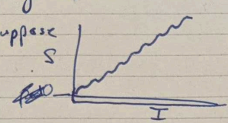
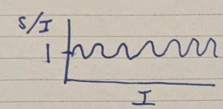
Digital BCM model 10/17/23

Osc Model

suppose



response is oscillatory



S is calibrated

$S = g(S_{raw} - ped0)$

~~$S = S_{raw} - ped0$~~

$S/I = a\sin(2\pi I/\lambda) + 1$

$S = I(1 + a\sin(2\pi I/\lambda))$

$A_S = \frac{S_0 - S_1}{S_1 + S_0} = \frac{I_0(1 + a\sin(2\pi I_0/\lambda)) - I_1(1 + a\sin(2\pi I_1/\lambda))}{I_0(1 + a\sin(2\pi I_0/\lambda)) + I_1(1 + a\sin(2\pi I_1/\lambda))}$

$= \frac{I_0 - I_1 + aI_0\sin(2\pi I_0/\lambda) - aI_1(1 + a\sin(2\pi I_1/\lambda))}{I_0 + I_1 + I_0 a\sin(2\pi I_0/\lambda) + aI_1\sin(2\pi I_1/\lambda)}$

$A_I = I_0 - I_1 / I_0 + I_1 \equiv \Delta/\Sigma$ $\quad I_0 = \frac{\Delta + \Sigma}{2}$ $\quad I_1 = \frac{\Sigma - \Delta}{2}$ $\quad \Delta = \Sigma A_I$

$\sin(x + \varepsilon) \approx \sin x + \varepsilon\cos x$ $\quad \sin(k(x+\varepsilon)) = \sin kx + k\varepsilon\cos kx$

$\sin\left(\frac{2\pi}{\lambda 2}(\Delta + \Sigma)\right) \approx \sin\left(\frac{2\pi\Sigma}{2\lambda}\right) + \Delta\cos\left(\frac{2\pi\Sigma}{2\lambda}\right)\pi/\lambda$

$\sin\left(\frac{-4\pi}{2\lambda}(\Sigma - \Delta)\right) \approx \sin\left(\frac{\pi\Sigma}{\lambda}\right) - \Delta\cos(\pi\Sigma/\lambda)\pi/\lambda$

$A_S \approx \frac{\Delta + a(\Delta+\Sigma)\frac{1}{2}\left(\sin\left(\frac{\pi\Sigma}{\lambda}\right) + \Delta\cos\left(\frac{\pi\Sigma}{\lambda}\right)\cdot\pi/\lambda\right) - a(\Sigma-\Delta)\frac{1}{2}\left(\sin\frac{\pi\Sigma}{\lambda} - \Delta\cos\frac{\pi\Sigma}{\lambda}\cdot\pi/\lambda\right)}{\Sigma + a(\Delta+\Sigma)\frac{1}{2}(\sin\pi\Sigma/\lambda + \Delta\cos\pi\Sigma/\lambda\cdot\pi/\lambda) + a(\Sigma-\Delta)\frac{1}{2}(\sin\pi\Sigma/\lambda - \Delta\cos\pi\Sigma/\lambda\cdot\pi/\lambda)}$

$\approx \frac{\Delta + a\Delta\sin\pi\Sigma/\lambda + a\Sigma\Delta\cos\pi\Sigma/\lambda\cdot\pi/\lambda}{\Sigma + a\Sigma\sin\pi\Sigma/\lambda + a\Delta^2\cos\pi\Sigma/\lambda\cdot\pi/\lambda}$

$\approx \Delta/\Sigma\left(\frac{1 + a\sin\pi\Sigma/\lambda + a\Sigma\cos\pi\Sigma/\lambda\cdot\pi/\lambda}{1 + a\sin\pi\Sigma/\lambda + a\frac{\Delta}{\Sigma}\Delta\cos\pi\Sigma/\lambda\cdot\pi/\lambda}\right)$

$\approx \Delta/\Sigma\left(1 + a\sin\pi\Sigma/\lambda + a\Sigma\cos\pi\Sigma/\lambda\cdot\pi/\lambda - a\sin\pi\Sigma/\lambda - a\frac{\Delta}{\Sigma}\Delta\cos\pi\Sigma/\lambda\cdot\pi/\lambda\right)$

$\approx \Delta/\Sigma\left(1 + a\Sigma\cos\pi\Sigma/\lambda\cdot\pi/\lambda - a\underbrace{\frac{\Delta}{\Sigma}\Delta}_{=\Delta^2/\Sigma = \Delta^2/\Sigma^2\,\Sigma}\cos\pi\Sigma/\lambda\cdot\pi/\lambda\right)$

$= A_I\left(1 + 2a\langle I\rangle\cos\left(\frac{2\pi\langle I\rangle}{\lambda}\right)\cdot\pi/\lambda - 2aA_I^2\langle I\rangle\cos\frac{2\pi\langle I\rangle}{\lambda}\cdot\pi/\lambda\right)$

$= A_I\left(1 + 2aI\cos\left(\frac{2\pi I}{\lambda}\right)\cdot\pi/\lambda - 2aA_I^2 I\cos\left(\frac{2\pi I}{\lambda}\right)\right)$

$A_S = A_I\left(1 + 2aI(1 - A_I^2)\cos\frac{2\pi I}{\lambda}\cdot\pi/\lambda\right)$

$A_S/A_I = 1 + 2aI(1 - A_I^2)\left(\cos\frac{2\pi I}{\lambda}\right)\pi/\lambda$

$A_S/A_I = 1 + (1 - A_I^2)\left(\frac{2\pi I}{\lambda}a\right)\left(\cos(2\pi I/\lambda)\right) \simeq 1 + \frac{2\pi I}{\lambda}a\cos\frac{2\pi I}{\lambda}$

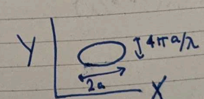
$\frac{(A_S/A_I - 1)}{(1 + \cancel{A_I^2})I} \simeq \frac{2\pi}{\lambda}a\cos(2\pi I/\lambda) \equiv Y$

$S/I - 1 = a\sin(2\pi I/\lambda) \equiv X$

$\simeq 4.15\,\mu A/70.2$

$\lambda \simeq \frac{69.6 - 7075}{70.2} \sim 16.4\,\mu A/ppm$

$a \sim 400\,ppm$



$2\pi a/\lambda \simeq 400\,ppm$

$a \simeq 200\,ppm$

$2\pi a/\lambda \approx \frac{2\pi\ 400}{16.4\,\mu} \simeq 2185$

$2a \sim 800\,ppm$

$S/I - 1 = a\sin\frac{2\pi I}{\lambda}$

$\frac{S_1 + S_0}{2} = \langle S\rangle = \bar{S}$

$\frac{I_1 + I_0}{2} = \langle I\rangle = 2\Sigma$

$\bar{S} = \frac{S_1 + S_0}{2} = \frac{I_0(1 + a\sin 2\pi I_0/\lambda) + I_1(1 + a\sin 2\pi I_1/\lambda)}{2}$

$= \frac{I_0 + I_1 + a(I_0\sin 2\pi I_0/\lambda + I_1\sin 2\pi I_1/\lambda)}{2}$

$\approx \frac{I_0 + I_1 + \frac{1}{2}a((\Sigma+\Delta)(\sin\pi\Sigma/\lambda + \frac{\pi}{\lambda}\Delta\cos\pi\Sigma/\lambda) + \frac{1}{2}a((\Sigma-\Delta)(\sin\pi\Sigma/\lambda - \frac{\pi}{\lambda}\Delta\cos\pi\Sigma/\lambda)}{2}$

$\approx \frac{\Sigma + \frac{2}{2}a\Sigma\sin\pi\Sigma/\lambda + \frac{2}{2}a\overset{\simeq 0}{\Delta^2}\frac{\pi}{\lambda}\cos\pi\Sigma/\lambda}{2}$

$\approx \frac{\Sigma + a\Sigma\sin\pi\Sigma/\lambda}{2}$

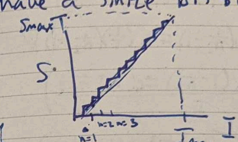
$\simeq \bar{I} + a\bar{I}\sin\pi\Sigma/\lambda$

$\bar{S} = \bar{I} + a\bar{I}\sin 2\pi\bar{I}/\lambda$

$\bar{S}/\bar{I} \approx 1 + a\sin(2\pi\bar{I}/\lambda)$

Bit Res Model

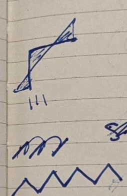
suppose since this is digital, I have a finite bit resolution.



$N_{bits} = \frac{S_{max}}{I_{max}}$

$\Delta I_{bit} = \frac{I_{max}}{N_{bits}}$

$\Delta S_{bit} = \frac{S_{max}}{N_{bits}}$



let $S = (S_{raw} - ped)\,gain$

so $S \sim I$ & $\Delta S_{bit} = \Delta I_{bit}$

$S = \Delta S_{bit}\,n = I$

$n = (I/\Delta I_{bit})$ rounded. $\equiv a_{rounded}$

$S - I = \Delta S_{bit}\,n - I = \Delta I_{bit}\,n - \Delta I_{bit}\,a$

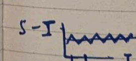
$= \Delta S_{bit}\left(\frac{I}{\Delta I_{bit}}\right)_{rounded} - I$

$= \Delta I_{bit}(n - a)$ $\quad n = a_{rounded}$

$= \Delta I_{bit}(a \pm \varepsilon - a)$ $\quad 0 < \varepsilon < 1/2$

$S - I = \Delta I_{bit} \pm \varepsilon$ $\quad 0 < \varepsilon < 1/2$

$S - I = \frac{I_{max}}{N_{bits}}(\pm\varepsilon)$ $\quad 0 < \varepsilon < 1/2$



$\varepsilon = Im + b$

$A_S = \frac{S_0 - S_1}{S_0 + S_1} = \frac{n_0 - n_1}{n_0 + n_1} = \frac{a_0 \pm \varepsilon_0 - (a_1 \pm \varepsilon_1)}{a_0 \pm \varepsilon_0 + a_1 \pm \varepsilon_1} = \frac{\overbrace{a_0 - a_1}^{\delta} \pm (\varepsilon_0 - \varepsilon_1)}{\underbrace{a_0 + a_1}_{2\bar{S}} \pm (\varepsilon_0 + \varepsilon_1)}$

$\approx \frac{\delta/\bar{S} \pm (\varepsilon_0 - \varepsilon_1)/\bar{S}}{\bar{S}/\bar{S} \pm (\varepsilon_0 + \varepsilon_1)/\bar{S}} = \frac{\delta}{\bar{S}}\left(1 \pm \frac{(\varepsilon_0 - \varepsilon_1)}{\delta} + \frac{(\varepsilon_0 + \varepsilon_1)}{\bar{S}}\right)$

$$\frac{\delta}{T} = A_I$$
$$\delta = A_I T$$
$$T = I_0 + I_1 = \langle I \rangle 2$$

$$A_S \approx \frac{\delta}{T}\left(1 \pm \frac{\varepsilon_0 - \varepsilon_1}{\delta} \mp \frac{\varepsilon_0 + \varepsilon_1}{T}\right)$$
$$\approx \frac{\delta}{T}\left(1 + \varepsilon_0\left(\pm\frac{1}{\delta} \mp \frac{1}{T}\right) + \varepsilon_1\left(\mp\frac{1}{\delta} \mp \frac{1}{T}\right)\right)$$
$$\approx \frac{\delta}{T}\left(1 \pm \varepsilon_0\left(\frac{1}{\delta} - \frac{1}{T}\right) \mp \varepsilon_1\left(\frac{1}{\delta} + \frac{1}{T}\right)\right)$$
$$\approx A_I\left(1 \pm \varepsilon_0\left(\frac{1}{T}\right)\left(\frac{1}{A_I} - 1\right) \mp \varepsilon_1\left(\frac{1}{T}\right)\left(\frac{1}{A_I} + 1\right)\right)$$
$$\approx A_I\left(1 - \frac{\pm\varepsilon_0\left(1 - \frac{1}{A_I}\right) \pm \varepsilon_1\left(1 + \frac{1}{A_I}\right)}{T}\right)$$
$$\approx A_I\left(1 - \frac{\pm\varepsilon_0\left(1 - \frac{1}{A_I}\right) \pm \varepsilon_1\left(1 + \frac{1}{A_I}\right)}{2\langle a \rangle}\right)$$
$$\approx A_I - \left(\frac{\pm\varepsilon_0(A_I - 1) \pm \varepsilon_1(A_I + 1)}{2\langle a \rangle_{bit}}\right)$$

if $\varepsilon_0 = 1/2$, $\varepsilon_1 = 1/2$

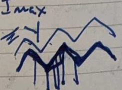
$$\frac{1/2\,(A_I - 1 + 1)}{2\langle a \rangle} = \frac{A_I}{2\langle a \rangle}$$

$$\langle a \rangle = \frac{\langle I \rangle}{\Delta I_{bits}} = \frac{\langle I \rangle N_{bits}}{I_{max}}$$

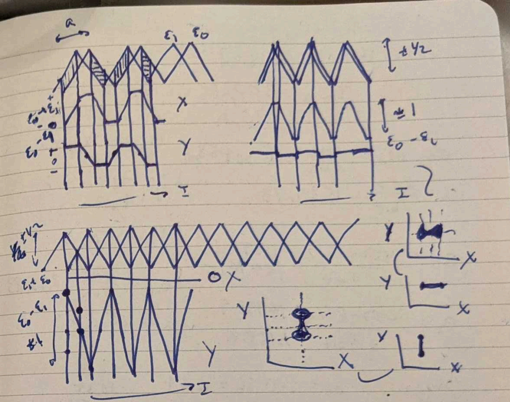
$$\approx A_I - \left(\frac{\pm\varepsilon_0(A_I - 1) \pm \varepsilon_1(A_I + 1)}{2\langle I \rangle N_{bits}}\right) I_{max}$$

$$A_S \approx A_I + \left(\frac{\pm\varepsilon_0 \mp \varepsilon_1}{2\langle I \rangle N_{bits}}\right) I_{max}$$

$$(A_S - A_I)\frac{\langle I \rangle N_{bits} 2}{I_{max}} = \pm\varepsilon_0 \mp \varepsilon_1 = \pm(\varepsilon_0 - \varepsilon_1)$$



$0 < \varepsilon_0 < 1/2$
$0 < \varepsilon_1 < 1/2$



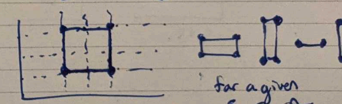
$$A_I \rightarrow \varepsilon_0 - \varepsilon_1 = \delta\varepsilon$$
$$I_0 - I_1 = \delta I$$

N_{bits}

$$\delta I / I_{max} = \delta a$$
$$\delta I / \Delta I_{bits} = \delta a$$
$$= \delta a \pm \varepsilon_0 \mp \varepsilon_1$$

$$(S_0 - I_0)\frac{N_{bits}}{I_{max}} = \pm\varepsilon_0 \quad 0 < \varepsilon < 1/2$$
$$(S_1 - I_1)\frac{N_{bits}}{I_{max}} = \pm\varepsilon_1$$
$$X \equiv (\bar{S} - \bar{I})\frac{N_{bits}}{I_{max}} 2 = \frac{\pm\varepsilon_0 \pm\varepsilon_1}{2} = \pm(\varepsilon_0 + \varepsilon_1)$$

$$\pm(\varepsilon_0 - \varepsilon_1) \equiv Y$$



for a given $\varepsilon_0, \varepsilon_1$ $\underline{A_I}$

$$X = \pm(\varepsilon_0 + \varepsilon_1)$$

16-bit $N_{bits} = 2^{16} = 65536$

$I_{max} = 300\,\mu A$
$\langle I \rangle = 70\,\mu A$

$A_{I_5} \sim 5$ kppm
$A_{I_{10}} \sim 10$ kppm

$$\Delta I_{bit} = \frac{I_{max}}{N_{bits}} = 0.00458\,\mu A$$

$$a = \frac{\langle I \rangle}{I_{bit}} = 15291.73$$

$$\delta I_{10} = A_I \cdot 2\langle I \rangle = 1.4\,\mu A$$

$$\delta I_5 = 0.7\,\mu A$$

$$a_0 = \frac{\langle I \rangle + \delta I/2}{I_{bit}} = \begin{matrix}15597.56\\15444.65\end{matrix}$$

$$\delta a_{10} = \frac{\delta I_{10}}{I_{bit}} = 305.83$$

$$a_1 = \frac{\langle I \rangle - \delta I/2}{I_{bit}} = \begin{matrix}14985.9\\15138.81\end{matrix}$$

$$\delta a_5 = 152.92$$

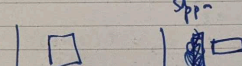
10 kppm: $\varepsilon_0 = 1 - 0.56 = 0.44$ $\quad \Delta 0.34$
$\varepsilon_1 = 1 - 0.9 = 0.1$ $\quad + 0.54$

5 kppm $\varepsilon_0 = 1 - 0.65 = 0.35$ $\quad \Delta 0.16$
$\varepsilon_1 = 1 - 0.81 = 0.19$ $\quad + 0.54$

10 kppm

5 ppm

ε_0 ε_1



$\varepsilon_0 + \varepsilon_1$

$0 < \varepsilon_0 < 1/2 \quad 0 < \varepsilon_1 < 1/2$

$$A_S = A_I + \left(\frac{\pm\varepsilon_0 \mp \varepsilon_1}{2\langle I \rangle N_{bits}}\right) I_{max}$$

$$A_S = A_I \pm (\pm\varepsilon_0 \mp \varepsilon_1)\, 32.7\,\text{ppm}$$

$$\frac{I_{max}}{2\langle I \rangle N_{bits}} = 32.7\,\text{ppm}$$

16 bit

14 bit: $N_{bits} = 2^{14}$

14 bit
$$A_S = A_I \pm (\varepsilon_0 \mp \varepsilon_1)\, 130\,\text{ppm}$$

$$\frac{I_{max}}{2\langle I \rangle N_{bits}} = 130\,\text{ppm}$$

12 bit
$$\frac{I_{max}}{2\langle I \rangle N_{bits}} = \underline{523\,\text{ppm}}$$

12 bit $A_S = A_I \pm (\pm\varepsilon_0 \pm \varepsilon_1)\, 523\,\text{ppm}$

$\Delta I_{bit} = 0.07\,\mu A$
$\approx 1000\,\text{ppm}$