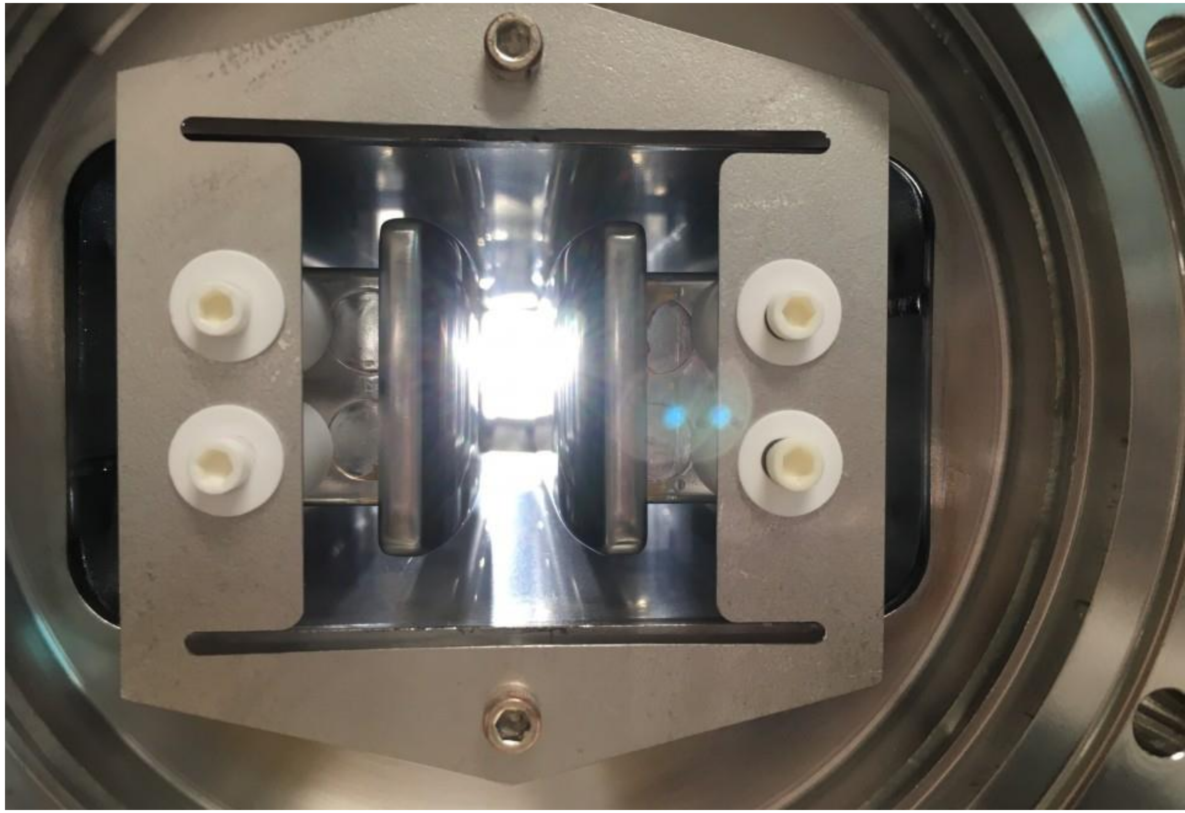
Wien Controls and Spin Rotator Calibration

Calibrate Vertical Wien, Spin Solenoids, and Horizontal Wien with Mott Polarimeter



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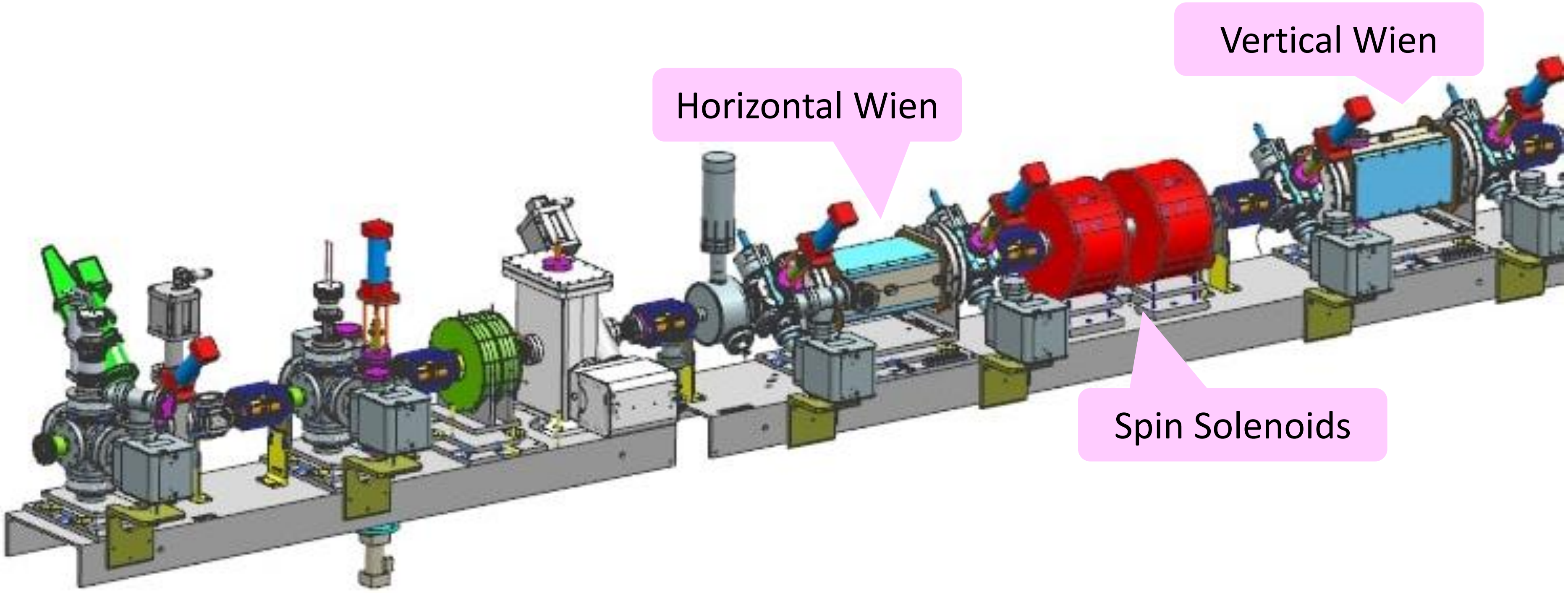
Monday, January 25, 2021

Calibration

CEBAF Spin Rotator



Polarization from Photocathode to Mott Polarimeter

- Polarization is longitudinal from photocathode: $\boldsymbol{P_i} = \begin{bmatrix} 0 \\ 0 \\ P_l \end{bmatrix}$

$$[\boldsymbol{Pol\ Mott}] = [\mathbf{12.5^\circ}\ \boldsymbol{Mott\ Dipole}]\left[\tfrac{\mathbf{1}}{\mathbf{4}}\boldsymbol{Unit}\right][\boldsymbol{HWien}][\boldsymbol{Solenoids}][\boldsymbol{VWien}][\mathbf{15^\circ}\ \boldsymbol{Dipole}][\boldsymbol{Gun\ E}]\ \boldsymbol{P_i}$$

Beam Request

- Beam Kinetic Energy: 5 MeV
- Beam Current: 1 μA
- Mott Au Foil: 1 μm
- Beam time for each angle: 20 minutes
- Calibration Plan – 3 shifts:
  - I. Vertical Wien: 0, 30, 60, 90,0,-30,-60,-90 (HW=0, S=0)
  - II. Solenoids: 0, 30, 60, 90,0,-30,-60,-90 (VW=90, HW=0)
  - III. Solenoids: 0, 30, 60, 90,0,-30,-60,-90 (VW=-90, HW=0)
  - IV. Horizontal Wien: 0, 30, 60, 90,0,-30,-60,-90 (VW=0, S=0)
  - V. Horizontal Wien: 0, 30, 60, 90,0,-30,-60,-90 (VW=90, S=90)
  - VI. Horizontal Wien: 0, 30, 60, 90,0,-30,-60,-90 (VW=90, S=-90)
  - VII. Horizontal Wien: 0, 30, 60, 90,0,-30,-60,-90 (VW=-90, S=90)
  - VIII. Horizontal Wien: 0, 30, 60, 90,0,-30,-60,-90 (VW=-90, S=-90)

T-BMT Spin Equation in Lab Frame

- Generalized Thomas-BMT equation in Lab Frame:

$$\frac{d\vec{S}}{dt} = \vec{\Omega}_s \times \vec{S}$$

Same as Jackson's Equation 11.170, p. 559
(Note: Jackson's is in **cgs** units)

$$\vec{\Omega}_s = -\frac{q}{m}\left[\left(a + \frac{1}{\gamma}\right)\vec{B} - \frac{a\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta} - (a + \frac{1}{\gamma+1})\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

$$\frac{d\vec{\beta}}{dt} = -\frac{q}{m\gamma c}[\vec{E} + c\vec{\beta}\times\vec{B} - (\vec{\beta}\cdot\vec{E})\vec{\beta}]$$

$$a = \frac{g}{2} - 1$$

Comparison of Spin Dynamics in the Cylindrical and Frenet–Serret Coordinate Systems
A. J. Silenko
https://link.springer.com/article/10.1134/S1547477115010197

Spin Equation Relative to Particle Momentum

- For CEBAF Spin Rotator, spin precession relative to electron momentum is given by:

$$\frac{d\vec{S}}{dt} = \Delta\vec{\Omega} \times \vec{S}$$

$$\Delta\vec{\Omega} = \vec{\Omega}_{spin} - \vec{\Omega}_{mom}$$

$$\Delta\vec{\Omega} = -\frac{q}{m}\left[a\vec{B}_{\perp} + \frac{1}{\gamma}(a+1)\vec{B}_{\parallel} - (a - \frac{1}{\gamma^2 - 1})\frac{\vec{\beta} \times \vec{E}}{c}\right]$$

- Note:$\gamma^2 - 1 = \beta^2\gamma^2$
- With electron charge, $q = -e$

$$a = \frac{g}{2} - 1$$

$$\Delta\vec{\Omega} = \frac{e}{m}\left[a\vec{B}_{\perp} + (\frac{1}{\gamma^2 - 1} - a)\frac{\vec{\beta} \times \vec{E}}{c}\right]$$

Spin Precession in Wien

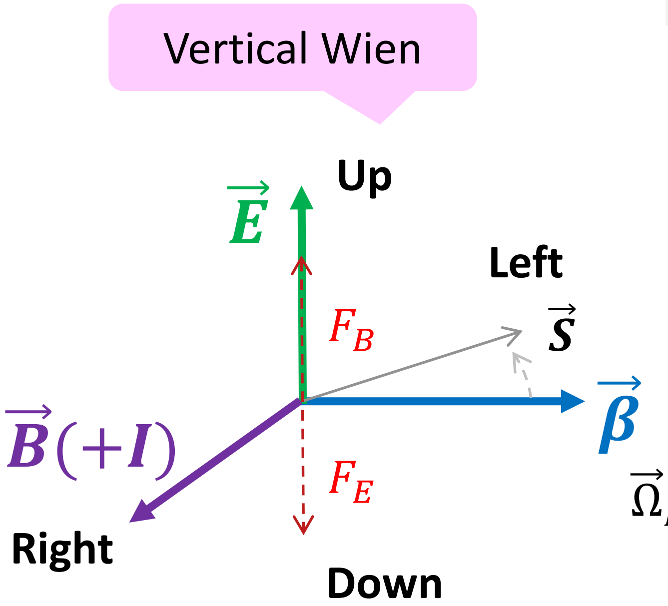
$$\Delta\vec{\Omega} = \frac{e}{m}\left[\frac{1}{\gamma}(a+1)\vec{B}_{\parallel}\right]$$

Spin Precession in Solenoid

Vertical Wien

Vertical Wien

$$\Delta\vec{\Omega} = \frac{e}{m}\left[a\vec{B}_{\perp} + (\frac{1}{\gamma^2 - 1} - a)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$



$$\Delta\vec{\Omega} = \vec{\Omega}_B + \vec{\Omega}_E$$

$\vec{\Omega}_B + \vec{\Omega}_E$ In β, E plane

- Spin Precession Angle: $\theta = \vec{\Omega}\, T$
- Time-of-Flight of electron beam in Wien determined by E-Field: $T = L_E/(\beta c)$

Electromagnetic Forces

Electron Charge, q	$-e$
Magnetic Force, F_B	$q\,\vec{\beta}c \times \vec{B}$
Electrostatic Force, F_E	$q\,\vec{E}$

$$B = E/(\beta c)$$

$$V = Eg/2$$

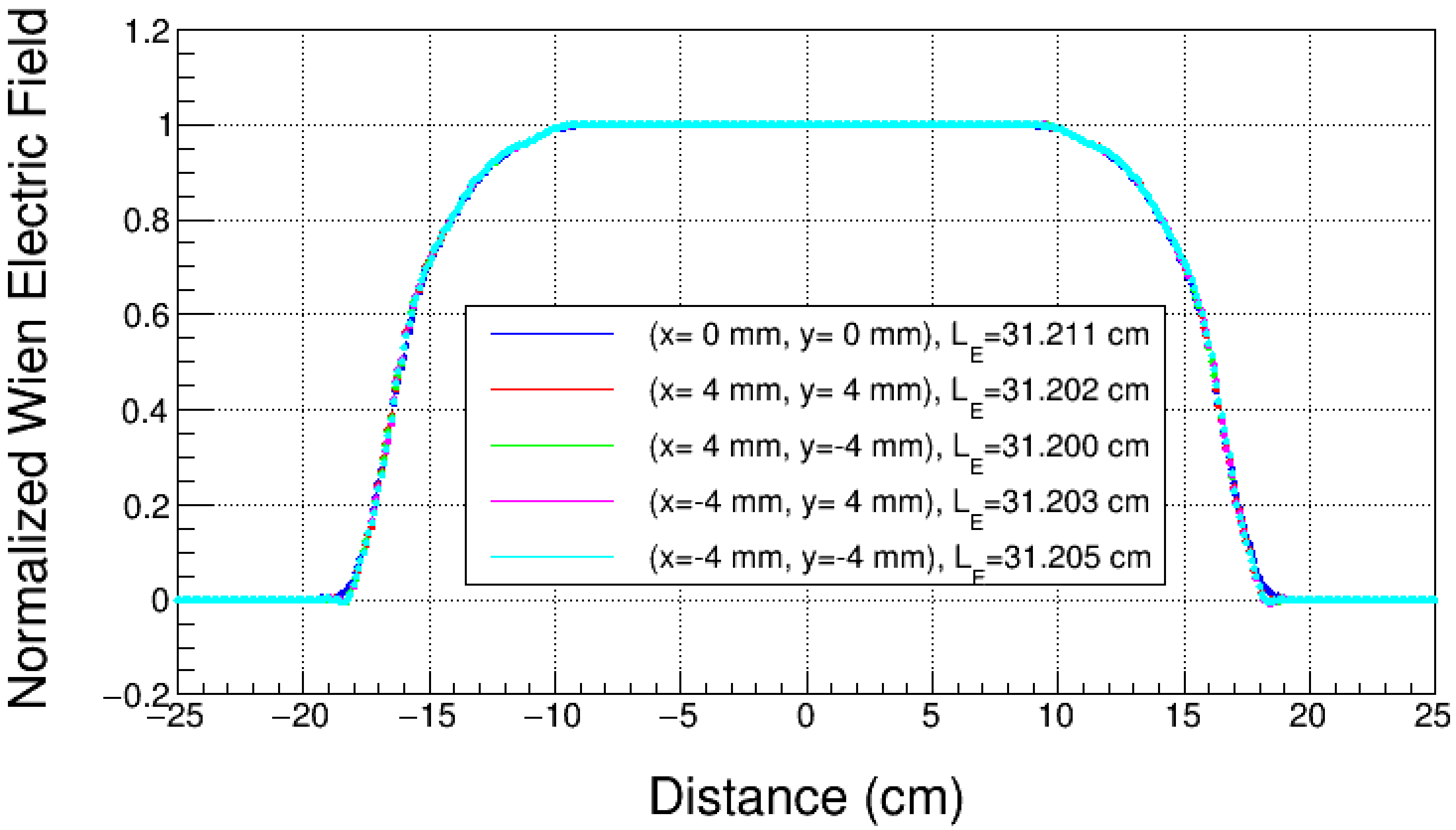
- c = 299792458.0 — Speed of Light (m/s)
- e = 1.602176e-19 — Electron Charge (C)
- mc^2 = 510998.950 — Electron Mass (eV)
- a = 0.00115965218091 — Electron Anomalous Magnetic Moment
- e/m = 1.758820088e+11 — Electron Cyclotron Frequency/Field (rad/(s T))
- L_E = 3.120e-01 — Electric Field Effective Length (m)
- L_B = 3.107e-01 — Vertical Magnetic Field Effective Length (m)
- g = 1.5e-02 — Wien Gap (m)
- E — Electric Field Strength (V/m or N/C)
- B — Magnetic Field (T or N·s/(C m))
- V — Plate Voltage on Beam Down

$$\theta = \frac{180}{\pi}\frac{e}{m}\frac{L_E}{\beta c}\left[a\frac{E}{\beta c} + (\frac{1}{\gamma^2 - 1} - a)\frac{\beta E}{c}\right]$$

Electric Field Effective Length

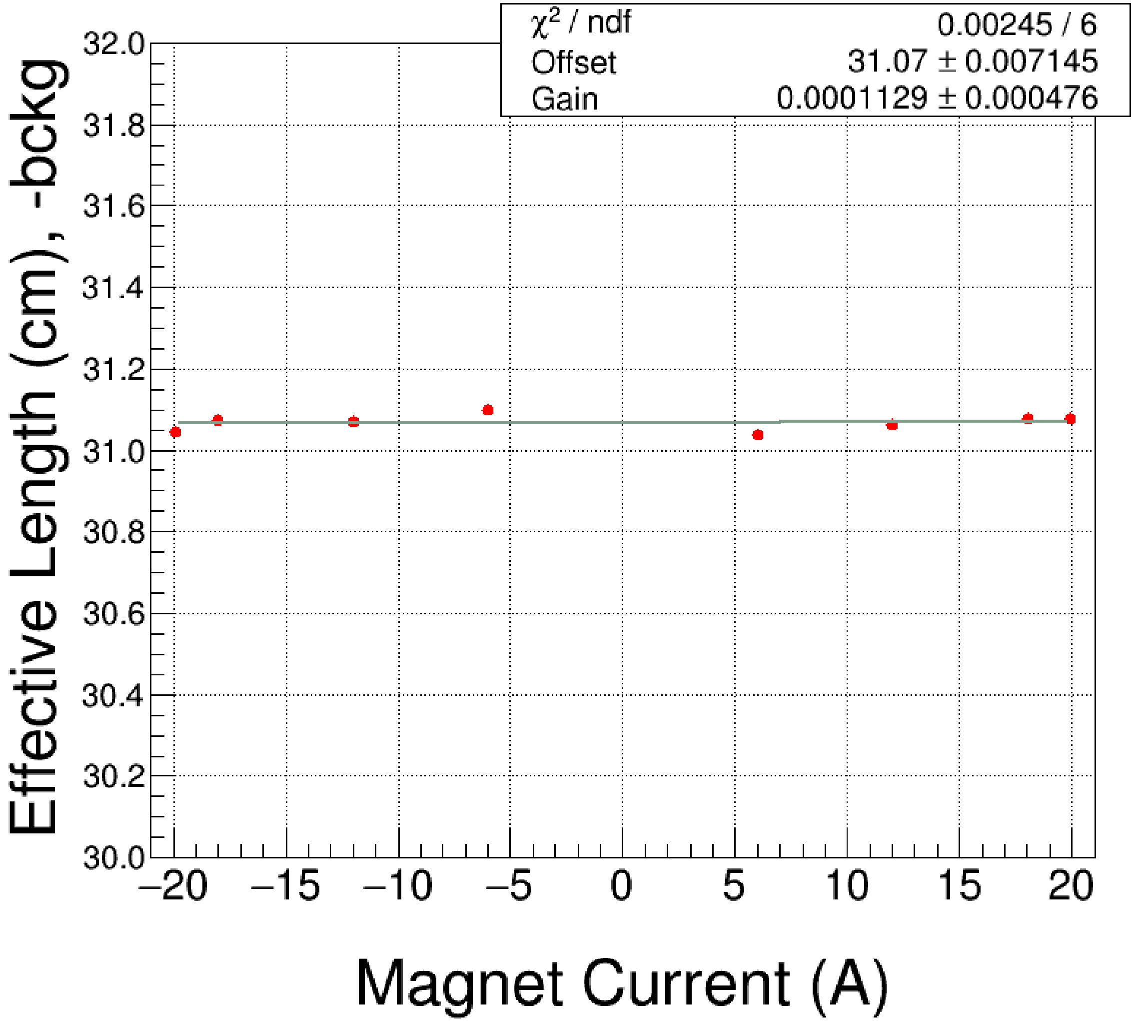
$$L_E = \frac{\int_{-25\,cm}^{25\,cm} E dz}{E_{max}}$$

CST model based on actual CEBAF drawings (faceted)

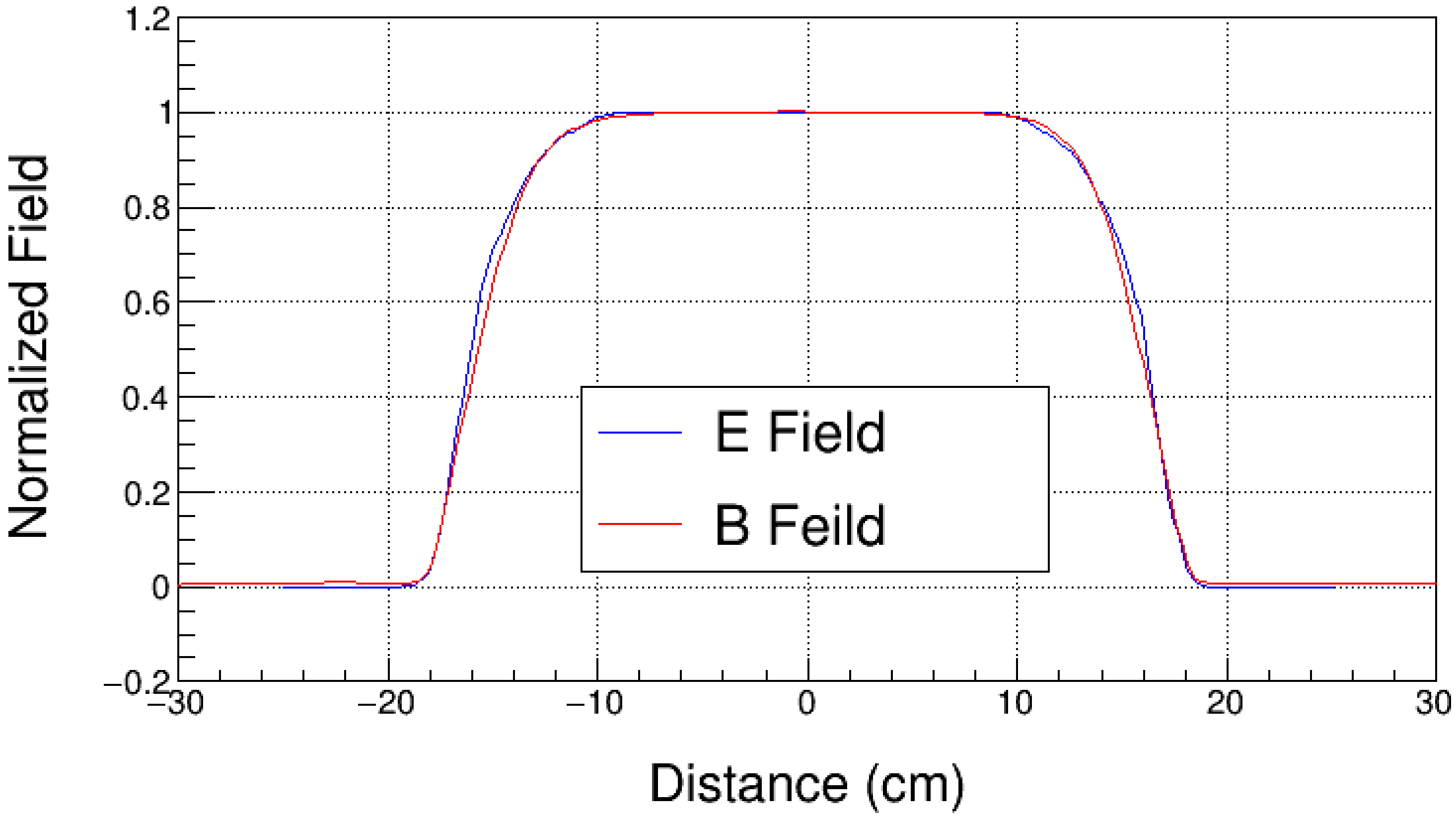


Vertical Magnetic Field Effective Length

$$L_B = \frac{\int_{-30\ cm}^{30\ cm} B dz}{B_{center}}$$



Electric and Magnetic Fields



Jefferson Lab

Wien Equations (Gun HV = 130 kV)

$B = E/(\beta c)$

$\mathrm{V} = Eg/2$

$$\theta = \frac{180}{\pi}\frac{e}{m}\frac{L_E}{\beta c}\left[a\frac{2}{g\beta c} + (\frac{1}{\gamma^2 - 1} - a)\frac{2\beta}{gc}\right] V$$

- Wien Angle: θ [deg] $= 0.00814225\, V$ [V]
- Plate Voltage: V [V] $= 122.816\, \theta$ [deg]
- Magnetic Field: Bdl [G cm] $= 0.228886\, V$ [V]

Wien Equations (Gun HV = 200 kV)

$B = E/(\beta c)$ $\mathrm{V} = Eg/2$

$$\theta = \frac{180}{\pi}\frac{e}{m}\frac{L_E}{\beta c}\left[a\frac{2}{g\beta c} + (\frac{1}{\gamma^2 - 1} - a)\frac{2\beta}{gc}\right] V$$

- Wien Angle: θ [deg] $= 0.00498929\ V$ [V]
- Plate Voltage: V [V] $= 200.429\ \theta$ [deg]
- Magnetic Field: Bdl [G cm] $= 0.198737\ V$ [V]

Vertical Wien Angle: Set and Readback (given here at 200 kV, as an example)

- **Set Wien Angle to $\boldsymbol{\theta}$** (angle rotation to beam Up is positive):
  - Beam Down Plate Voltage: V [V] $= 200.429\ \theta$ [deg]
  - Sign of voltage is same as sign of angle
  - Magnetic Field: Bdl [G cm] $= 0.198609\ V$ [V]
  - Sign of voltage at beam Up plate is opposite, $(-V)$

- **Read Wien Angle:**
  - Wien Angle: θ [deg] $= c_0 + c_1 \times 0.00498929\ V$ [V] $+ c_2\ V^2$
  - c_0, c_1, c_2: are determined by fitting Mott Asymmetry vs Plate Voltage
  - Ideally, $c_0 = \theta_0 = 0$, $c_1 = 1$, $c_2 = 0$, otherwise we will have to change "Set Wien Angle" procedure

Vertical Wien Settings

130 keV

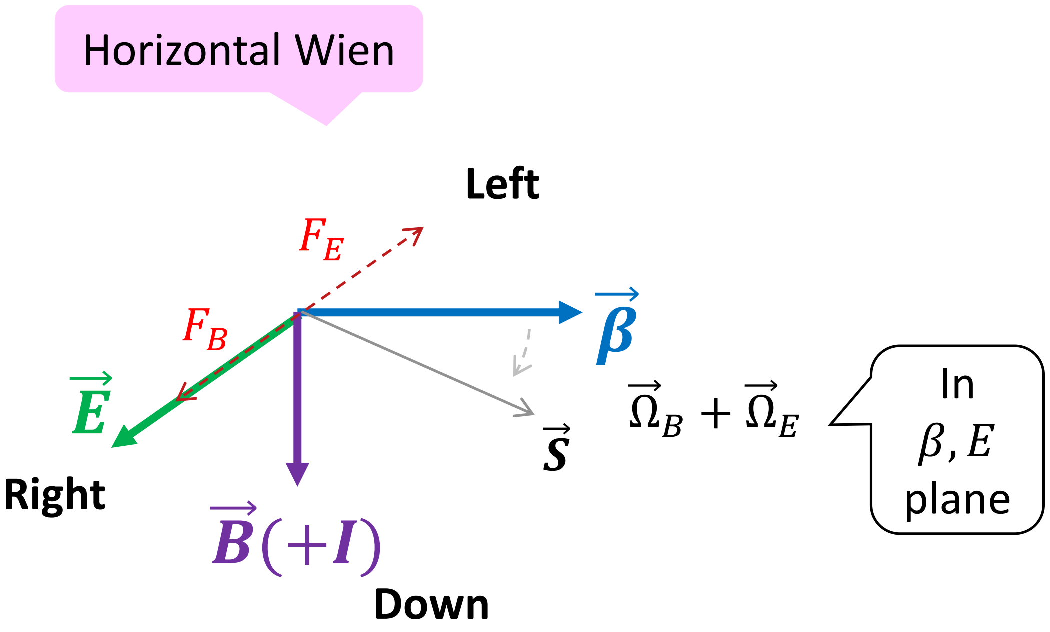
Wien Angle (degree)	Down Plate Voltage (V)	Up Plate Voltage (V)	Magnetic Field (G cm)
90.0	11053.5	-11053.5	25230.0
45.0	5526.7	-5526.7	1265.0
0.0	0.0	0.0	0.0
-45.0	-5526.7	5526.7	-1265.0
-90.0	-11053.5	11053.5	-2530.0

200 keV

Wien Angle (degree)	Down Plate Voltage (V)	Up Plate Voltage (V)	Magnetic Field (G cm)
90.0	18038.6	-18038.6	3584.9
45.0	9019.3	-9019.3	1792.5
0.0	0.0	0.0	0.0
-45.0	-9019.3	9019.3	-1792.5
-90.0	-18038.6	18038.6	-3584.9

Horizontal Wien

Horizontal Wien



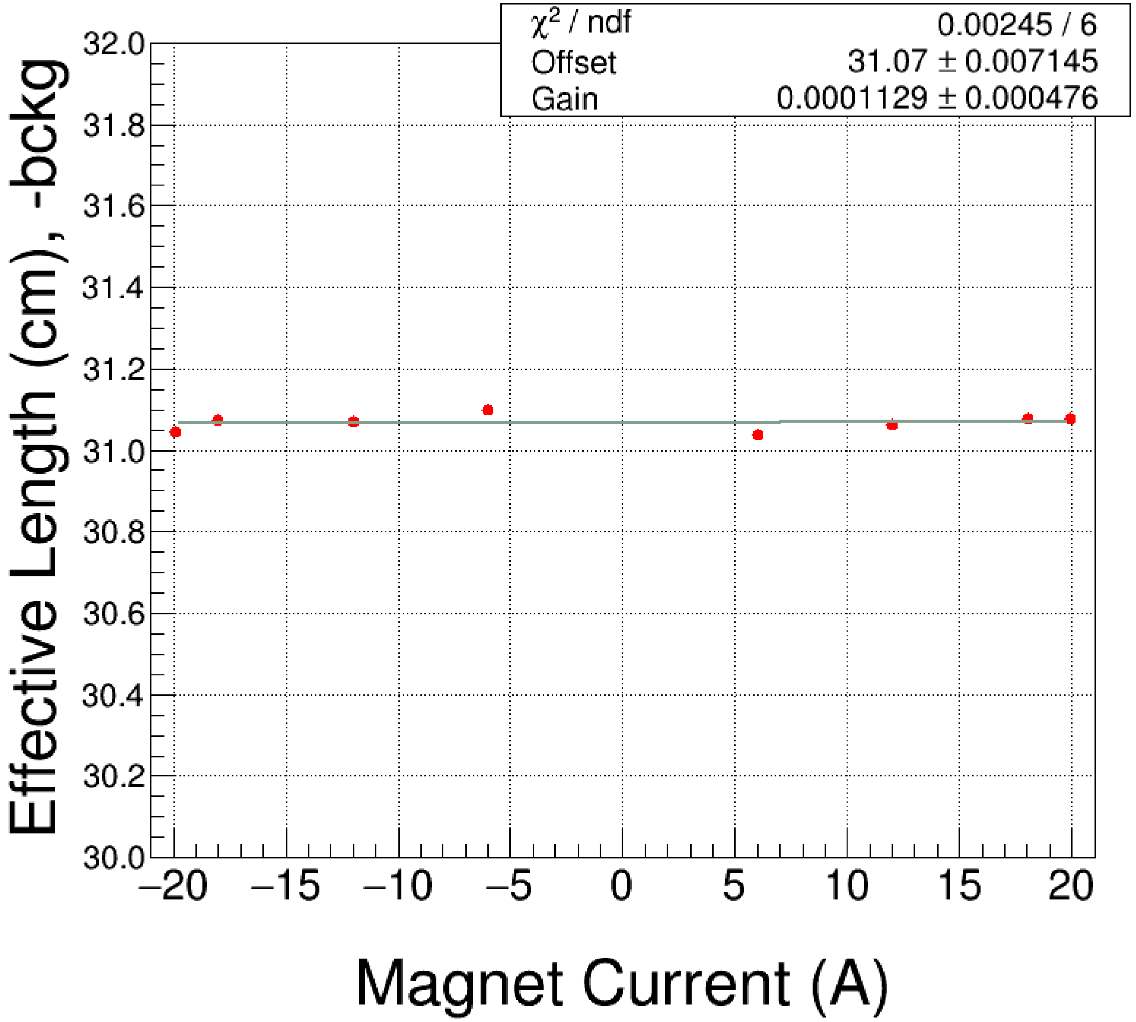
Gun HV 130 kV:

- Wien Angle: θ [deg] $= 0.00814225\, V$ [V]
- Plate Voltage: V [V] $= 122.816\, \theta$ [deg]
- Magnetic Field: Bdl [G cm] $= 0.228886\, V$ [V]

Gun HV 200 kV:

- Wien Angle: θ [deg] $= 0.00498929\, V$ [V]
- Plate Voltage: V [V] $= 200.429\, \theta$ [deg]
- Magnetic Field: Bdl [G cm] $= 0.198737\, V$ [V]

Horizontal Magnetic Field Effective Length



Horizontal Wien Angle: Set and Readback (given here at 200 kV, as an example)

- **Set Wien Angle to $\boldsymbol{\theta}$** (angle rotation to beam Right is positive):
  - Beam Left Plate Voltage: V [V] $= 200.429\ \theta$ [deg]
  - Sign of voltage is same as sign of angle
  - Magnetic Field: Bdl [G cm] $= 0.198737\ V$ [V]
  - Sign of voltage at beam Right plate is opposite, ($-V$)

- **Read Wien Angle:**
  - Wien Angle: θ [deg] $= c_0 + c_1 \times 0.00498929\ V$ [V] $+ c_2\ V^2$
  - c_0, c_1, c_2: are determined by fitting Mott Asymmetry vs Plate Voltage
  - Ideally, $c_0 = \theta_0 = 0$, $c_1 = 1$, $c_2 = 0$, otherwise we will have to change "Set Wien Angle" procedure

Horizontal Wien Settings

130 keV

Wien Angle (degree)	Left Plate Voltage (V)	Right Plate Voltage (V)	Magnetic Field (G cm)
90.0	11053.5	-11053.5	25230.0
45.0	5526.7	-5526.7	1265.0
0.0	0.0	0.0	0.0
-45.0	-5526.7	5526.7	-1265.0
-90.0	-11053.5	11053.5	-2530.0

200 keV

Wien Angle (degree)	Left Plate Voltage (V)	Right Plate Voltage (V)	Magnetic Field (G cm)
90.0	18038.6	-18038.6	3584.9
45.0	9019.3	-9019.3	1792.5
0.0	0.0	0.0	0.0
-45.0	-9019.3	9019.3	-1792.5
-90.0	-18038.6	18038.6	-3584.9

Spin Solenoids

Spin Solenoids

$$\Delta\vec{\Omega} = \frac{e}{m}\left[\frac{1}{\gamma}(a+1)\vec{B}_{\parallel}\right]$$

Dipoles

15° Dipole and Mott 12.5° Dipole

- Lab Frame:

$$\vec{\Omega}_{spin} = \frac{e}{m}\left[\left(a + \frac{1}{\gamma}\right)\vec{B}_{\perp}\right]$$

- Relative to Particle Momentum:

$$\Delta\vec{\Omega} = \frac{e}{m}\left[aB_{\perp}\right]$$

Mott Polarimeter

Mott Polarimeter

- Simultaneously measures beam transverse polarization components: P_x and P_y
- $P = \sqrt{P_x^2 + P_y^2 + P_z^2}$