A detailed examination of the MDL field map and the TOSCA model of this "5 MeV" dipole

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Abstract

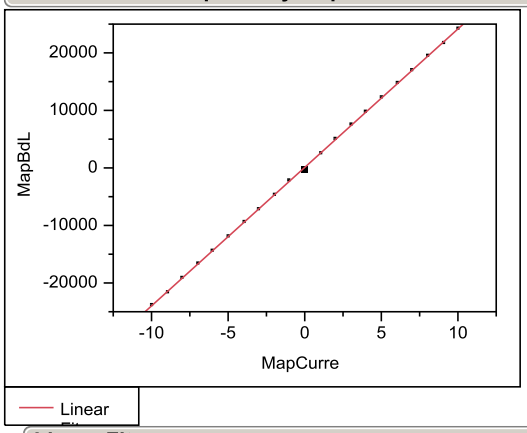
The MDL field map from EPICS and the TOSCA model of the dipole have been examined in great detail in support of the upcoming bubble chamber experiment which is aiming for 0.1% energy accuracy. The TOSCA model with minimum allowable gap allowed by drawing and the TOSCA default BH curve agrees with the EPICS field map at the 0.1% level in the linear approximation. Fifth order fits are required to reduce residuals to small levels and make them normal for either field map or model. BdL vs P and BdL vs KE tables were generated from the model for three beam lines: -30^o, $+25^o$ and -12.5^o. These also require higher order fits if residuals are to be normal. These tables or fits provide a much more accurate method for setting BdL(P) than the hitherto used approximation to the usual formula for long magnets, $B*\rho(T\text{-}m) = 3.335641*P(GeV/c)$. The MDL dipole and its predecessor are both 102 mm long and have ~26 mm gaps, so length/gap is small.

EPICS field map

Magnet ID: DL 001, location 0L02

Amps	Hall Probe BdL(G-cm)
-9.992	-23944.2
-8.996	-21569.6
-7.991	-19169.0
-6.990	-16769.5
-5.990	-14360.7
-4.993	-11954.6
-3.994	-9542.8
-2.989	-7116.1
-1.989	-4698.2
-0.990	-2283.6
0.003	126.0
1.009	2548.4
2.009	4960.8
3.009	7374.6
4.010	9785.8
5.010	12192.0
6.010	14589.8
7.011	16980.4
8.013	19360.4
9.015	21720.5
10.014	24038.1

Bivariate Fit of MapBdL By MapCurrent



Linear Fit

MapBdL = 87.408503 +

Summary of Fit

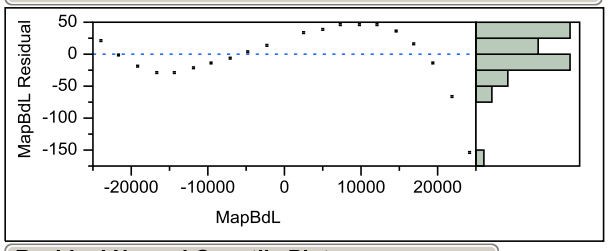
RSquare	0.999993
RSquare Adj	0.999993
Root Mean Square Error	40.54256
Mean of Response	-506.49
Observations (or Sum Wgts)	39

Parameter Estimates

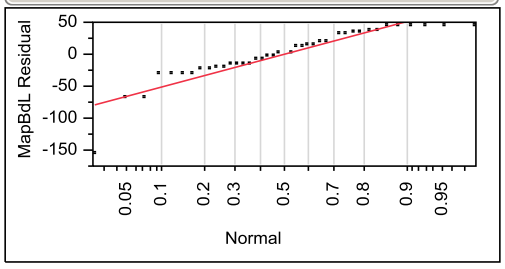
Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	87.408503	6.497361	13.45	<.0001 *
MapCurrent	2407.1951	1.06905	2251.7	<.0001 *

Diagnostics Plots

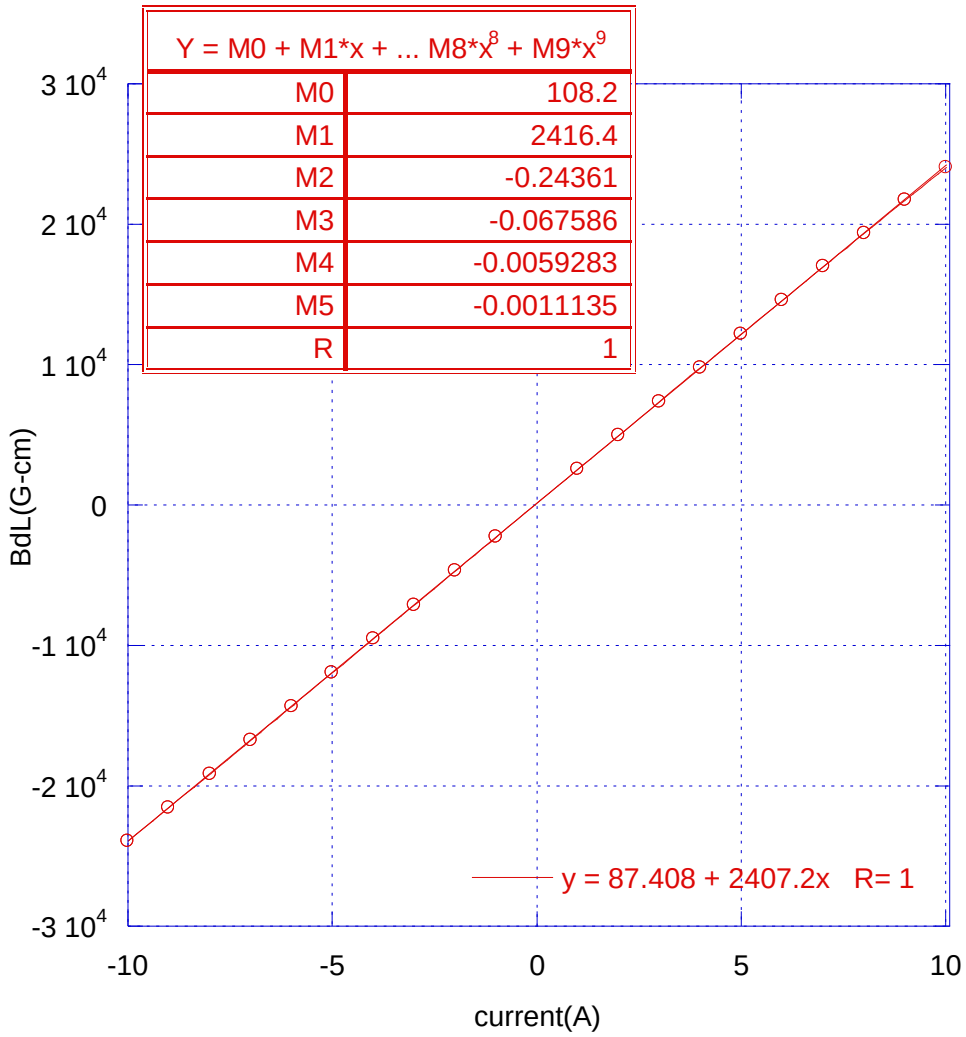
Residual by Predicted Plot



Residual Normal Quantile Plot



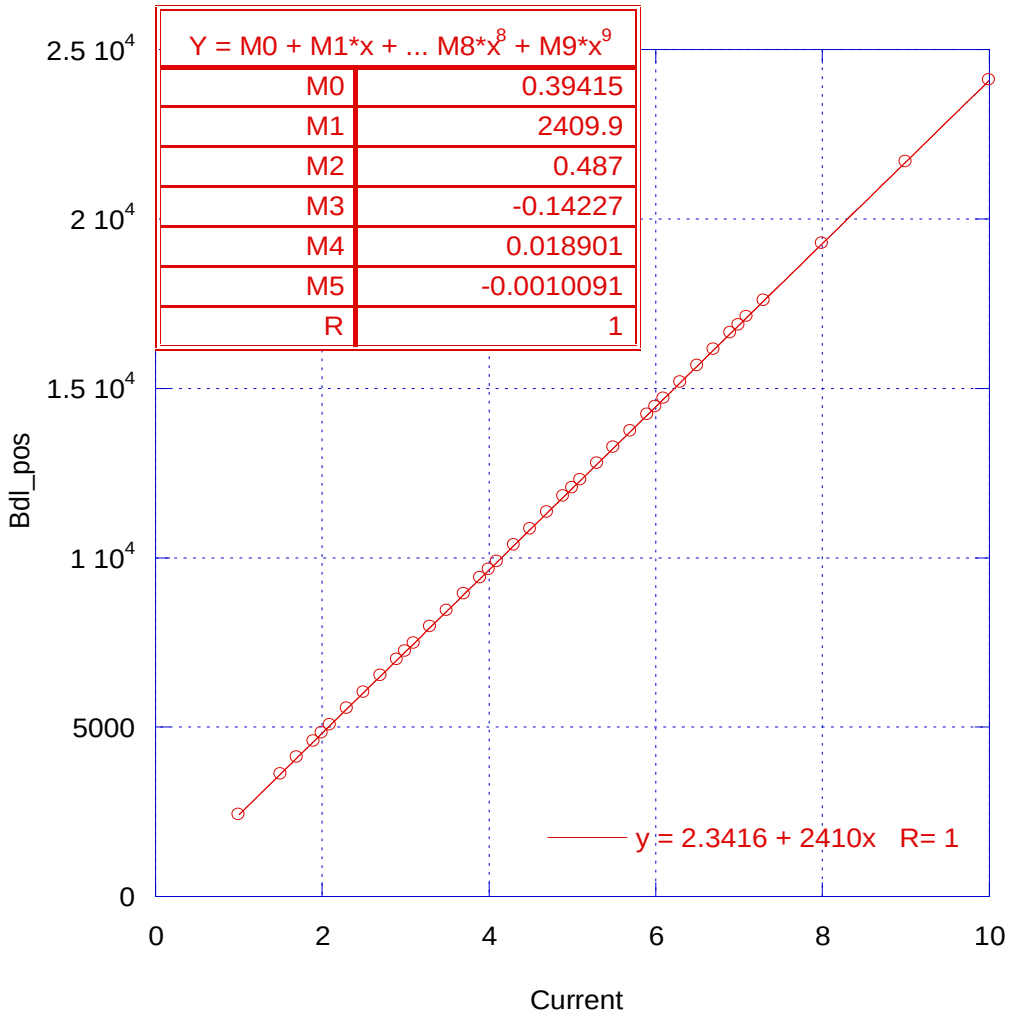
Point at zero current excluded from these plots as residuals were off the smooth curve. I will now fit the data with third and fifth order polynomials in another program because of the curves in the residual by predicted plot. I will compute residuals for those fits and examine for normality.



Y = M0 + M1*x + ... M8*x^8+ M9*x^9	
M0	114.24
M1	2419.3
M2	-0.77555
M3	-0.19544
R	1

Fit	Mean residual	Standard deviation	Residuals normal?
First order	-3.87	47.08	No
Third order	-0.72	8.48	No
Fifth order	-0.11	1.92	yes

EPICS field map data fits



Y = M0 + M1*x + ... M8*x^8 + M9*x^9	
M0	2.0659
M1	2408.6
M2	0.58064
M3	-0.051551
R	1

Three fits to model data for BdL(I), no hysteresis. The slopes of the first order fits for EPICS map (2407.2) and model (2410) are in the ratio 1:1.001, hence abstract claim of agreement at 0.1% level.

Fit to model Bdl(I)	Mean residual	Standard deviation	Residuals normal?
First order	-0.207	1.605	No
Third order	0.010	0.255	No
Fifth order	-0.004	0.026	Yes

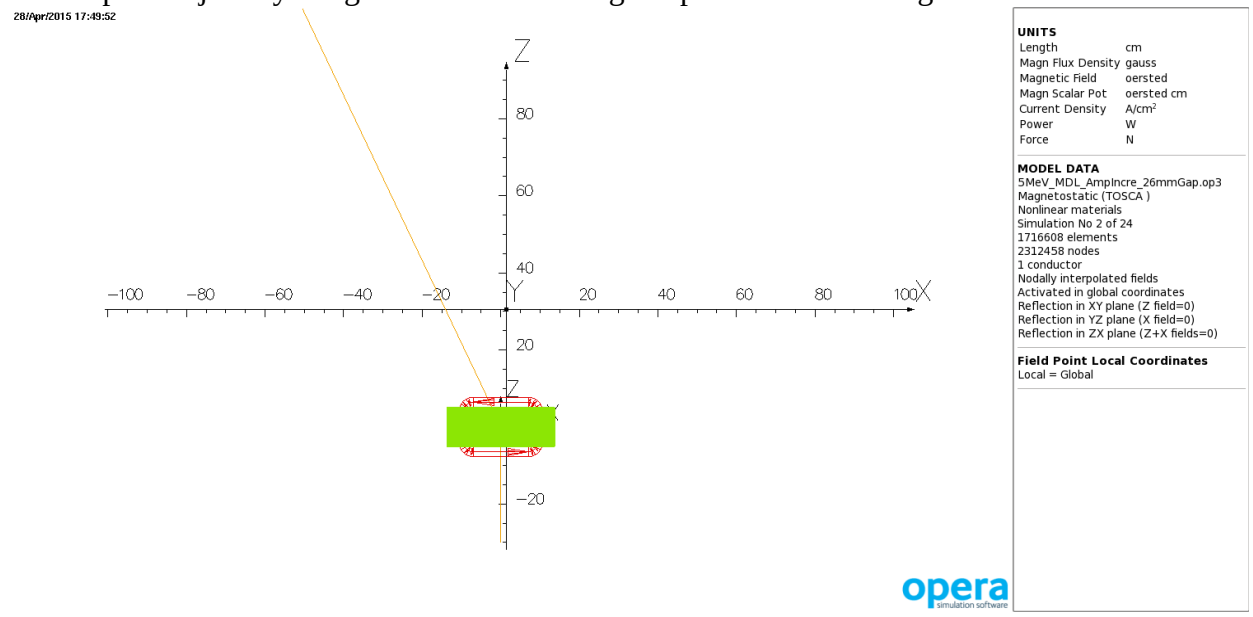
Straight BdL vs momenta of particles bent into various beam lines

There is one beam line into the 5 MeV dipole, normal to the pole face. There are four exiting the dipole, the 5D line at 25^o, the straight-through line to CEBAF, the 3D Mott line at -12.5^o and the 2D spectrometer line at -30^o . Even though the MDL is 160 mm wide vs 102 mm for the old BV, the field still falls off at the extreme angles. Simulations were run at 40 different currents encompassing most of the momenta expected to be possible even with the new quarter cryomodule. The simulation at the lowest current, 1A, stands out in residuals for linear fits but is included in all fits shown below.

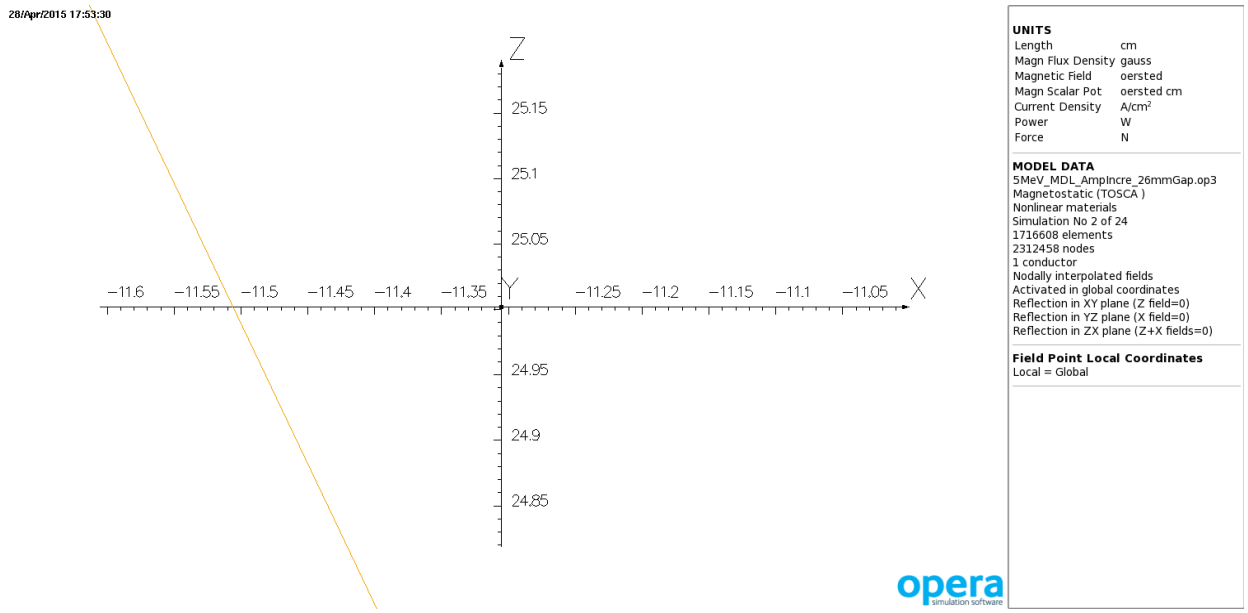
The bubble chamber experiment will be mounted on the 5D beam line. One needs to be able to set the magnet to a current derived from the EPICS map BdL(I) which is measured straight through the magnet, normal to both poles. Electrons were tracked through simulations. Energy was varied until the desired angles were reached for each simulation. Momentum was then calculated from this kinetic energy value assuming electron mass 511 keV. Values obtained by this procedure are shown in the table and plots below. Because the model does not have hysteresis effects, as seen in the low constant terms in the fits on the previous page, the electrons were always bent to the left as seen from below in the model. This doesn't matter for the model but does matter for the real magnet. In the machine, the 2D and 3D lines (beam left) require negative currents and the 5D line (beam right) requires a positive current. Appropriate BdL signs are used in the plots but not in the table which immediately follows. One will therefore be able to use the fits on the plots to set the (straight) BdL via EPICS to get the desired momentum.

The question then becomes how good is the field map? The work on pages 2 and 3 suggest it's not bad. Piece-wise linear extrapolation between points, used in EPICS, has not been examined. It is suggested that a new EPICS field map be created by evaluating the fifth order fit on page 3 at 0.2A intervals, replacing the measured map (page 1).

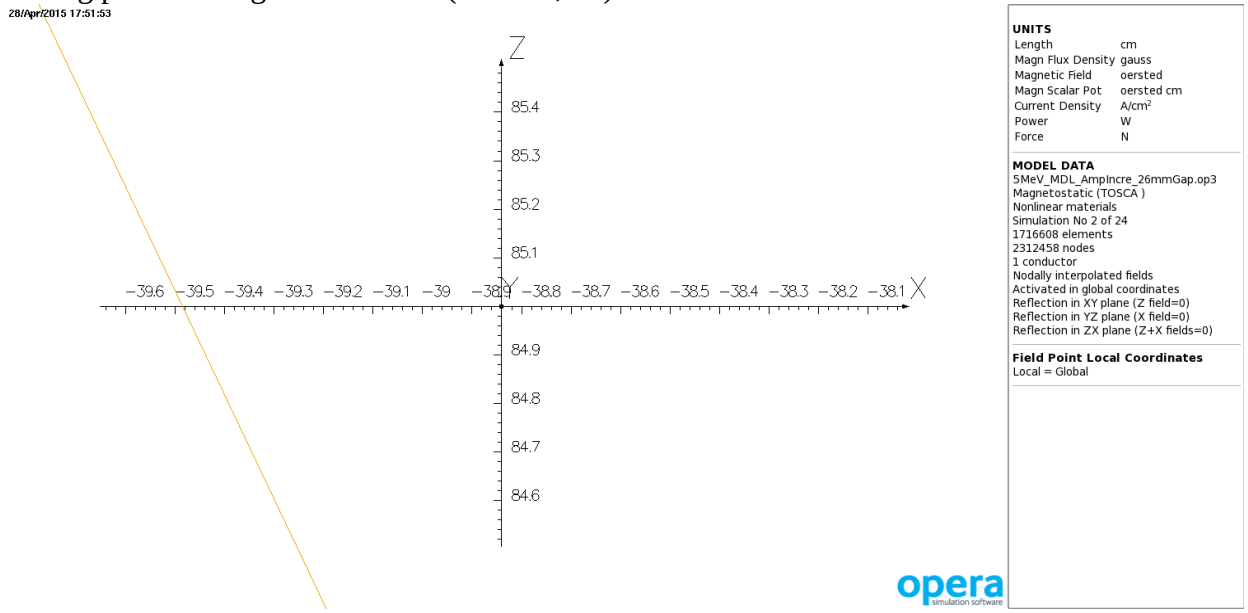
Three Opera trajectory images follow illustrating the procedure at 25 degrees.



Overall path of particle with 2.9816 MeV kinetic energy launched at (0,0,-30).



Starting point for angle calculation (-11.505, 25)



End point for angle calculation (-39.485, 85). Angle 25.001 degrees

The multipoles for the MDL model for 6.3 MeV KE to 30 degrees are:

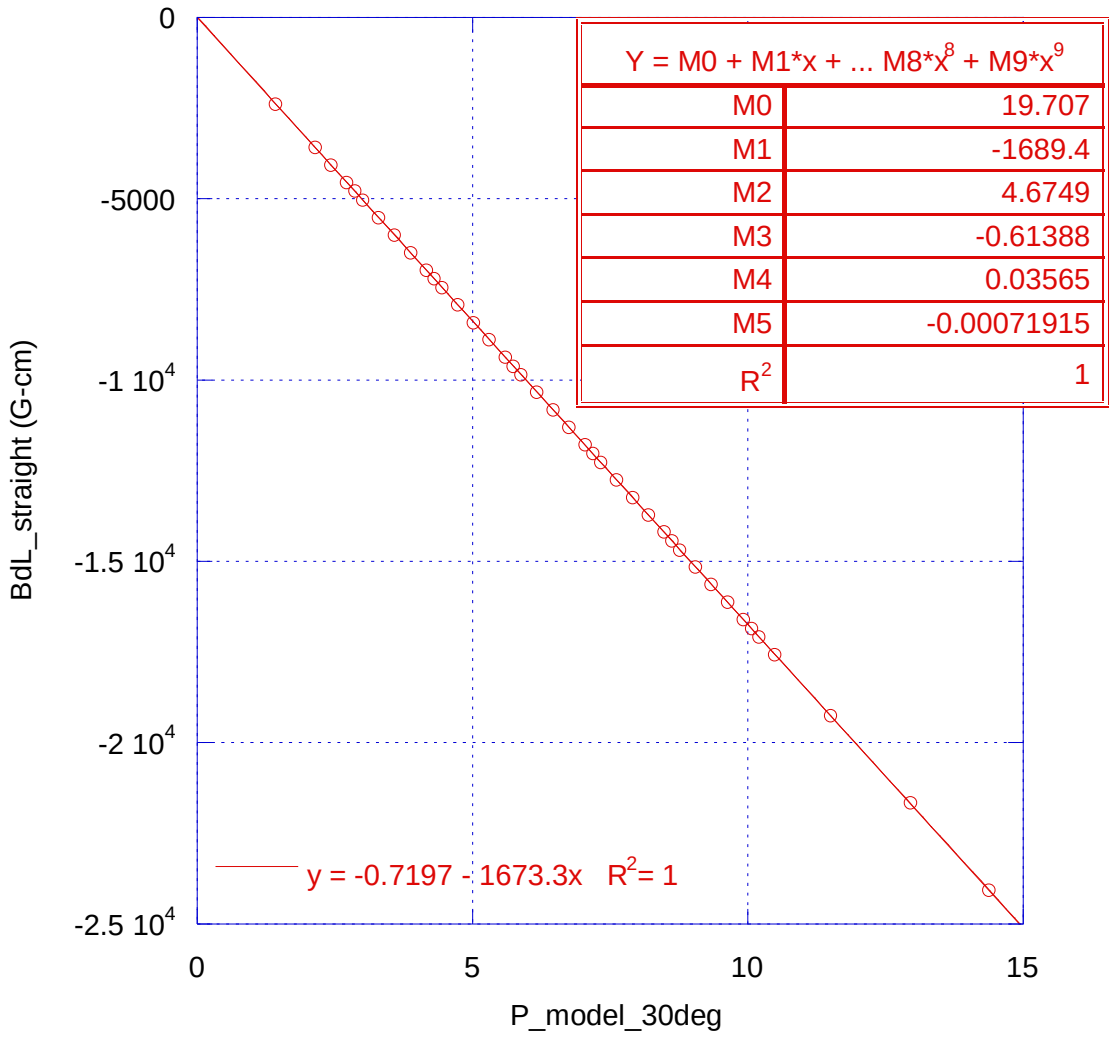
Orbit angle (degrees)	Dipole	Quadrupole	sextupole	Octupole	Decapole
29.995	-11831.28	-16.26	3.67	-0.91	-0.64
25.002	-11677.54	-9.91	2.14	-1.09	-0.52
12.51	-11415.70	-3.09	0.06	-0.37	0.37

evaluated on 1 cm radius circles.

corrected

BdL_neg_str	P_model_30deg	P_model_25deg	P_model_12.5deg
-2410.64	1.4411	1.7073	3.3381
-3615.96	2.1619	2.5609	5.0060
-4098.08	2.4503	2.9024	5.6730
-4580.21	2.7384	3.2438	6.3404
-4821.28	2.8820	3.4146	6.6765
-5062.34	3.0270	3.5853	7.0064
-5544.47	3.3151	3.9266	7.6740
-6026.59	3.6029	4.2680	8.3434
-6508.72	3.8905	4.6088	9.0115
-6990.84	4.1759	4.9507	9.6795
-7231.9	4.3239	5.1218	10.0150
-7472.97	4.4633	5.2924	10.3464
-7955.09	4.7510	5.6339	11.0182
-8437.2	5.0389	5.9752	11.6848
-8919.32	5.3268	6.3164	12.3544
-9401.43	5.6148	6.6574	13.0220
-9642.49	5.7644	6.8291	13.3532
-9883.54	5.9027	6.9999	
-10365.6	6.1906	7.3402	
-10847.7	6.4786	7.6820	
-11329.8	6.7738	8.0242	15.6907
-11811.9	7.0615	8.3659	
-12053	7.2059	8.5363	
-12294	7.3503	8.7065	
-12776.1	7.6379	9.0478	
-13258.1	7.9265	9.3891	
-13740.2	8.2151	9.7306	
-14222.2	8.5037	10.0720	
-14463.3	8.6469	10.2433	
-14704.3	8.7821	10.4145	
-15186.3	9.0697	10.7539	
-15668.3	9.3576	11.0952	
-16150.2	9.6455	11.4366	
-16632.2	9.9333	11.7779	
-16873.2	10.0881	11.9501	
-17114.1	10.2211	12.1192	
-17596	10.5089	12.4605	
-19282.3	11.5277	13.6564	
-21689.6	12.9669	15.3615	
-24093.9	14.4049	17.0634	

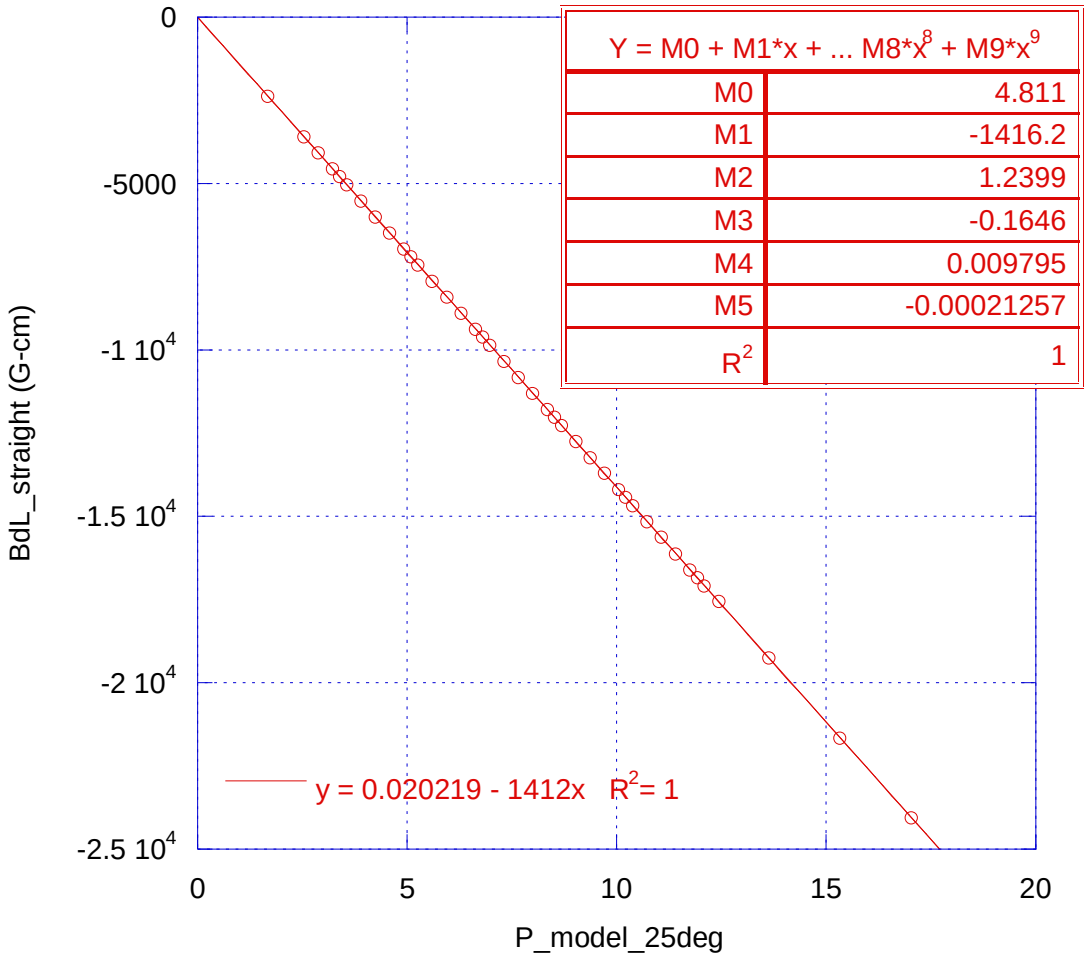
The kinetic energy range 7.85-10 MeV is most important for the bubble chamber experiment so simulation currents were chosen most densely there and where CEBAF normally runs. Blank entries in the table above can be filled in upon request.



Y = M0 + M1*x + ... M8*x^8 + M9*x^9	
M0	-0.83108
M1	-1671.6
M2	-0.4441
M3	0.026604
R^2	1

First, third and fifth order fits for BdL(P), G-cm(MeV/c), for 2D line at 30 degrees.

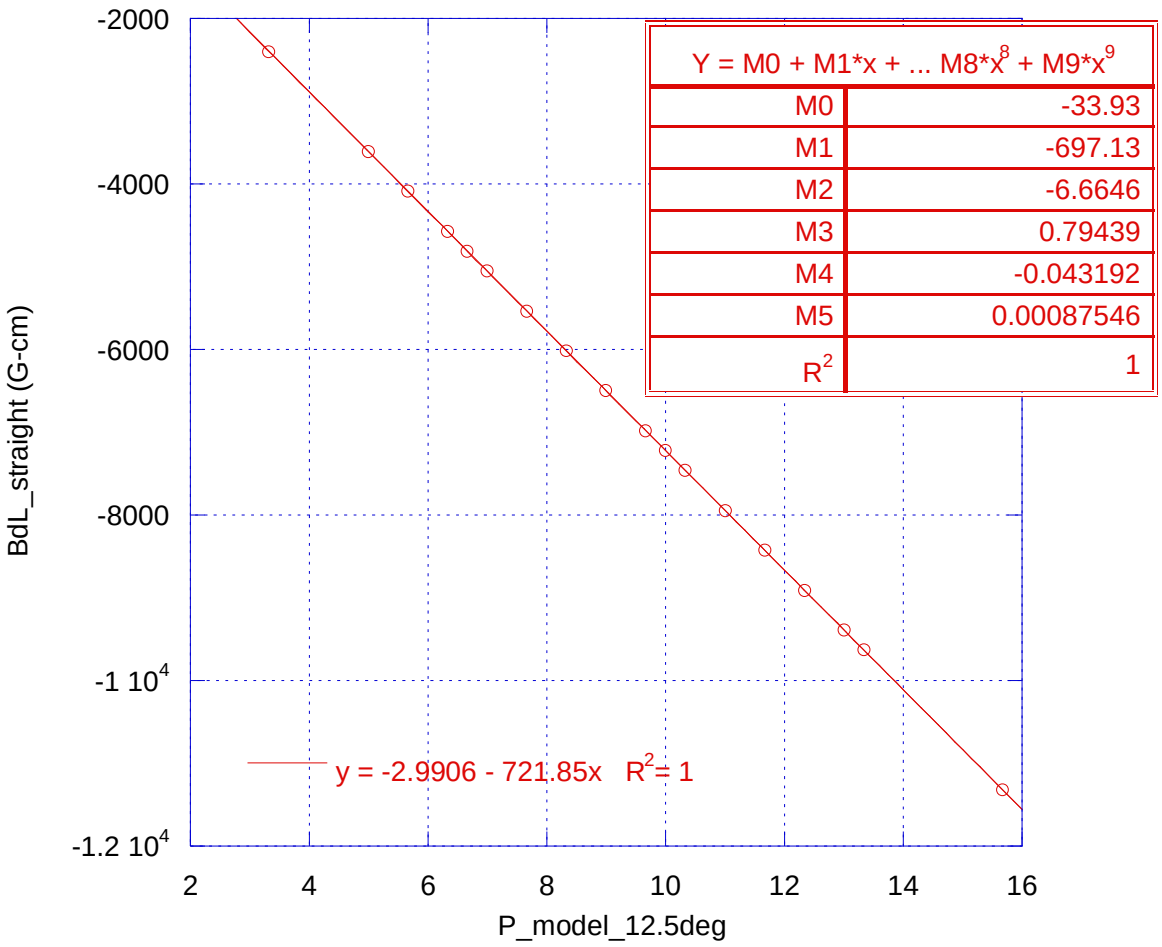
Fit to model Bdl(I)	Mean residual	Standard deviation	Residuals normal?
First order	0.1568	6.4678	No, binormal
Third order	-0.3122	6.0392	No, binormal
Fifth order	0.1364	5.9401	No, binormal



Y = M0 + M1*x + ... M8*x^8 + M9*x^9	
M0	-0.25585
M1	-1411.8
M2	-0.040947
M3	0.0018509
R^2	1

First, third and fifth order fits for BdL(P), G-cm(MeV/c), for 5D line at 25 degrees.

Fit to model Bdl(I)	Mean residual	Standard deviation	Residuals normal?
First order	-0.3414	0.7818	No
Third order	-0.1102	0.7386	marginal
Fifth order	0.2714	0.7118	a bit better



Y = M0 + M1*x + ... M8*x^8 + M9*x^9	
M0	9.9117
M1	-726.42
M2	0.48399
M3	-0.01571
R^2	1

First, third and fifth order fits for BdL(P), G-cm(MeV/c), for 3D line at 12.5 degrees.

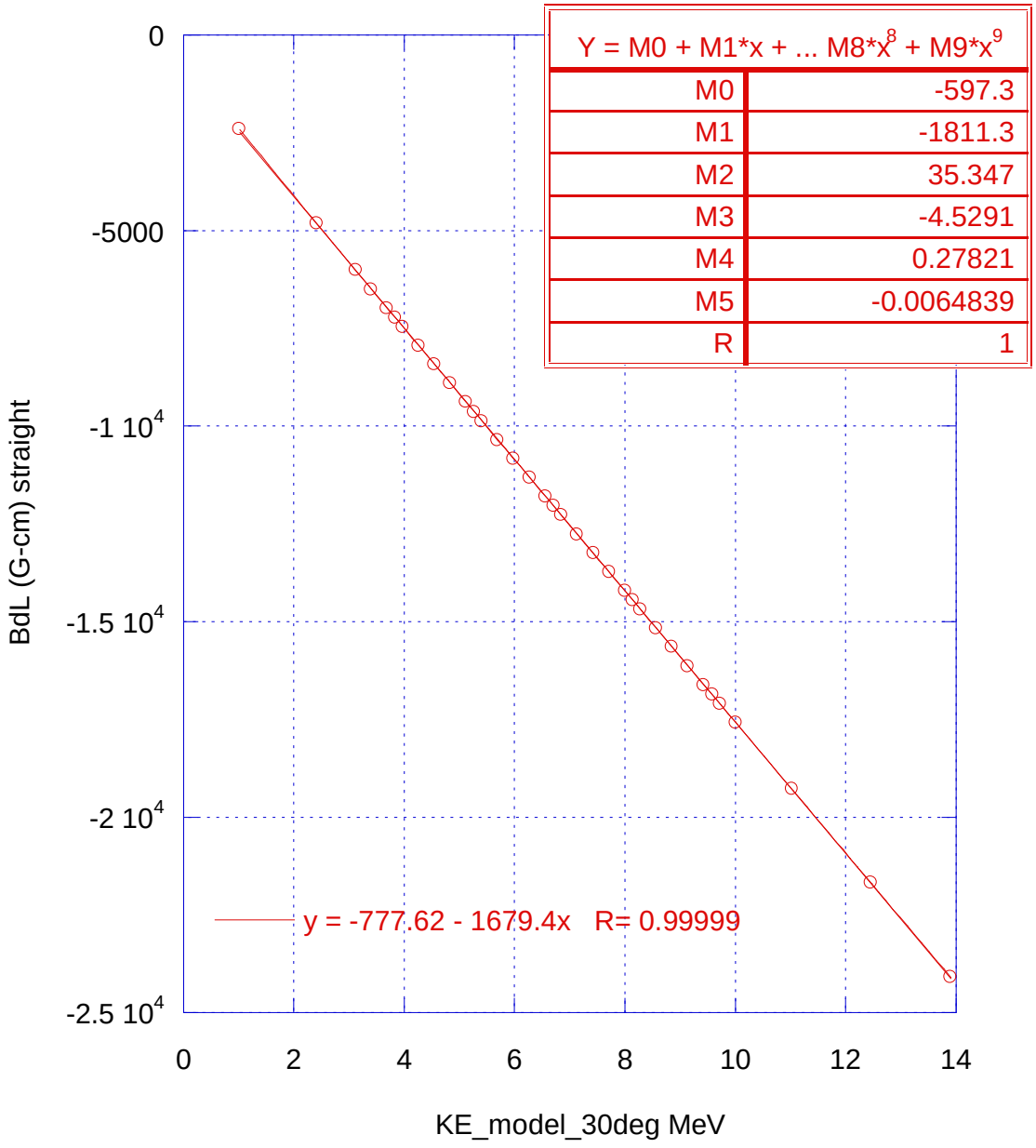
Fit to model Bdl(I)	Mean residual	Standard deviation	Residuals normal?
First order	-0.4911	0.0095	yes
Third order	-0.0315	0.8919	yes
Fifth order	-0.0181	0.8061	yes

Now that I'm calculating P properly from KE, the linear fits all have near-zero intercepts.

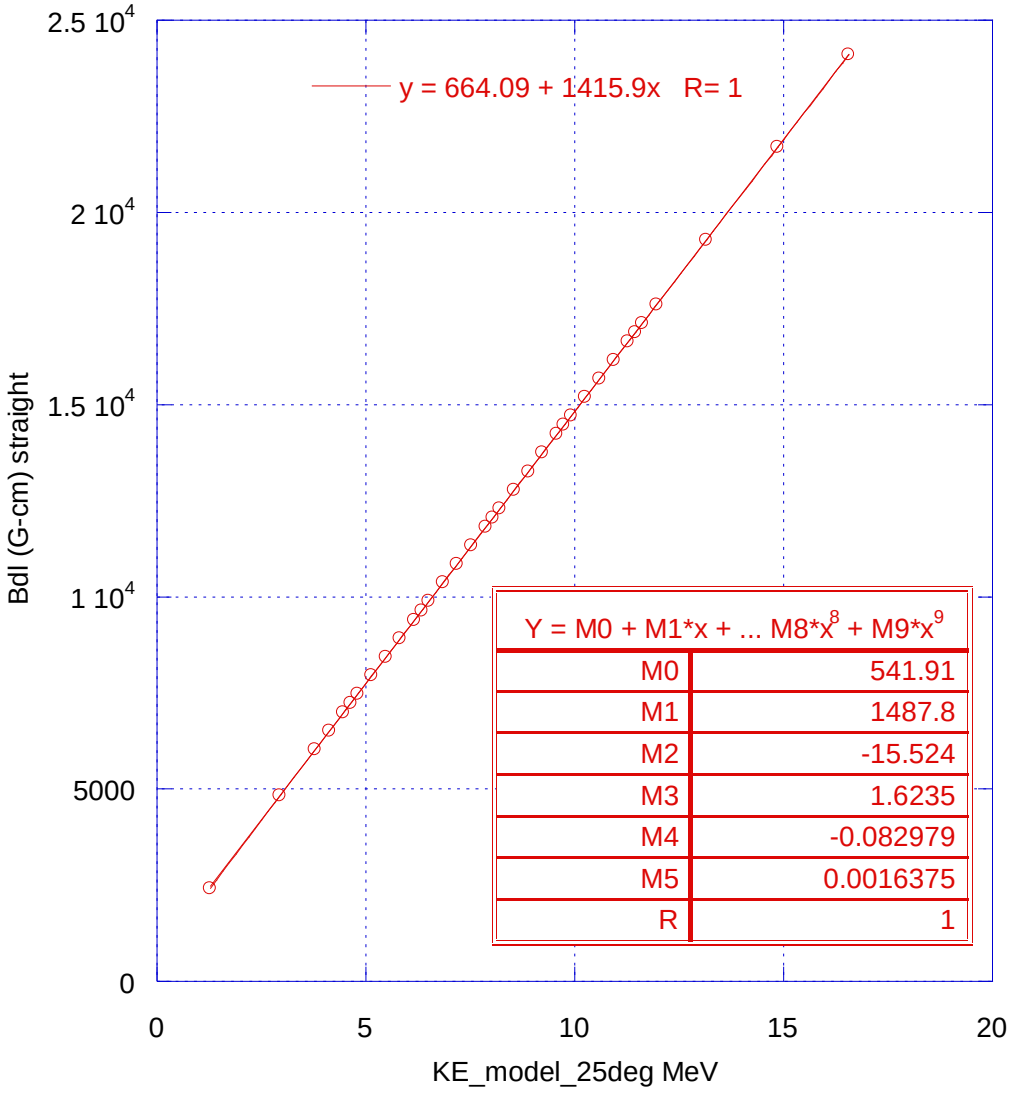
Kinetic energy data and plots

I am told that the Optim decks have 6.3 MeV kinetic energy, not momentum, so I add the KE info that I actually obtained from the models, and fits thereto.

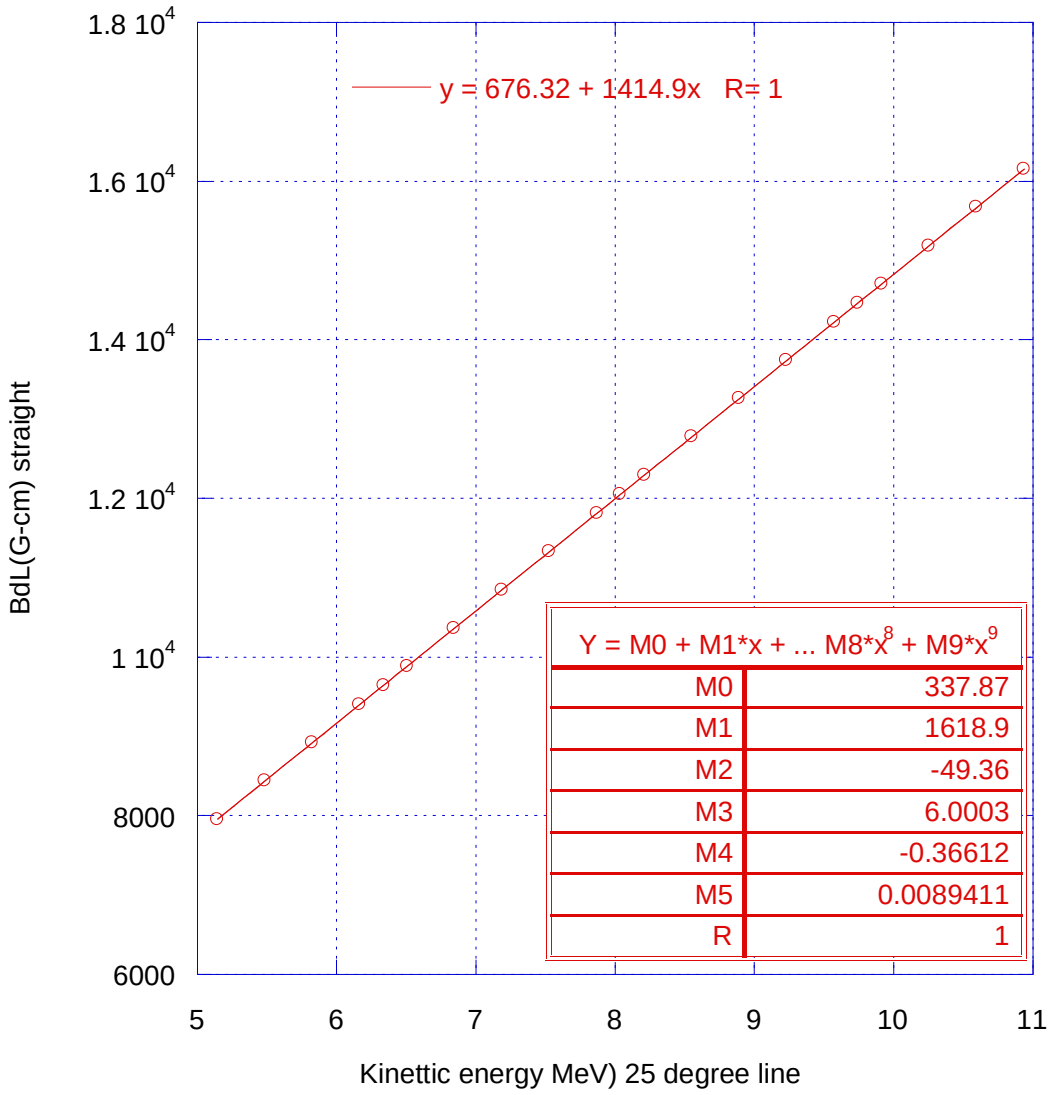
BdL_neg_str	KE_model_30deg	KE_model_25deg	KE_model_12.5deg
-2410.64	1.018	1.27112	2.866
-3615.96			4.521
-4098.08			5.185
-4580.21			5.85
-4821.28	2.416	2.9416	6.185
-5062.34			6.514
-5544.47			7.18
-6026.59	3.128	3.7875	7.848
-6508.72	3.4129	4.126	8.515
-6990.84	3.696	4.466	9.182
-7231.9	3.843	4.63625	9.517
-7472.97	3.9815	4.806	9.848
-7955.09	4.2674	5.146	10.519
-8437.2	4.5537	5.486	11.185
-8919.32	4.84025	5.826	11.854
-9401.43	5.127	6.166	12.521
-9642.49	5.276	6.3372	12.852
-9883.54	5.4138	6.5075	
-10365.6	5.7007	6.847	
-10847.7	5.9877	7.188	
-11329.8	6.282	7.5295	15.188
-11811.9	6.569	7.8705	
-12053	6.713	8.0406	
-12294	6.857	8.2105	
-12776.1	7.144	8.5512	
-13258.1	7.432	8.892	
-13740.2	7.72	9.233	
-14222.2	8.008	9.574	
-14463.3	8.151	9.745	
-14704.3	8.286	9.916	
-15186.3	8.5731	10.255	
-15668.3	8.8605	10.596	
-16150.2	9.148	10.937	
-16632.2	9.4354	11.278	
-16873.2	9.59	11.45	
-17114.1	9.7229	11.619	
-17596	10.0103	11.96	
-19282.3	11.028	13.155	
-21689.6	12.466	14.859	
-24093.9	13.903	16.56	



Straight-through BdL and the kinetic energy which bends 30 degrees left in each model. I show only the linear and fifth order fits because those are the only two one might use, the linear fit as adequate for most purposes and the fifth order for best available precision. I haven't calculated and checked residuals.



Straight-through BdL and the kinetic energy which bends 25 degrees right in each model. I show only the linear and fifth order fits. This fit could be used for the bubble chamber experiment.



Straight-through BdL and the kinetic energy which bends 25 degrees right in each model using only the central region of the previous graph. I show the linear and fifth order fits. This alternate fit could be used for the bubble chamber experiment. I will compare residuals for the two fits to 25 degree KE results to make a recommendation.

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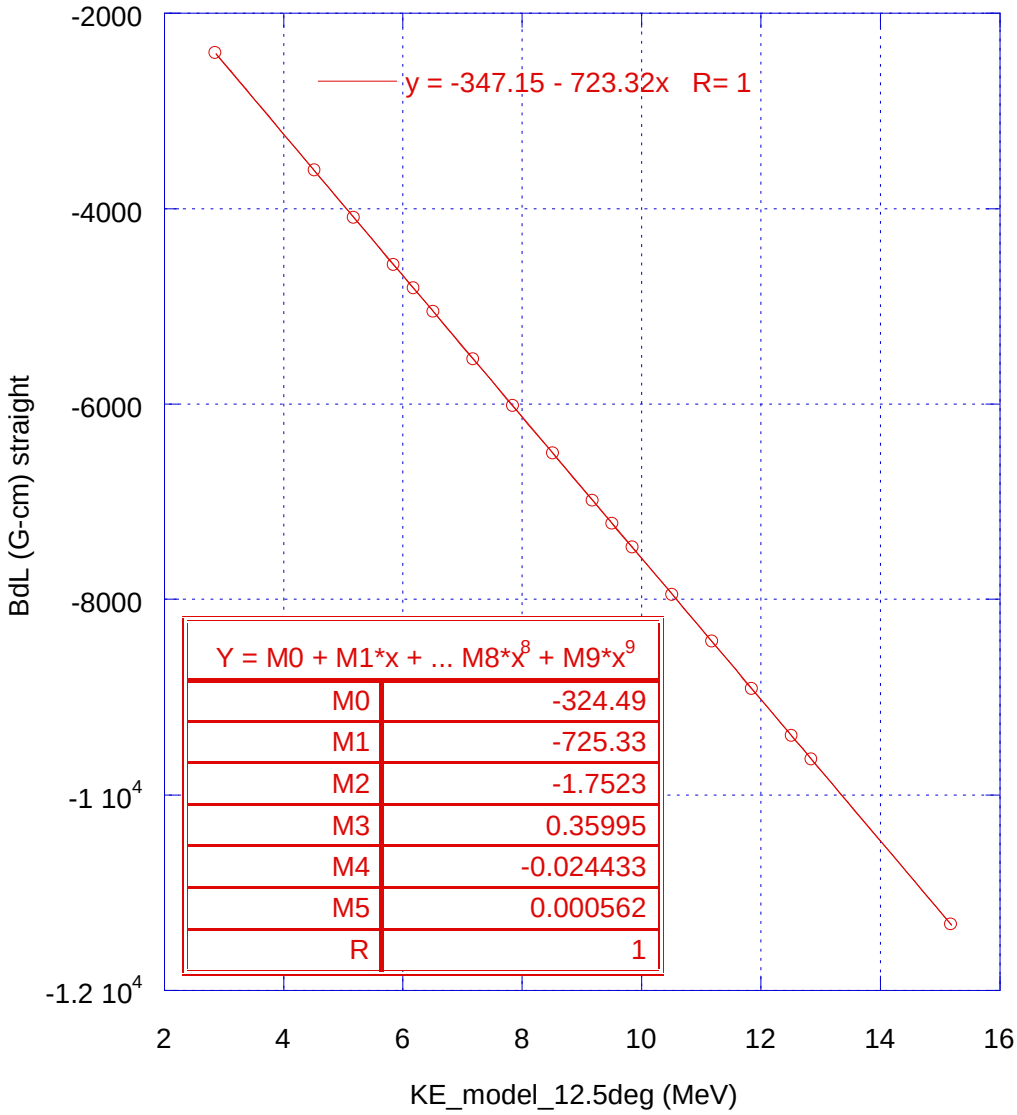


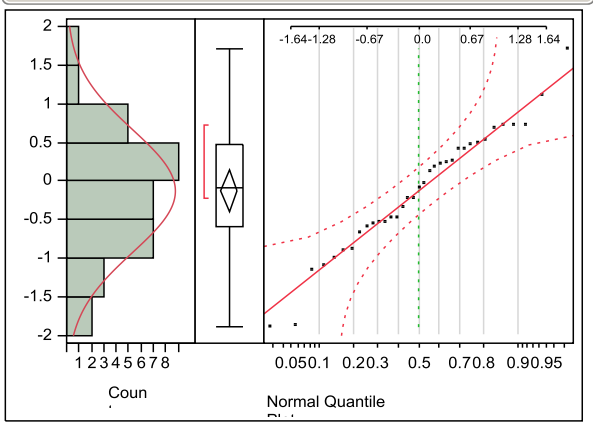
Straight-through BdL and the kinetic energy which bends 12.5 degrees left in each model, to the Mott polarimeter. I show only the linear and fifth order fits.

Bdl (G-cm) straight	KE_model 25deg	full_range_fit	Residual of full_range_fit	central range fit	Residual of central_fit
2410.64	1.27112	2411.12	-0.4827		
3615.96					
4098.08					
4580.21					
4821.28	2.9416	4819.56	1.7151		
5062.34					
5544.47					
6026.59	3.7875	6026.67	-0.0773		
6508.72	4.126	6508.24	0.4807		
6990.84	4.466	6991.31	-0.4690		
7231.9	4.63625	7233.00	-1.0968		
7472.97	4.806	7473.86	-0.8885		
7955.09	5.146	7955.99	-0.9012	7954.81	0.2801
8437.2	5.486	8437.80	-0.5960	8437.11	0.0921
8919.32	5.826	8919.35	-0.0289	8918.95	0.3699
9401.43	6.166	9400.71	0.7162	9400.47	0.9568
9642.49	6.3372	9643.04	-0.5510	9642.85	-0.3573
9883.54	6.5075	9884.07	-0.5259	9883.90	-0.3633
10365.6	6.847	10364.49	1.1061	10364.36	1.2398
10847.7	7.188	10846.98	0.7166	10846.85	0.8529
11329.8	7.5295	11330.14	-0.3430	11329.98	-0.1792
11811.9	7.8705	11812.57	-0.6743	11812.36	-0.4578
12053	8.0406	12053.22	-0.2193	12052.97	0.0336
12294	8.2105	12293.58	0.4208	12293.28	0.7175
12776.1	8.5512	12775.57	0.5340	12775.16	0.9401
13258.1	8.892	13257.68	0.4159	13257.14	0.9565
13740.2	9.233	13740.07	0.1310	13739.38	0.8179
14222.2	9.574	14222.43	-0.2283	14221.61	0.5895
14463.3	9.745	14464.30	-1.0020	14463.44	-0.1376
14704.3	9.916	14706.16	-1.8648	14705.28	-0.9759
15186.3	10.255	15185.61	0.6921	15184.77	1.5264
15668.3	10.596	15667.82	0.4844	15667.25	1.0470
16150.2	10.937	16149.95	0.2544	16149.99	0.2065
16632.2	11.278	16631.99	0.2142		
16873.2	11.45	16875.09	-1.8888		
17114.1	11.619	17113.93	0.1741		
17596	11.96	17595.76	0.2410		
19282.3	13.155	19283.45	-1.1506		
21689.6	14.859	21688.88	0.7210		
24093.9	16.56	24094.44	-0.5402		
		mean	-0.1289		
		stdev	0.7945		
mean central range			-0.0697		0.3883
stdev central range			0.7239		0.6523

Looking at the numbers, the fifth order fit over the full range of kinetic energy might be better for the bubble chamber experiment because the mean residual is closer to zero and the difference in standard deviation is not large. The experiment will run 11800-14800 G-cm; the worst case residual of the full range fit in this restricted BdL range is 126 ppm - OK.

Distributions

residual_full_fit



Normal(-

Moments

Mean	-0.128886
Std Dev	0.7945218
Std Err Mean	0.1342987
Upper 95% Mean	0.1440421
Lower 95% Mean	-0.401813
N	35

Fitted Normal

Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Location	μ	-0.128886	-0.401813	0.1440421
Dispersion	σ	0.7945218	0.6426665	1.0409841

-2log(Likelihood) =

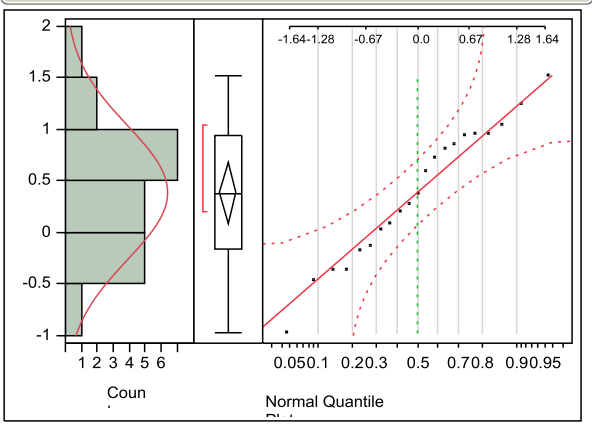
Goodness-of-Fit Test

Shapiro-Wilk W

W	Prob<W
0.980067	0.7613

Note: Ho = The data is from the Normal distribution. Small p-values reject Ho.

residual_central_fit



Normal

Moments

Mean	0.3883571
Std Dev	0.652266
Std Err Mean	0.1423361
Upper 95% Mean	0.6852651
Lower 95% Mean	0.0914492
N	21

Fitted Normal

Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Location	μ	0.3883571	0.0914492	0.6852651
Dispersion	σ	0.652266	0.4990221	0.9419174

-2log(Likelihood) =

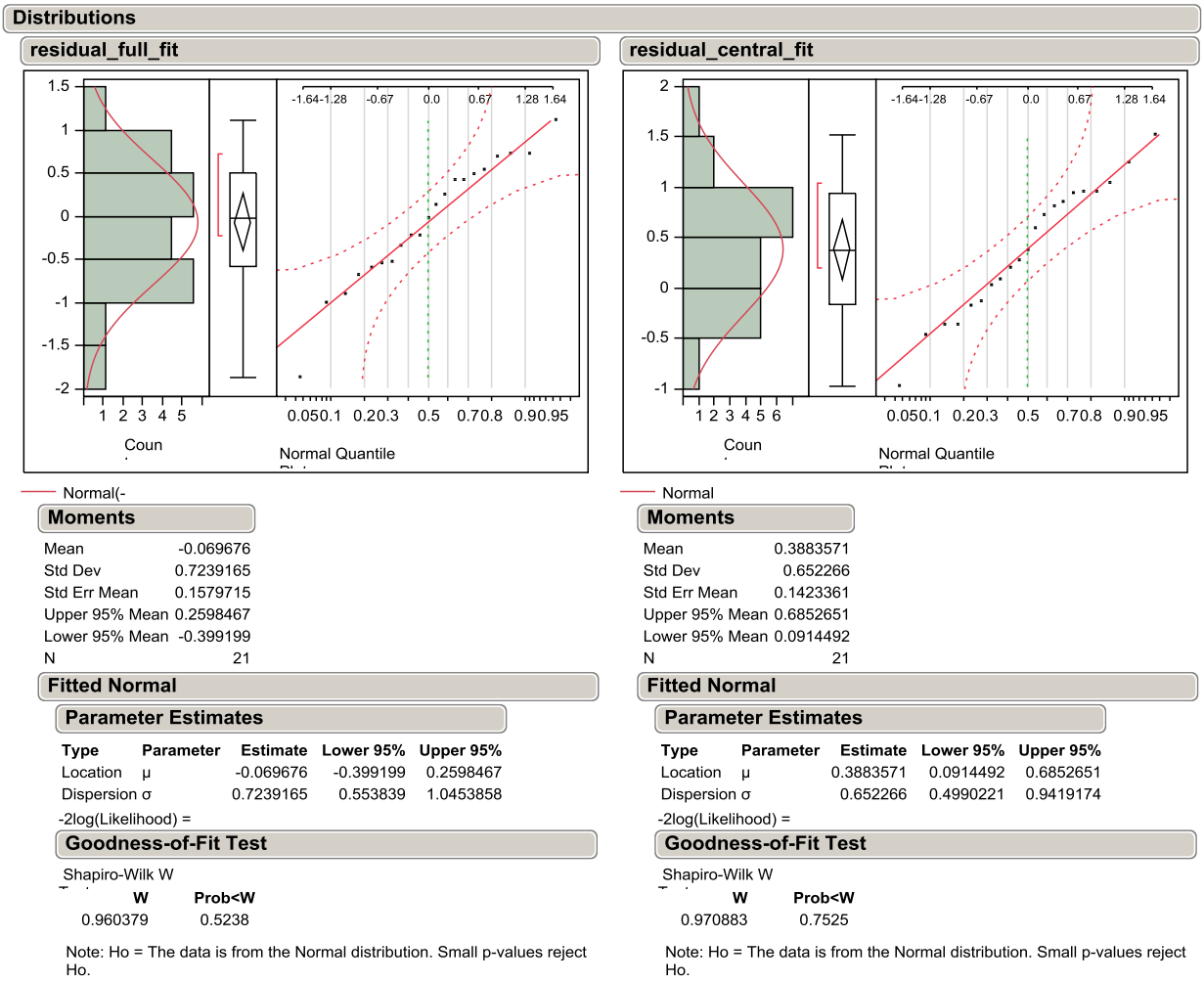
Goodness-of-Fit Test

Shapiro-Wilk W

W	Prob<W
0.970883	0.7525

Note: Ho = The data is from the Normal distribution. Small p-values reject Ho.

Plots of the all of the residuals in the preceeding table. Both are consistent with normality. The mean of the residuals in the left plot is consistent with zero at the 95% level while this is not true for the right plot, which leads me to prefer the left.



Residuals for the same 21 kinetic energy points, 5-11 MeV, with fit using all 35 points (left) and only 21 points (right). The mean of the residuals being consistent with zero in the left plot is persuasive that this fit is better.

Conclusion

For the bubble chamber experiment I'd use the fifth order fit encompassing all simulations as the residuals are centered about zero and the difference in the span of residuals is not significant for the desired energy resolution. For normal machine setup even the linear fit would be better than 1% and much better than the approximation which has been in use.

Acknowledgement

Joe Grames's question about the large intercepts in the previous P(BdL) fits led to my determination that I had erred in calculating P from KE.