Positron Collection system

Sami Habet

IJCLab.

Jefferson Laboratory.

April 2022

Plan

1. Positron collection system
2. Quarter wave transformer
3. Adiabatic matching device
4. Conclusion & Questions

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Jefferson Lab

Plan

1. Positron collection system
2. Quarter wave transformer
3. Adiabatic matching device
4. Conclusion & Questions

Why?

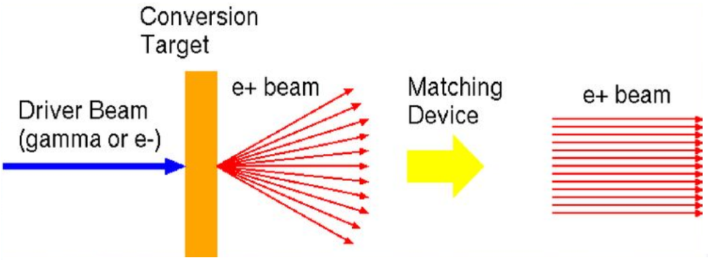


Figure: schematic and purpose of the positron collection system symbolized here by the matching device item.

The solenoidal magnetic field

- The phase space in a solenoid is described by :

$$\left(\frac{eB}{2}\right)^2 (x^2 + y^2) + (p_x^2 + p_y^2) = \mathrm{Cte} \quad (1)$$

  Where :

  - e is the particle charge.
  - B is the magnetic field in the solenoid.
  - x, y Spacial coordinates the solenoid.
  - p_x, p_y are the particles momenta.

- This equation define the **Hyper ellipsoid in the phase space.**

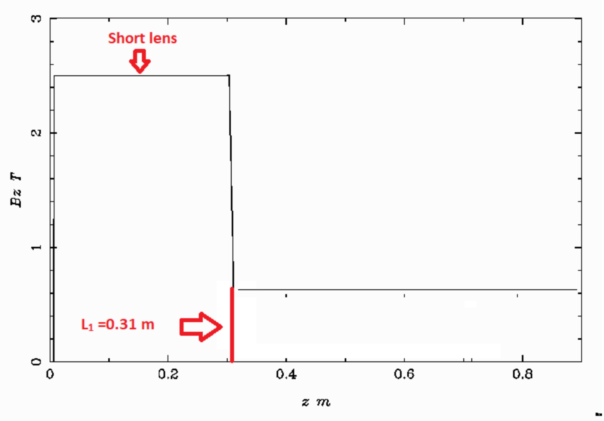
Plan

1. Positron collection system
2. Quarter wave transformer
3. Adiabatic matching device
4. Conclusion & Questions

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Jefferson Lab

Quarter Wave Transformer



- The position vector $\vec{q}(x, p_x, y, p_y)$
- The transfer matrix from the target z_0 to the exit of the solenoid according to the longitudinal position z_s can be written :

$$\vec{q}_B = M\ (z_s|z_0)\ \vec{q}_0$$

$$M\ (z_s|z_0) = R_2\, M_2\, R_1\, M_1$$

Where:

  - M_1 Transfer matrix in the first solenoid.
  - M_2 Transfer matrix in the second solenoid.
  - R_1 and R_2 are the rotation matrix.
- After decoupling:

$$\begin{pmatrix} X \\ P_X \end{pmatrix} = e^{-i(\xi_1+\xi_2)} \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} X_0 \\ P_{X_0} \end{pmatrix} \quad (2)$$

where

$$M_{11} = \cos\chi_1 \cos\chi_2 - \frac{B_1}{B_2}\sin\chi_1 \sin\chi_2 \quad (3)$$

$$M_{12} = \frac{2}{eB_1}\sin\chi_1 \cos\chi_2 + \frac{2}{eB_2}\cos\chi_1 \sin\chi_2 \quad (4)$$

$$M_{21} = -\frac{eB_2}{2}\cos\chi_1 \sin\chi_2 - \frac{eB_1}{2}\sin\chi_1 \cos\chi_2 \quad (5)$$

$$M_{22} = -\frac{B_2}{B_1}\sin\chi_1 \sin\chi_2 + \cos\chi_1 \cos\chi_2\,. \quad (6)$$

- **The rotation angle in the (x, y) plane**

$$\chi = \int_0^l \frac{eB}{2p}dz \quad (7)$$

$$XX^* + \left(\frac{2}{eB_2}\right)^2 P_X P_X^* = \left[\cos^2\chi_1 + \left(\frac{B_1}{B_2}\right)^2 \sin^2\chi_1\right] x_0 x_0^* + \left[\left(\frac{2}{eB_1}\right)^2 \sin^2\chi_1 + \left(\frac{2}{eB_2}\right)^2 \cos^2\chi_1\right] p_{x_0} p_{x_0}^* + \frac{2}{eB_1} \sin\chi_1 \cos\chi_1 \left[1 - \left(\frac{B_1}{B_2}\right)^2\right] \left(x_0^* p_{x_0} + x_0 p_{x_0}^*\right) \quad (8)$$

- $X = x + iy$ and $P_x = p_x + ip_y$
- $X^* = x - iy$ and $P_x^* = p_x - ip_y$
- $XX^* = x^2 + y^2$
- $P_x P_x^* = p_x^2 + p_y^2$

After symplification we get:

$$\left(\frac{eB_2}{2}\right)^2 XX^* + P_X P_X^* = \left(\frac{eB_2}{2}\right)^2 (x^2 + y^2) + (p_x^2 + p_y^2) = \mathrm{Cte} \tag{9}$$

We can get :

$$XX^* + \left(\frac{2}{eB_2}\right)^2 P_X P_X^* = \mathrm{Cte}\,.$$

- The acceptance condition :

$$XX^* \leq a^2$$

- **a is the aperture at the end of the long solenoid**
- **We can write :**

$$\mathrm{Cte} - \left(\frac{2}{eB_2}\right)^2 P_X P_X^* \leq a^2\,.$$

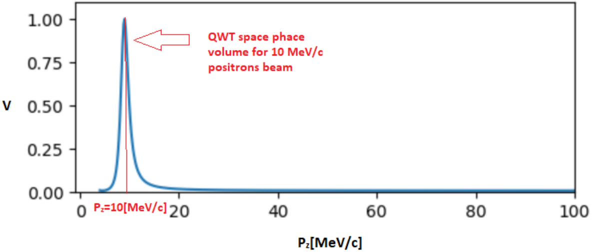
QWT Volume acceptance

The volume acceptance V of the QWT is then defined as:

$$V_{QWT} = \int_{XX^* \leq a^2} dx\, dy\, dp_x\, dp_y$$

and can be written :

$$V(\chi_1) = \frac{2\pi^2}{3}\left(\frac{eB_2 a^2}{2}\right)^2 \left[1 - \left(1 - \frac{1}{\sin^2\chi_1 + \left(\frac{B_1}{B_2}\right)^2 \cos^2\chi_1}\right)^{\frac{3}{2}}\right].$$



Volume acceptance

The momentum p_m at maximum transmission is obtained from the equation

$$\frac{dV(\chi_1)}{d\chi_1} = 0 \tag{10}$$

Will get:

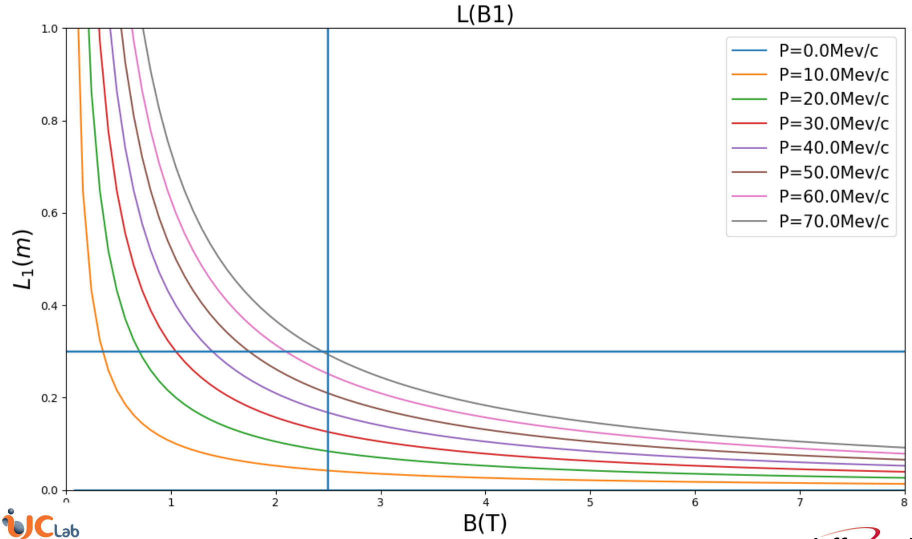
$$\chi_1 = \pi/2 \tag{11}$$

Which leads to:

$$p_m = \frac{eB_1 L_1}{\pi} \tag{12}$$

- Correlation between the length of the high field and the magnetic field intensity.

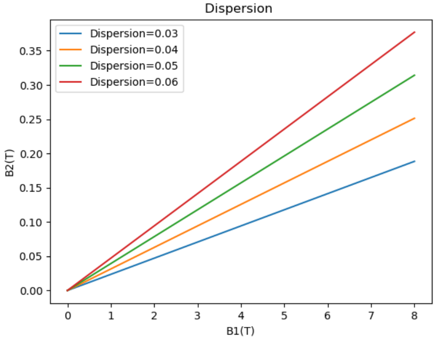
QWT characteristique



The accepted momentum dispersion in the QWT

- At the maximum volume acceptance:

$$\frac{\delta p}{p} = \frac{4}{\pi}\frac{B_2}{B_1}$$



Transverse acceptance

- Replacing the rotation angle $\chi_1 = \pi/2$ which lead to maximize the volume acceptance:

$$XX^* + \left[\frac{2}{eB_2}\right]^2 = \left[\frac{B_1}{B_2}\right]^2 X_0\, X_0^* + \left[\frac{2}{eB_1}\right]^2 P_{X_0}\, P_{X_0^*} = \mathrm{Cte} \qquad (13)$$

- Cylindrical coordinates:

$$\left[\frac{B_1}{B_2}\right]^2 r_0^2 + \left[\frac{2}{eB_1}\right]^2 \left[P_{r_0}^2 + \frac{P_{\phi 0}}{r^2}\right] = \mathrm{Cte} \qquad (14)$$

- With

$$\mathrm{Cte} - \left(\frac{2}{eB_2}\right)^2 P_X\, P_X^* \leq a^2\,. \qquad (15)$$

- where $(r_0, P_{r_0}, P_{\phi_0})$ are the coordinates of the particles at target.

$$\left[\frac{B_1}{B_2}\right]^2 r_0^2 + \left[\frac{2}{eB_1}\right]^2 \left[P_{r_0}^2 + \frac{P_{\phi 0}^2}{r^2}\right] = a^2 + \left[\frac{2}{eB_2}\right]^2 \frac{P_{\phi 0}^2}{a^2} \qquad (16)$$

Transverse acceptance

- The maximum extension in the spacial demension for an ellipsoid equation is deteminded if:

$$P_{r_0} = P_{\phi_0} = 0$$

- Providing the radial accepptance of the QWT:

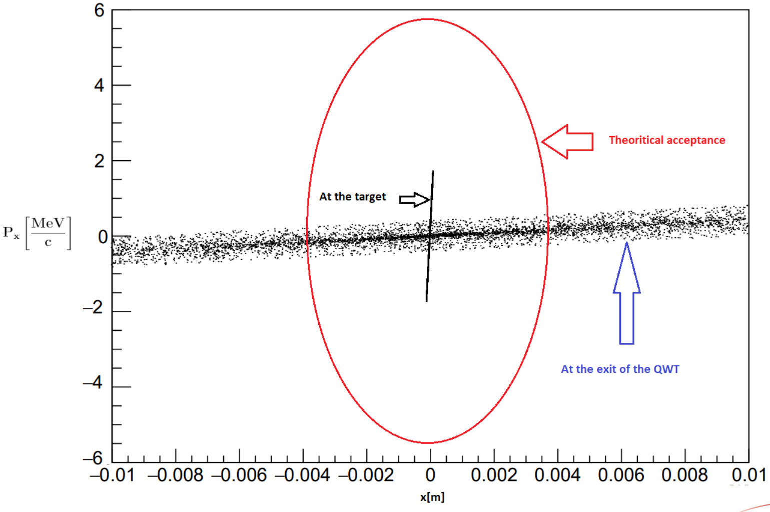
$$r_0^{max} = \frac{B_2}{B_1} a \,. \tag{18}$$

- Example: Considering the case (B_1, B_2, a)=(2.5 T,0.5 T,20 mm), we obtain

$$r_0^{max} = 4\,\mathrm{mm} \tag{19}$$

$$P_{r_0}^{max} = 5.7\,\mathrm{MeV}/c \,. \tag{20}$$

Transverse acceptance



Plan

1. Positron collection system
2. Quarter wave transformer
3. Adiabatic matching device
4. Conclusion & Questions

Adiabatic matching device

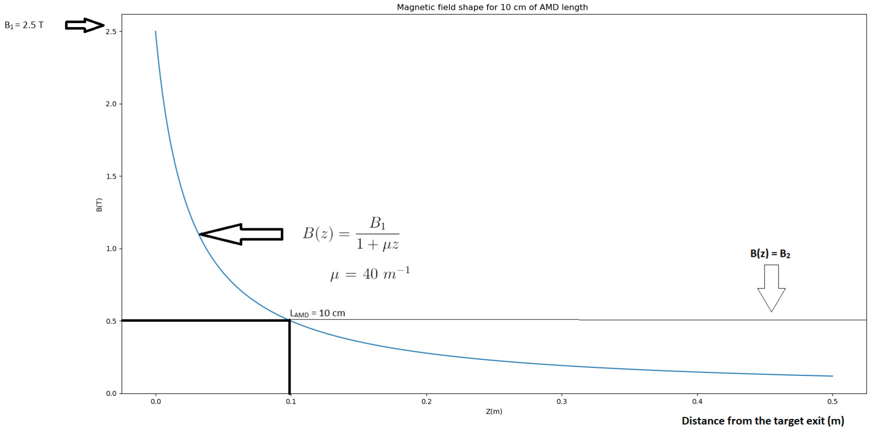


Figure: longitudinal profile of the magnetic field of an AMD.

AMD Volume acceptance

- The accepted phase space under AMD magnetic field is :

$$XX^* + \left[\frac{2}{eB_2}\right]^2 P_X P_X^* = \frac{B_1}{B_2} X_0 X_0^* + \frac{4}{e^2 B_1 B_2} P_{X_0} P_{X_0}^* = \mathrm{Cte}\,. \qquad (21)$$

- The corresponding volume acceptance for such a device :

$$V_{AMD} = \int_{XX^* \leq a^2} dx\, dy\, dp_x\, dp_y = \frac{2\pi^2}{3} \left[\frac{eB_2 a^2}{2}\right]^2 . \qquad (22)$$

- Where B_2 is the end value of the magnetic field at the end of the AMD
-

$$B(z) = \frac{B_1}{1 + \mu z} \qquad (23)$$

where

$$\mu = \frac{\epsilon B_1}{P_0} \qquad (24)$$

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Jefferson Lab

AMD Volume acceptance

- where P_0 is a particular central scalar momentum. The value of the smallness parameter

$$\epsilon = \frac{P_0}{B_2}\frac{dB}{dz} \tag{25}$$

- is a characteristic constant of the adiabatic operation ($\epsilon \ll 1$). From

$$\int_0^{L_1} \frac{\epsilon}{P_0}\, dz = \int_{B_1}^{B_2} \frac{dB}{B_2} \tag{26}$$

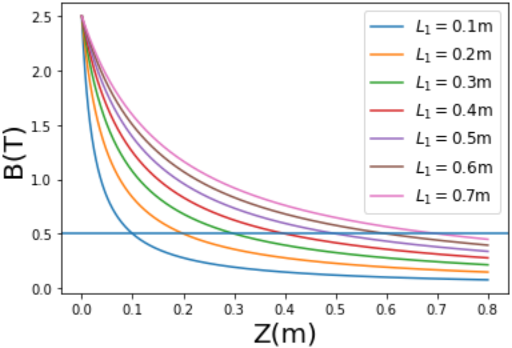
- We obtain

$$B(z) = \frac{B_1 B_2 L_1}{B_2 L_1 + (B_1 - B_2)\, z}\,. \tag{27}$$

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Jefferson Lab

AMD Volume acceptance



AMD transverse acceptance

- Cylindrical coordinates:

$$[\frac{B_1}{B_2}](\frac{r_0}{a})^2 + (\frac{P_{r0}}{\frac{1}{2}e\sqrt{B_1 B_2}a})^2 + (\frac{P_{\phi 0}}{\frac{1}{2}}eB_2 a^2)^2[\frac{B_1}{B_2}\frac{1}{[\frac{r_0}{a}]^2} - 1] \leq 1 \quad (28)$$

- Making $P_{r0} = 0$ and $P_{\phi 0} = 0$ in (28), we get the radial acceptance for the AMD:

$$r_0^{max} = \sqrt{\frac{B_2}{B_1}}a \quad (29)$$

Making $r_0 = 0$ and $P_{\phi 0} = 0$ in (28), we get the transverse momentum acceptance for the AMD::

$$p_{x0}^{max} = \frac{1}{2}e\ a\sqrt{B_2 B_1} \quad (30)$$

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Jefferson Lab

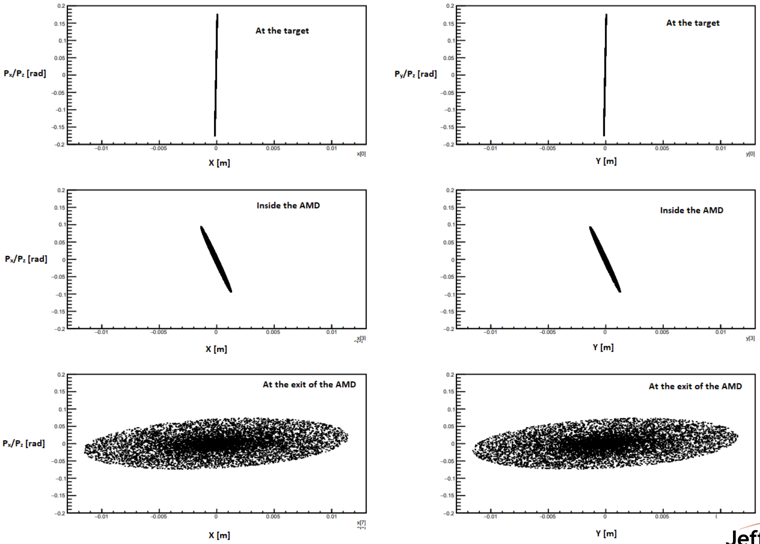
AMD Transverse acceptance

- For $B_1 = 2.5$ T and $B_2 = 0.5$ T with $\mu = 40\ m^{-1}$ and $L = 10$ cm the acceptance caracteristique are :

$$r_0^{max} = \sqrt{\frac{B_2}{B_1}} a = \sqrt{\frac{0.5}{2.5}} \times 20mm \approx 8.9mm \quad (31)$$

$$p_{x0}^{max} = \frac{1}{2} ea\sqrt{B_2 B_1} = \frac{1}{2} e \times 20mm\sqrt{2.5 \times 0.5} \leq 3.34 \frac{MeV}{c} \quad (32)$$

Transverse acceptance AMD



Plan

1. Positron collection system
2. Quarter wave transformer
3. Adiabatic matching device
4. **Conclusion & Questions**

Conclusion

- With a suitable length for the QWT we got a momentum acceptance which is much larger than the momentum acceptance in the AMD. Moreover, the wide momentum acceptance allow us to use a small radius aperture or large angle for the positron distribution at the target.
- The radial acceptance in AMD is larger than the radial acceptance in QWT.
- The volume of the phase space of the AMD is independent of the longitudinal momentum.

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Jefferson Lab