GTS Emittance Measurement with Single Slit and Viewscreen

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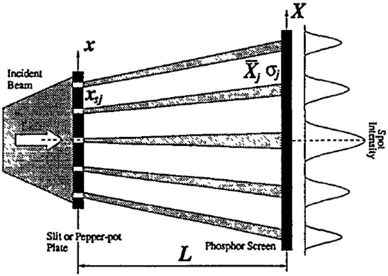
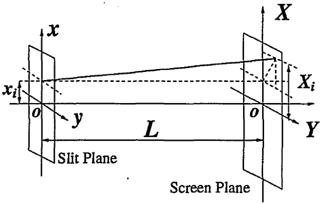
Single Slit and View Screen Setup





- Drift Length (L) $= 500$ mm
- Constant step size $w \approx 0.76$ mm
- Measurement steps $m = 1, ..., N$: $N = 21$

Using Mathematica for Calculations

- Mathematica code allows for quick analysis of data with little to no prior calculations
- Data should be a $(n+1) \times 4$ matrix: The first row contains the column descriptions: Beamlet position on V2 (in pixels), Sigma Y (in pixels), Beamlet Intensity (unitless), Beam position on V1 (in pixels). The subsequent rows contain data for the n measurements.
- Parameters necessary (these need to be input directly into the script itself):
  - mm/pixel conversion factors for each viewer
  - Laser size (mm)
  - Drift Length (mm)
  - Value of $\beta\gamma$ (1.23)
- Everything else is calculated using Mathematica as follows...

Calculation of Slit Positions x_{sj} using Gaussian Fit

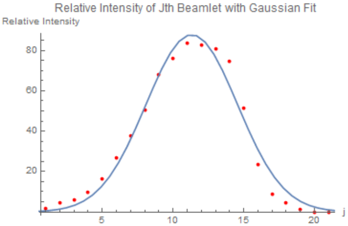


Figure: Plot of Beam Intensity of the jth beamlet (orange) with Gaussian fit (blue)

- The position of the mean of the Gaussian fit (i.e. centroid) is rounded to the nearest integer value. Mathematica finds the data point with its j-value closest to this value, then sets it to be the center beamlet position x_c
- The beamlet position data on V2, x_j is then centered about x_c using $x_{j,centered} = x_j - x_c$
- The slit image positions on V2, X_j, are given by $X_j = x_{j,centered} + j \times stepsize$
- The slit positions on V1 (if V1 were a multislit mask), x_{sj} , are given by $x_{sj} = j \times stepsize$
- The stepsize is calculated by averaging the intervals between successive beam positions on V1

Calculations

$$\bar{x} = \frac{1}{N}\sum_{j=1}^{p} n_j x_{sj},\ \bar{X}_j = \frac{1}{n_j}\sum_{i=1}^{n_j} X_{ji}$$

$$\bar{x'_j} = \frac{\bar{X}_j - x_{sj}}{L},\ \bar{x'} = \frac{1}{N}\sum_{j=1}^{p} n_j \bar{x'_j}$$

$$\sigma_{x'_j} = \frac{\sigma_j}{L}$$

$$\langle x^2 \rangle = \frac{1}{N}\sum_{j=1}^{p} n_j \left(x_{sj} - \bar{x}\right)^2$$

$$\langle x'^2 \rangle = \frac{1}{N}\sum_{j=1}^{p} \left[n_j \sigma_{x'_j}^2 + n_j \left(\bar{x'_j} - \bar{x'}\right)^2\right]$$

$$\langle xx' \rangle = \frac{1}{N}\left(\sum_{j=1}^{p} n_j x_{sj} \bar{x'_j} - N\bar{x}\bar{x'}\right)$$

$$\varepsilon_x^2 \equiv \langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2$$

$$= \frac{1}{N^2}\left\{\left[\sum_{j=1}^{p} n_j \left(x_{sj} - \bar{x}\right)^2\right]\left[\sum_{j=1}^{p}\left[n_j \sigma_{x'_j}^2 + n_j \left(\bar{x'_j} - \bar{x'}\right)^2\right]\right] - \left[n_j x_{sj} \bar{x'_j} - N\bar{x}\bar{x'}\right]^2\right\}$$

x_{sj} = j-th slit's position
$\bar{x}$ = Mean position of all beamlets (just after V1)
$\bar{X}_j$ = Mean position of the j-th spot on the screen (V2)
$\bar{x'_j}$ = Mean divergence of j-th beamlet
$\bar{x'}$ = Mean divergence of all beamlets
$\sigma_{x'_j}$ = RMS divergence of all beamlets
j = Slit number
p = Total number of slits (measurements)
n_j = Intensity of j-th beamlet (number of particles passing through the j-th slit)
N = Sum of all beamlet intensities n_j
ε_x = RMS emittance

Results

First Sum	4.19E+03
Second Sum	1.68E+04
Third Sum	-8.40E+03
Emittance (mm-mrad)	$(6.11E-01)\,i$
Normalized Emitance (mm-mrad)	$(7.51E-01)\,i$
Thermal Angle (mrad)	$7.09i$

Table: Calculation of Emittance and Thermal Angle