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Recombination Rate Calculation

1 Purpose

Calculate Recombination Rate based on QE of photocathode.

2 Derivation

By definition,

$$QE = \frac{N_{e^-}}{N_\gamma} \tag{1}$$

where N_{e^-} is the number of electrons emitted and N_γ is the number of photons incident on the photocathode. For a uniform laser (fixed wavelength λ and power P), the laser power in some time interval t is given by

$$P\,(W) = \frac{EN_\gamma}{t} = \frac{hc}{\lambda}\frac{N_\gamma}{t} \tag{2}$$

since $E_\gamma = \dfrac{hc}{\lambda}$ is the energy of a photon with wavelength λ. Solving for N_γ yields

$$N_\gamma = \frac{P\lambda t}{hc} \tag{3}$$

When the photons strike the photocathode in some area, some number N_{e^-} of electrons will be emitted. The SI unit of current I is the ampere, which is one coulomb per second, and can be related to electrons per second when noting that one coulomb is equivalent to the charge of 6.242×10^{18} electrons and so

$$N_{e^-} = I\,(\text{A})\,t\,(\text{s}) \times \left(\frac{6.242 \times 10^{18} \text{ electrons}}{1 \text{ Coulomb}}\right) \tag{4}$$

Dividing (4)by (3) yields the equation for QE:

$$\begin{aligned} QE = \frac{N_{e^-}}{N_\gamma} &= I\,(\text{A})\,t\,(\text{s})\left(6.242 \times 10^{18}\right) \times \frac{hc}{P\lambda t} \\ &= I\,(\text{A})\,t\,(\text{s})\left(6.242 \times 10^{18}\right) \times \frac{1240\text{eV nm}}{P\left(\frac{\text{eV}}{\text{s}}\right)\lambda\,(\text{nm})\,t\,(\text{s})} \\ &= \frac{I\,(\text{A})\left(6.242 \times 10^{18}\right)}{P\left(\frac{\text{eV}}{\text{s}}\right)}\frac{1240\text{eV nm}}{\lambda\,(\text{nm})} \end{aligned}$$

We can solve this for power P and plug in some typical numbers for the ghost beam: $I = 4\text{nA} = 4 \times 10^{-9}\text{A}$, $\lambda = 122\text{nm}$ (highest intensity spectral line of hydrogen gas), $QE = 0.2\,(= 20\%)$. Thus, the total power of all photons incident on the photocathode that produce the ghost beam is

$$\begin{aligned} P\left(\frac{\text{eV}}{\text{s}}\right) &= \frac{\left(4 \times 10^{-9}\text{A}\right)\left(6.242 \times 10^{18}\right)(1240\text{eV nm})}{(0.2)\,(122\text{nm})} \\ &\approx 1.27 \times 10^{12}\frac{\text{eV}}{\text{s}} \\ &= 0.203\mu\text{W} \end{aligned}$$

Based on this power, the number of photons per second is:

$$\begin{aligned} \frac{N_\gamma}{t} = \frac{P\lambda}{hc} &= \frac{\left(1.27 \times 10^{12}\frac{\text{eV}}{\text{s}}\right)(122\text{nm})}{(1240\text{eV nm})} \\ &= 1.25 \times 10^{11}\frac{photons}{s} \end{aligned}$$

Not all of the photons produced due to recombination actually make it to the photocathode. Only photons that are within a solid angle that encompasses the photocathode can produce the ghost beam. Assume that all of these recombinations take place at the center of the beamline 0.15m away from the photocathode. This corresponds to somewhere between the anode and the middle of the magnetizing solenoid, i.e. where we think we are trapping ions and electrons. The photocathode has a radius of 6.4mm. Thus, the angle θ subtended by the solid angle of the photocathode is

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{6.4 \times 10^{-3}\text{m}}{0.15\text{m}}\right) \\ &= 0.0426\text{radians}\end{aligned}$$

The solid angle Ω on a unit sphere corresponding to this θ is

$$\begin{aligned}\Omega &= 4\pi \sin\left(\frac{0.0426\text{radians}}{2}\right) \\ &= 0.2676\end{aligned}$$

(If $\theta = \pi$ radians, then $\Omega = 4\pi$). The fraction of surface area taken up by the solid angle compared to the total surface area of the unit sphere is

$$\frac{\Omega}{4\pi} = \frac{0.2676}{4\pi} \approx 0.0213 = 2.13\%$$

Note that the solid angle is *independent of radius*. Thus, if $1.25 \times 10^{11} \frac{photons}{s}$ equates to 2.13% of all photons produced in the trap, then by proportionality, the total number of photons produced per second within the trap (independent of direction) is

$$\begin{aligned}\frac{1.25 \times 10^{11} \frac{photons}{s}}{2.13\%} &= \frac{x}{100\%} \\ x &\approx 5.87 \times 10^{12} \frac{photons}{s}\end{aligned}$$

Since in each radiative recombination, it is assumed that one photon is emitted, this is equal to the recombination rate.