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Ionization Theory of Ghost Beam

1 Purpose/Intro

Breaking down the Ionization Theory of the Ghost Beam into each process, deriving the theory behind each process, and plugging in numbers from measurements in order to validate or invalidate the theory. I'll use the Ghost Beam current and intensity measurements taken on 11/20/18 in order to plug in numbers to see if the theory holds up.

2 Process 1: Real electron beam passes through residual gas and ionizes it

On 11/20/18, we ran 100µA electron beam for 10 minutes with the following parameters in Table 1:

Beam Current	100µA
Beam Duration	10 minutes
Gun HV	100kV
Gun Solenoid Current	150A
Anode Bias	+1kV
First Solenoid Lens Current	0.732A
Second Solenoid Lens Current	0.723A

Table 1: Electron Beam & Beamline Parameters

While the electron beam is on, residual gases such as H_2 can be ionized by the high energy electrons. The number of hydrogen ions produced per second depends the densities of the hydrogen gas n_g, the density of the electron beam n_e, the ionization cross section σ_g, and the relative velocity of the electron beam $\beta_e c$:

$$\frac{dn_i}{dt} = n_g n_e \sigma_g \beta_e c \tag{1}$$

It is assumed that the electrons are moving much faster than the residual gas molecules such that $\beta_e c$ is essentially the velocity of the electrons.

Because ions have the opposite charge of the electrons, they can become trapped by the electron beam potential and thus occupy the same volume V as the electrons. Thus, we can rewrite (1) as

$$\begin{aligned} \frac{1}{V}\frac{dN_i}{dt} &= \frac{N_e N_g}{V^2}\sigma_g \beta_e c \\ \frac{dN_i}{dt} &= \frac{N_e N_g}{V}\sigma_g \beta_e c \end{aligned} \tag{2}$$

where N_i, N_e, and N_g are now the number of ions, electrons, and gas molecules respectively.

The ionization cross section σ_g is derived from Bethe's theory:

$$\sigma_g = \frac{8a_0^2 \pi I_R A_1}{m_e c^2 \beta_e^2} f(\beta) \left(\ln \frac{2A_2 m_e c^2 \beta_e^2 \gamma^2}{I_R} - \beta_e^2 \right) \tag{3}$$

Numerically, this can be rewritten as:

$$\sigma_g \left(\mathrm{m}^2\right) = \frac{1.872 \times 10^{-24} A_1}{\beta_e^2} f(\beta_e) \left[\ln \left(7.515 \times 10^4 A_2 \beta_e^2 \gamma^2 \right) - \beta_e^2 \right] \tag{4}$$

$$f(\beta) = \frac{I_i}{T_e}\left(\frac{T_e}{I_i} - 1\right) = \frac{2I_i}{m_e c^2 \beta^2}\left(\frac{m_e c^2 \beta^2}{2I_i} - 1\right) \tag{5}$$

Here, A_1 and A_2 are empirical constants that depend on the gas species, $\gamma = \left(1-\beta_e^2\right)^{-\frac{1}{2}}$ is the Lorentz factor, I_i is the ionization energy, and T_e is the electron kinetic energy.

We can relate the number of gas molecules N_g of a certain species to its pressure P_g and temperature T_g assuming it is an ideal gas:

$$N_g = \frac{P_g V}{k_B T_g} \tag{6}$$

where k_B is the Boltzmann constant and T_g is the residual gas temperature, assumed to be room temperature (293.15K). Plugging this into (2) yields

$$\frac{dN_i}{dt} = \frac{\sigma_g \beta_e c}{k_B T_g} P_g N_e \tag{7}$$

Assume for simplicity that the 100µA electron beam is a uniform cylinder. The number of electrons passing through a given cross section of the beam in one second is

$$N_e = 100 \times 10^{-6}\text{A} \times \frac{1 \text{ electron}}{1.602 \times 10^{-19}\text{C}} \times 1\text{s} = 6.24 \times 10^{14}\text{electrons}$$

The electrons have kinetic energy $T_e = 100\text{keV}$ after passing through the anode. Their velocity $\beta_e c$ can be derived from the relativistic kinetic energy equation:

$$T_e = (\gamma - 1)\, m_e c^2$$
$$\beta_e c = c\sqrt{1 - \left(\frac{1}{\frac{T_e}{m_e c^2} + 1}\right)^2}$$

Using the electron rest mass energy $m_e c^2 = 511 \times 10^3\text{eV}$, we have that $\beta_e c \approx 0.548c$. Assuming that the accelerator vacuum is composed only of hydrogen (H_2), then using the 11/20/18 data, the maximum hydrogen gas pressure is $P_g = 1.8 \times 10^{-8}\text{Pa}$ $\left(= 1.3 \times 10^{-10}\text{torr}\right)$ (note that the gas pressure continually changed during this run). Using eq. (4), the ionization cross section for hydrogen gas for a 100keV electron beam is $\sigma_{H_2} = 4.49 \times 10^{-23}\text{m}^2$. With $k_B = 1.38 \times 10^{-23}\frac{\text{J}}{\text{K}}$, $c = 299,792,458\frac{\text{m}}{\text{s}}$, and $T_g = 293.15\text{K}$, the hydrogen ion production rate using eq. (7) is $\frac{dN_{H_2}}{dt} = 2.05 \times 10^{13}$ H_2^+/s. Note that when H_2 is ionized by an electron, an electron is released from the molecule. This ejected electron is called a secondary electron. Thus, the hydrogen ion production rate is equal to the secondary electron production rate.

3 Process 2: Hydrogen ions and secondary electrons are collected and trapped by the gun solenoid field

3.1 Magnetic field map

A plot of the longitudinal z-component of the magnetic field of the gun solenoid as a function of z is shown below in Figure 1 using the field maps of the gun solenoid and solenoid lenses with the magnetic field strengths from Table 1.

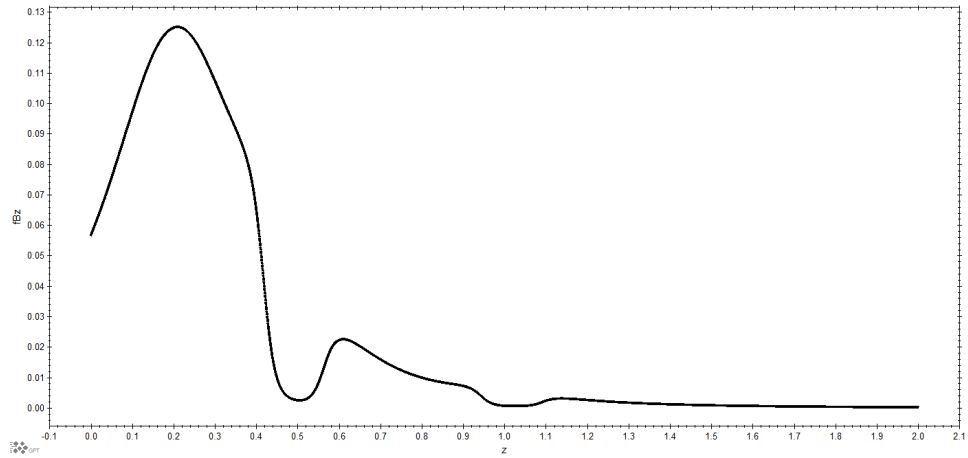

Figure 1: The z-component of magnetic field experienced by electrons along the central axis of the accelerator. The y-axis is in Tesla and the x-axis is in meters.

Electrons and ions with sufficiently low energy can be trapped within the two magnetic "wells" located at $z \approx 0.5$m and $z \approx 1$m via the magnetic mirror effect due to the sharp gradients in magnetic field, as explained below.

3.2 Magnetic Mirror Effect

Suppose we have a charged particle in a relatively uniform magnetic field that varies in magnitude with z, as is the case with two coaxial solenoids. In cylindrical coordinates we can write the components of the magnetic field:

$$\vec{B} = B_r\hat{r} + B_\theta\hat{\theta} + B_z\hat{z} \tag{8}$$

In the case of coaxial solenoids, $\vec{B}$ has azimuthal symmetry, thus $B_\theta = 0$. From $\nabla \cdot \vec{B} = 0$, we can obtain B_r:

$$\begin{aligned}
\nabla \cdot \vec{B} &= 0 \\
\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{\partial B_z}{\partial z} &= 0 \\
\frac{\partial}{\partial r}(rB_r) &= -r\frac{\partial B_z}{\partial z} \\
rB_r &= -\int_0^r r'\frac{\partial B_z}{dz}dr'
\end{aligned}$$

If we assume that $\dfrac{\partial B_z}{\partial z}$ is known at $r = 0$ and does not vary significantly with r (a valid assumption in our case), then

$$\begin{aligned}
rB_r &\approx -\frac{1}{2}r^2\left[\frac{\partial B_z}{\partial z}\right]_{r=0} \\
B_r &= -\frac{r}{2}\left[\frac{\partial B_z}{\partial z}\right]_{r=0}
\end{aligned} \tag{9}$$

Now, in the absence of electric fields, the Lorentz force is

$$\begin{aligned}
\vec{F}_B &= q\vec{v} \times \vec{B} \\
&= q\left[(v_\theta B_z - v_z B_\theta)\,\hat{r} - (v_r B_z - v_z B_r)\,\hat{\theta} + (v_r B_\theta - v_\theta B_r)\,\hat{z}\right]
\end{aligned}$$

Since $B_\theta = 0$,

$$\vec{F}_B = q\left[v_\theta B_z\hat{r} + (v_z B_r - v_r B_z)\,\hat{\theta} - v_\theta B_r\hat{z}\right] \tag{10}$$

For the magnetic mirror, we are mainly concerned with F_z. Using (9), we have:

$$\begin{aligned} F_z &= -qv_\theta B_r \\ &= \frac{qrv_\theta}{2}\frac{\partial B_z}{\partial z} \end{aligned}$$

Averaging over one Larmor period, we have

$$F_{z,avg} = \pm\frac{qr_L v_\perp}{2}\frac{\partial B_z}{\partial z}$$

where $r_L = \dfrac{mv_\perp}{|q|\,B}$ is the Larmor radius. Plugging this in:

$$F_{z,avg} = -\frac{1}{2}\frac{mv_\perp^2}{B}\frac{\partial B_z}{\partial z}$$

But $\dfrac{1}{2}\dfrac{mv_\perp^2}{B} = \mu$, which is the magnetic moment of the particle. Thus,

$$F_{z,avg} = -\mu\frac{\partial B_z}{\partial z} \tag{11}$$

Equation (11) is the "mirror force". If we consider a line element ds along B, then we can extend $F_{z,avg}$ into 3D:

$$F_\parallel = -\mu\frac{dB_z}{ds} = -\mu\nabla_\parallel B$$

$F_\parallel$ is the mirror force parallel to B. Now,

$$\begin{aligned} F_\parallel = m\frac{dv_\parallel}{dt} &= -\mu\frac{dB}{ds} \\ mv_\parallel\frac{dv_\parallel}{dt} &= -\mu v_\parallel\frac{dB}{ds} \end{aligned}$$

Now, $\dfrac{d}{dt}\left(\dfrac{1}{2}mv_\parallel^2\right) = \dfrac{m}{2}\dfrac{d}{dt}\left(v_\parallel^2\right) = \dfrac{m}{2}\left(2v_\parallel\dfrac{dv_\parallel}{dt}\right) = mv_\parallel\dfrac{dv_\parallel}{dt}$, so

$$\frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right) = -\mu v_\parallel\frac{dB}{ds}$$

Since $v_\parallel = \dfrac{ds}{dt}$,

$$\frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right) = -\mu\frac{ds}{dt}\frac{dB}{ds} = -\mu\frac{dB}{dt}$$

Since the particle's energy is conserved in the absence of electric fields, $\dfrac{dE}{dt} = \dfrac{d}{dt}\left(\dfrac{1}{2}mv_\parallel^2 + \dfrac{1}{2}mv_\perp^2\right) = \dfrac{d}{dt}\left(\dfrac{1}{2}mv_\parallel^2 + \mu B\right) = 0 \rightarrow \dfrac{d}{dt}\left(\dfrac{1}{2}mv_\parallel^2\right) = -\dfrac{d}{dt}(\mu B)$. Thus,

$$\begin{aligned} \mu\frac{dB}{dt} - \frac{d(\mu B)}{dt} &= 0 \\ \mu\frac{dB}{dt} - \left(B\frac{d\mu}{dt} + \mu\frac{dB}{dt}\right) &= 0 \\ B\frac{d\mu}{dt} &= 0 \end{aligned} \tag{12}$$

Since $B \neq 0$ by assumption, $\dfrac{d\mu}{dt} = 0$, so μ is a constant of the motion.

Let's apply this to the case of two coaxial solenoids. Suppose we have a charged particle between two coaxial solenoids that are relatively close together. Let the z-axis be the along the central axis of the solenoids with $z = 0$ at their midpoint. At two different locations z_1 and z_2, the strength of the magnetic field is B_1 and B_2 respectively, and the particle has transverse velocities $v_{\perp 1}$ and $v_{\perp 2}$ respectively. (Note that since there is azimuthal symmetry and we assume that the particle is close enough to the axis to feel the effects of the solenoidal magnetic fields, we only need to specify their z-component). Invoking the invariance of μ, we have

$$\begin{aligned} \mu_1 &= \mu_2 \\ \frac{mv_{\perp 1}^2}{2B_1} &= \frac{mv_{\perp 2}^2}{2B_2} \\ v_{\perp 1}^2 &= v_{\perp 2}^2 \frac{B_1}{B_2} \end{aligned} \tag{13}$$

Also, since $\dfrac{dE}{dt} = 0$,

$$\begin{aligned} \frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2 + \frac{1}{2}mv_\perp^2\right) &= 0 \\ \frac{1}{2}mv_\parallel^2 + \frac{1}{2}mv_\perp^2 &= \varepsilon \\ v_\parallel^2 + v_\perp^2 &= \frac{2\varepsilon}{m} \end{aligned} \tag{14}$$

where ε is a constant. Suppose the particle moves from z_1 to z_2. If $B_1 < B_2$, then in order for equation (13) to be true, $v_{\perp 2} > v_{\perp 1}$, meaning that the particle's transverse velocity *increases*. By equation (14), this means that $v_\parallel$ *decreases* and the particle slows down. Solving (14) for $v_\perp^2$ and inserting it into (13), we have

$$\begin{aligned} \frac{2\varepsilon}{m} - v_{\parallel 1}^2 &= \left(\frac{2\varepsilon}{m} - v_{\parallel 2}^2\right)\frac{B_1}{B_2} \\ v_{\parallel 1}^2 &= \frac{2\varepsilon}{m} - \left(\frac{2\varepsilon}{m} - v_{\parallel 2}^2\right)\frac{B_1}{B_2} \\ v_{\parallel 1}^2 &= \frac{2\varepsilon}{m}\left(1 - \frac{B_1}{B_2}\right) + v_{\parallel 2}^2\frac{B_1}{B_2} \end{aligned}$$

The case when $v_{\parallel 2} = 0$ corresponds to the case when the particle's velocity is solely *transverse* at z_2, i.e. the particle has no z-component of velocity. In this case,

$$v_{\parallel 1}^2 = \frac{2\varepsilon}{m}\left(1 - \frac{B_1}{B_2}\right)$$

where $B_1 < B_2$ by assumption so that the r.h.s. is positive. Define $\sin\theta = \dfrac{v_\perp}{v}$ where θ can be thought of as the pitch angle of the particle. Note that $\theta \in [0°, 90°]$. From (13),

$$\sin^2\theta = \frac{B_1}{B_2} \tag{15}$$

Equation (15) denotes the threshold for mirroring. Particles can mirror so long as $\sin^2\theta \geq \dfrac{B_1}{B_2}$, i.e. θ is large and $B_2 > B_1$ However, if θ is too small, the particle slows down, but does not mirror. $v_\parallel$ remains constant when $B_2 = B_1$ and increases when $B_2 < B_1$; it cannot decrease to zero.

Define B_m as the maximum magnetic field and R_m as the mirror ratio $R_m = \dfrac{B_m}{B_1}$. The smallest value that θ can be with a particle mirroring is given by

$$\sin^2\theta_m = \frac{B_1}{B_m} = \frac{1}{R_m} \tag{16}$$

So long as $\theta > \theta_m$, the particle will mirror. Particles with $\theta \leq \theta_m$ escape the mirror.

3.3 Plugging in numbers: Thresholds for trapping

To get a sense of magnitudes, lets plug in numbers using the magnetic field map shown in Figure 1. We'll first consider the magnetic well at solenoid lens 1, located at $z \approx 0.5$m. At this location, the z-component of magnetic field is $B_{0.5} \approx 2.7\times10^{-3}$T. The upstream magnetic field maximum, located at $z \approx 0.21$m, is $B_{0.21} \approx 0.13$T. The downstream maximum, located at $z \approx 0.61$m is $B_{0.61} \approx 2.3 \times 10^{-2}$T. It is important to note that the magnetic field wells at the solenoid lenses are not due to the lenses themselves, but rather due to the steel shielding surrounding the lenses, which sharply lowers the magnetic field at the edges of the solenoid lenses. The magnetic fields due to the solenoid lenses themselves are negligible compared to the magnetic field of the gun solenoid.

Consider a particle intially in the middle of lens 1 ($z = 0.5$m) moving towards the gun solenoid ($z = 0.21$m) . Using eq. (16), the upstream threshold for mirroring at lens 1 is

$$\sin^2\theta_{1,up} = \frac{B_{0.5}}{B_{0.21}} \approx 0.0208$$
$$\theta_{1,up} = \sin^{-1}\left(\sqrt{\frac{B_{0.5}}{B_{0.21}}}\right) \approx 8.29^\circ$$

The downstream threshold for lens 1 is

$$\theta_{1,down} = \sin^{-1}\left(\sqrt{\frac{B_{0.5}}{B_{0.61}}}\right) \approx 20.0^\circ$$

We see that $\theta_{1,up} \neq \theta_{1,down}$, indicating that the magnetic well is *not* symmetric, as we clearly see from Figure (1). As a result, a particle with $\theta = 10^\circ$ would get reflected approaching the gun solenoid, but would not get reflected approaching lens 2, since $\theta_{1,up} < \theta < \theta_{2,up}$. Thus, in order for particles to remain within lens 1, θ must be greater than the maximum of the upstream and downstream thresholds, that is, greater than 20.0° at lens 1.

It's easier to think in terms of particle kinetic energy T rather than pitch angle θ, thus we can derive an equation for T in terms of θ. However, a pitch angle does not uniquely define T. We can instead derive a range of kinetic energies using the fact that a particle cannot have a larger Larmor radius than the radius of the beam pipe $R = 0.034$m.

Let's first consider an electron initially in the middle of lens 1 moving towards the gun solenoid and recall that the Larmor radius of a particle *decreases* with increasing magnetic field. The maximum transverse velocity would be the transverse velocity when the Larmor radius equals the beam pipe radius at the center of the well

$$v_{\perp,max} = \frac{qB_{0.5}R}{m}$$

Using $v = \dfrac{v_\perp}{\sin\theta}$, the velocity of an electron with $r_L = R$ at the threshold pitch angle is

$$v_{1,up,max} = \frac{v_{\perp,max}}{\sin\theta_{1,up}} = \frac{\left(1.602\times10^{-19}\text{C}\right)\left(2.7\times10^{-3}\text{T}\right)(0.034\text{m})}{(9.11\times10^{-31}\text{kg})\sin(8.29^\circ)} = 1.12\times10^8\,\frac{\text{m}}{\text{s}}$$

Note that if $\theta_{1,up} < \theta < 90^\circ$, the velocity of the electron would be less than this value, thus $v_{1,up,max}$ is a maximum velocity. The corresponding maximum electron kinetic energy is

$$\begin{aligned} T_{1,up,max} &= (\gamma - 1)\,m_e c^2 \\ &= \left(\frac{1}{\sqrt{1-\dfrac{1.12\times10^8\,\frac{\text{m}}{\text{s}}}{c^2}}} - 1\right)(511\text{keV}) \\ &\approx 39.9\text{keV} \end{aligned}$$

If the electron were to instead move towards lens 2, then the maximum velocity would be $v_{1,down,max} = 4.72\times10^7\,\frac{\text{m}}{\text{s}}$, with a corresponding kinetic energy of $T_{1,down,max} = 6.5$keV. This means that electrons with kinetic energies higher than 6.5keV cannot remain trapped within lens 1.

Now let's consider the magnetic well at lens 2. The magnetic field in the middle of the well, at $z \approx 1.03\text{m}$ is $B_{1.03} \approx 7.8 \times 10^{-4}\text{T}$. The downstream maximum magnetic field at $z \approx 1.13\text{m}$ is $B_{1.13} \approx 3.2 \times 10^{-3}\text{T}$ and the upstream maximum magnetic field at $z \approx 0.9\text{m}$ is $B_{0.9} = 7.4 \times 10^{-3}\text{T}$. The threshold pitch angles are

$$\theta_{2,up} = \sin^{-1}\left(\sqrt{\frac{B_{1.03}}{B_{0.9}}}\right) \approx 19^\circ$$
$$\theta_{2,down} = \sin^{-1}\left(\sqrt{\frac{B_{1.03}}{B_{1.13}}}\right) \approx 30^\circ$$

The maximum velocities of electrons corresponding to these angles are

$$v_{2,up,max} \approx 1.43 \times 10^7 \frac{\text{m}}{\text{s}}$$
$$v_{2,down,max} \approx 9.33 \times 10^6 \frac{\text{m}}{\text{s}}$$

The corresponding maximum kinetic energies are

$$T_{2,up,max} \approx 582\text{eV}$$
$$T_{2,down,max} \approx 248\text{eV}$$

Thus, electrons with kinetic energies higher than 248eV cannot remain trapped within lens 2.

It is important to realize that the upstream magnetic maximum at $z \approx 0.9\text{m}$ is not really a maximum, as the magnetic field is higher upstream of $z = 0.9\text{m}$. Thus, electrons with certain pitch angles at the center of lens 2 moving towards lens 1 may pass $z = 0.9\text{m}$ before turning around. These electrons would have a smaller Larmor radius than those that turn around at $z = 0.9\text{m}$.

What if the particles were hydrogen gas ions, H_2^+? In this case, $m_{H_2^+} \approx 3.35 \times 10^{-27}\text{kg}$. The maximum velocities and kinetic energies for lens 1 are

$$v_{1,up,max} \approx \frac{\left(1.602 \times 10^{-19}\text{C}\right)\left(2.7 \times 10^{-3}\text{T}\right)(0.034\text{m})}{(3.35 \times 10^{-27}\text{kg})\sin(8.29^\circ)} = 3.05 \times 10^4 \frac{\text{m}}{\text{s}}$$
$$v_{1,down,max} \approx 1.28 \times 10^4 \frac{\text{m}}{\text{s}}$$
$$T_{1,up,max} = (\gamma - 1)\, m_{H_2^+} c^2 \approx 9.725\text{eV}$$
$$T_{1,down,max} \approx 1.713\text{eV}$$

The maximum velocities and kinetic energies of H_2^+ for lens 2 are

$$v_{2,up,max} \approx 3.90 \times 10^3 \frac{\text{m}}{\text{s}}$$
$$v_{2,down,max} \approx 2.54 \times 10^3 \frac{\text{m}}{\text{s}}$$
$$T_{2,up,max} \approx 1.59 \times 10^{-1}\text{eV}$$
$$T_{2,down,max} \approx 6.75 \times 10^{-2}\text{eV}$$

It is important to note that the maximum of all of the maximum kinetic energy calculations above is 39.9keV, which is much less than the kinetic energy of the electrons in the electron beam. This means that nowhere can 100keV electrons get trapped within the solenoid lenses. Thus, electrons that get trapped must be either ions or low-energy secondary electrons.

3.4 Secondary Electrons

See table by Opal, Beaty, and Peterson, 1972. Some general trends:

- The 3D graphs of the secondary electron cross section for a given angle and energy for a given gas (Helium or Nitrogen, e.g.) show that the shape of the graph is seemingly independent of the primary electron energy. Clearly the secondary electron cannot have a greater energy than the primary electron, which is why the cutoff energy of each graph is around the primary electron energy.

- In all graphs (regardless of gas species), the general trend is that the smaller the secondary electron energy, the higher the cross section. Thus, it is much more likely for a secondary electron to be produced with only a few eV rather than tens or hundreds of eV.
- In all graphs (regardless of gas species), the likeliest angle of the secondary electron is ~45° for most energies above 10eV. At secondary electron energies below 10eV, it is more likely that the secondary electron would be scattered at large angles ($130° - 150°$). Again, these two trends are independent of primary energy.

The important conclusion is that the most likely occurence is the production of a secondary electron with a few eV of kinetic energy with a velocity vector between 130° and 150° with respect to the primary electron velocity. In our experiment, this means that the secondary electrons will be produced with a small kinetic energy T on the order of a few eV and with θ between 30° to 60°. Since these angles are above the threshold values for θ for both lenses, it is likely that these secondary electrons become trapped within the lenses.

However, at these kinetic energies, the secondary elecvtrons do not have sufficient energy to ionize a second gas molecule, as the minimum ionization energy is for hydrogen gas, which is 13.6eV. Thus, it is much more likely that hydrogen gas will be ionized than any other gas, and so we would expect to see hydrogen spectral lines above any other line with a spectral analyzer.

3.5 Recombination and Spectral Lines

When relative velocity between an electron and a positive ions is small and they are in close proximity, it is possible for the electron to recombine with the ion. In doing so, the electron loses energy in the form of light. Since the energy levels of an atom are quantized, only discrete wavelengths of light can be emitted during recombination and depend on gas species itself. The spectral lines of hydrogen gas are well known. The highest intensity spectral lines are listed below in Table 2. Since hydrogen gas is expected to predominate any other residual gas species, these spectral lines should be more prominent than spectral lines from other gas species. Note that we might not be able to see the 121.57nm spectral line through a transmission window.

Wavelength (nm)	Color	Relative Intensity	Energy (eV)
121.57	Far UV	8.4×10^5	10.20
388.9	Near UV	7.0×10^4	3.18
410.17	Violet	7.0×10^4	3.02
434.04	Blue	9.0×10^4	2.86
486.1	Green (Aqua)	1.8×10^5	2.55
656.3	Red	5.0×10^5	1.89
1875	IR	5.1×10^4	0.66

Table 2: Hydrogen spectral line data taken from NIST website