10.8	10.5
8.1	9.3	9.1
8.2	8.0	7.1
8.3	7.0	6.3
8.4	6.3	5.8
8.5	5.6	5.2

This is the cross section dependence on energy

- Accounted for dN_{ij} due to energy error when calculating dy_i

$$\approx \frac{\delta E}{i\Delta}$$

$$[dN_{ij} / N_{ij}] = \begin{bmatrix} 0.100 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.058 & 0.050 & 0 & 0 & 0 & 0 & 0 \\ 0.041 & 0.039 & 0.033 & 0 & 0 & 0 & 0 \\ 0.031 & 0.031 & 0.029 & 0.025 & 0 & 0 & 0 \\ 0.025 & 0.025 & 0.025 & 0.023 & 0.020 & 0 & 0 \\ 0.021 & 0.021 & 0.021 & 0.021 & 0.020 & 0.017 & 0 \\ 0.018 & 0.018 & 0.018 & 0.018 & 0.018 & 0.017 & 0.022 \end{bmatrix}$$

- With:

$$[B] = [N]^{-1}$$

$$[\sigma] = [B] \bullet [Y]$$

- Then:

$$[d\sigma^2] = [B] \bullet ([dY^2] + [dN^2] \bullet [\sigma^2]) \bullet [B]^T$$

- Where:

Note: Correlation Coefficient =1

$$\text{var}(y_i, y_i) = y_i$$

$$\text{cov}(y_i, y_j) = \rho_{ij} \sqrt{y_i} \sqrt{y_j}$$

$$[dY^2] = \begin{bmatrix} y_1 & \sqrt{y_1} \sqrt{y_2} & \cdots & \sqrt{y_1} \sqrt{y_n} \\ \sqrt{y_2} \sqrt{y_1} & y_2 & \cdots & \sqrt{y_n} \sqrt{y_n} \\ \vdots & \ddots & \ddots & \vdots \\ \sqrt{y_n} \sqrt{y_1} & \sqrt{y_n} \sqrt{y_2} & \cdots & y_n \end{bmatrix}$$

$$[d\sigma^2] = \begin{bmatrix} d\sigma_1^2 & \text{cov}(\sigma_1, \sigma_2) & \cdots & \text{cov}(\sigma_1, \sigma_n) \\ \text{cov}(\sigma_2, \sigma_1) & d\sigma_2^2 & \cdots & \text{cov}(\sigma_2, \sigma_n) \\ \vdots & \ddots & \ddots & \vdots \\ \text{cov}(\sigma_n, \sigma_1) & \text{cov}(\sigma_n, \sigma_2) & \cdots & d\sigma_n^2 \end{bmatrix}$$

$$[dN^2] = \begin{bmatrix} (dN_{11})^2 & 0 & \cdots & 0 \\ (dN_{21})^2 & (dN_{22})^2 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ (dN_{n1})^2 & (dN_{n2})^2 & \cdots & (dN_{nn})^2 \end{bmatrix}$$

$$[\sigma^2] = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n^2 \end{bmatrix}$$

Systematic Error Propagation (2)

$$\begin{aligned}(d\sigma_i)^2 = & \frac{1}{N_{ii}^2} \left[dy_i^2 - 2dy_i \sum_{j=1}^{i-1} N_{ij} d\sigma_j \right. \\ & + \sum_{j=1}^{i-1} (N_{ij} d\sigma_j)^2 + \sum_{k=1}^{i-1} \sum_{l=1}^{i-1} N_{ik} \operatorname{cov}(\sigma_k, \sigma_l) N_{il} \\ & \left. + \sum_{j=1}^{i-1} (dN_{ij} \sigma_j)^2 + (dN_{ii} d\sigma_i)^2 \right]\end{aligned}$$

$\operatorname{cov}(y_i, y_j) \neq 0,$
 $\operatorname{cov}(\sigma_i, \sigma_j) \neq 0$

Other Systematic Errors

Beam Current, $\delta I/I$	Absolute	3%
Photon Flux, $\delta\phi/\phi$	Absolute	5%
Radiator Thickness, $\delta R/R$	Relative	3%
Bubble Chamber Thickness, $\delta T/T$	Relative	3%
Bubble Chamber Efficiency, ε	Relative	5%

Simulation

- Then:

$$(dy_i)^2 = (dy_i(\delta E))^2 + \left[\left(\frac{\delta I}{I} \right)^2 + \left(\frac{\delta R}{R} \right)^2 + \left(\frac{\delta T}{T} \right)^2 + \varepsilon^2 \right] y_i^2$$

$$(dN_{ij})^2 = \left(\frac{\delta \phi}{\phi} \right)^2 N_{ij}^2$$

Systematic Errors Results

Electron Beam K. E.	Cross Section (nb)	Sys Error (Energy, %)	Sys Error (Total, %)
7.9	0.046	12.5	19.5
8.0	0.185	10.2	21.1
8.1	0.58	8.3	22.1
8.2	1.53	7.0	23.6
8.3	3.49	6.0	25.4
8.4	7.2	5.3	27.7
8.5	13.6	4.7	30.1