

# Mott Polarimeter Upgrade at Jefferson Lab

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## Abstract

The Mott polarimeter at Jefferson Lab is being upgraded. The task this summer has been divided into three parts. The first task was to optimize the electron beam at the target in the polarimeter. The second task was to investigate multiple scattering of the electron beam within the target foils in the target ladder and determine scattering angle distributions. The last task was to begin investigating the power distribution on the dump plate.

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# 1 Introduction

## 1.1 Mott Scattering

As a general overview it would be beneficial to understand what exactly Mott scattering is even though it is not a large part of what was done this summer. Mott scattering is also know as spin dependent elastic Coulomb scattering. The scattering distribution is given by;

$$\sigma = I(\theta)[1 + S(\vec{P} \cdot \hat{n})] \tag{1}$$

where  $I(\theta)$  is the unpolarized portion, S is the Sherman Function or analyzing power, and P is the polarization. The product of S and P gives the polarized portion of the equation. In order for an assymetry to take place the polarization needs to have a perpendicular component to the motion of the electron beam. This is because if it does not, the dot product,  $\vec{P} \cdot \hat{n}$ , will be zero, therefore canceling out the polarized portion of the distribution. To use Mott scattering to determine overall polarization one looks at an assymetry. The assymetry is given by;

$$A = \frac{N_L - N_R}{N_L + N_R} = PS \tag{2}$$

where P and S are the polarization and analyzing power respectively, and  $N_L$  and  $N_R$  are the number of counts in the left and right detectors on the polarimeter. By dividing the assymetry by the analyzing power, one can find the polarization of the beam. The detector at Jefferson Lab is made of 4 detectors up, down, left, right where the up and down detectors provide  $P_x$  and the left and right detectors provide  $P_y$ . Normally, experiments conducted in the experimental halls require the beam's polarization to be in the direction of travel, but as stated before, the polarimeter will not work if that is the case. So, a series of magnets before the polarimeter alter the beams polarization so that it is perpendicular to the motion of travel at the Mott [1].

## 1.2 Power Distribution and Backscatter

One of the important things to know is how the power is distributed into the dump plate after it scatters through the foil. If there is too big a concentration of power on a small area of the plate the beam could heat up the plate enough to bore through. An accurate description of the power distribution will help in figuring out how to best cool the plate as the beam illuminates it. The description of how the beam's power disperses gives valuable insight into the amount of the beam that is actually hitting the plate. Due to the large scattering

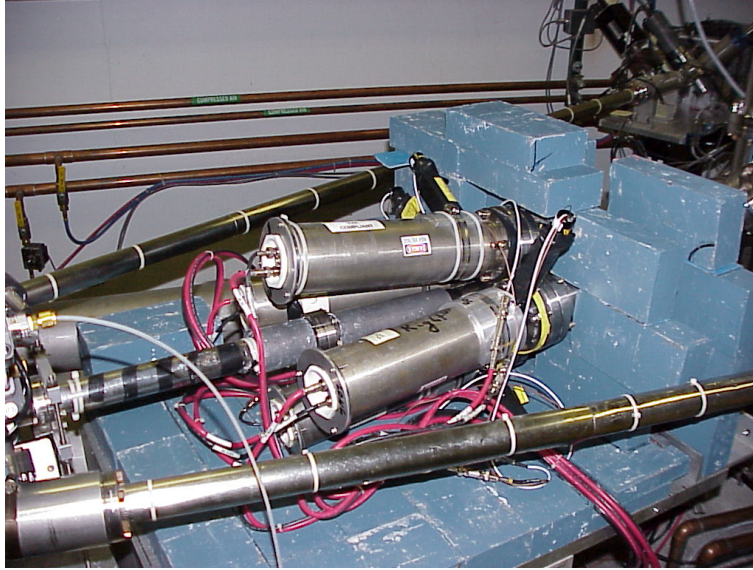


Figure 1: Mott Polarimeter at Jefferson Lab.

angles that thick targets produce, a large amount of the beam's power ends up hitting the beam pipe's wall. It is important to know the power per area here as well because overheating of the beam pipe would be dangerous and if it bored through the pipe it would be costly to repair. In addition to knowing the power distribution on the plate, it is valuable to know how much backscatter off of the dump is occurring. Backscatter from the dump plate can affect the counts in the polarimeter and skew the data. By looking at the backscatter off of the plate one could determine which material would best cut down on backscatter. The plate is currently made out of aluminum and an element with a lower  $Z$  would help reduce backscatter. Preliminary plans are to make the new plate out of beryllium which has an atomic number of 4 compared to the atomic number of aluminum which is 13.

### 1.3 Goals

The main goal this summer was to accomplish three things. The first thing was to optimize beam sizes at the target of the Mott Polarimeter using an electron beam tracking program called *elegant*. The second thing to accomplish was the modeling of multiple coulomb scattering through the target foil in polarimeter. Using a few different methods, the task was to look at the scattering angle of the electrons as they come out of the target foil. Getting the scattering angle from different methods allowed for a verification of software called *G4Beamline*, which is a program that models particle interactions and runs off of *Geant4*.

The third thing to do was to help design a new dump plate for the end of the Mott line that the polarimeter sits on using the data from the beam optimization process and the determined scattering angles. The last goal was to look at power distribution of the beam on the dump plate using the scattering distributions calculated using *G4Beamline*

## 2 Motivation

The Mott Polarimeter at Jefferson Lab has gone through an upgrade along with the rest of the CEBAF accelerator. All of the steps that have been taken this summer were in the hopes of designing a new dump plate for the end of the Mott line. The dump plate on that section of the beamline is currently made of aluminum that is approximately 1.5 inches thick. The problem with the plate right now is that aluminum creates a few problems for the Mott Polarimeter. When the beam goes through the target and then encounters the dump plate about 2 meters away a variety of things occur. One of these things that can occur is back scattering. The problem with back scattering is that it can back scatter all the way back to the detector and provide a source of error for the experiments the Mott group runs. Another thing that can occur is that the thermal energy of the beam on the aluminum dump plate could eventually lead to the beam boring through the dump plate and then entering into the tunnel of the CEBAF accelerator. For obvious reasons this would be incredibly bad. By looking at spot sizes at the target and dispersion of the beam through the target one can determine the distribution of the beam over the dump plate. This in turn allows one to test different materials for replacement of the aluminum dump plate. The hope is that different materials will reduce back scattering and also handle the thermal energy distribution better than aluminum.

## 3 Methodology

### 3.1 Beam Optimization

Beam optimization was the first thing to be completed at Jefferson Lab this summer. In order to do this the beam needed to be redefined and redirected in the particle tracking software, *elegant* [3]. It was originally being directed towards a dipole magnet, which bends the beam through an angle, and then bent at an angle of 25 degrees into a device for an experiment called PEPPo. Before going into the details of beam optimization, it would be beneficial to look at how *elegant* works. A file that is run by *elegant* comes as two different files. One of these files is called a lattice file. A lattice file is where key parts of the beam line are defined. For instance, different bending and focusing magnets would be defined as well as different drifts where the beam is just drifting through the vacuum. All of the lengths of these different components are in units of meters. The second file is an elegant file. The elegant file defines initial conditions of the beam, such as beam momenta and size, as the beam enters a specific section of

the beamline and is where commands are placed to manipulate the beam. The elegant file also reads the lattice file and when run, the program spits out tons of data about the characteristics of the beam through the section of the line for which you have run the program. Knowing all those things about how elegant is run allows one to then manipulate the beam to give certain conditions at a specific point.

The first step of the optimization process was to redirect the beam down the right line to the Mott Polarimeter. This part of the task was relatively easy and consisted of changing the bend angle in the lattice file from 25 degrees to an angle of -12.5 degrees. The next part consisted of defining the beam elements and distances on the Mott line. Because the beam was being bent through a different angle, the length the beam traveled through the dipole magnet changed and needed to be calculated using the equations below.

$$L = \rho * \alpha \quad (3)$$

$$l = \rho \sin(\alpha) \quad (4)$$

In equation (3)  $L$  is the arc length in meters,  $\rho$  is the bend radius in meters, and  $\alpha$  is the bend angle in radians. In equation (4)  $l$  is the effective length of the dipole magnet. A dipole magnet in reality increases in strength as the beam approaches, stays constant inside, and then decreases as it exits and moves away. However, the dipole is often modeled using the effective length, which is a length where one can say that the beam is not influenced by the magnet outside of that length. So, the beam feels the full force of the magnet at the start of  $l$ , and then as soon as it moves through  $l$  it no longer feels any of the force. Using the arc length of the 25 degree bend it was possible to solve for the effective length. Then using that effective length, the problem could be worked through and the arc length of the -12.5 degree bend could be found. After this step the rest of the Mott line had to be defined and this was done by going down into the tunnel and measuring the lengths of the different components with a tape measure.

Once the lengths were defined it was just a matter of optimizing the beam to be a desired spot size at the target. This was done using an optimization command in Elegant. The optimization command was used to optimize two quadrupole magnets to get a certain spot size. In the program, quadrupole magnets are modeled using a parameter known as a K1 value. The K1 value determines the strength of the quadrupole magnet. In order to optimize the quadrupole it was important to know the maximum K1 value that the two quadrupole magnets used could achieve. The "QJ" quadrupoles, which were used, achieve a maximum magnetic field of 4000 G/m at 10 amps of current [2]. Using that information it is possible to calculate the maximum K1 value using equation (5) below,

$$K1 = 119.92/p \quad (5)$$

where  $p$  is the beam momentum and the units of K1 are  $\frac{1}{m}$ . Knowing now what the maximum K1 for beam momenta ranging from 3 to 8 MeV at Mott

polarimeter, it was possible to optimize the quadrupoles to specific beam sizes at the target foil.

### 3.2 Multiple Scattering

This step had two overall goals, the first was to determine the scattering angle of the beam due to multiple scattering and the second was to test the validity of *G4Beamline* by comparing its output to analytical calculations of the scattering angle that used Gaussian approximations. Using *G4Beamline* is relatively simple. Using an input file that defined the beamline and only changing beam momenta and target thickness the program ran and produced ROOT files for each target thickness at each beam momentum. A program called HistoROOT allowed for plotting histograms of the data. In order to find the scattering angle from the histograms it was important to know which variable to plot. The easiest way to find the scattering angle was to look at the momentum in the beam. Most of the momentum before the interaction with the target is in the positive z direction. Once the beam passes through the target the beam gains momentum in the perpendicular directions. Using this knowledge, one can take the inverse tangent of  $\frac{P_x}{P_z}$ . Making a histogram of that expression provides the RMS of the angular distribution.

The data from *G4Beamline* was then compared to a script prepared in *Mathematica* using the method of finding scattering angle described in W.R. Leo's book, *Techniques for Nuclear and Particle Physics Experiments*[4].

$$\langle \theta^2 \rangle = 2 \frac{\chi_c^2}{1 + F^2} \left( \frac{1 + \nu}{\nu} \ln(1 + \nu) - 1 \right) \text{rad}^2 \quad (6)$$

$$\nu = 0.5 \frac{\Omega}{1 - F} \quad (7)$$

$$\Omega = \frac{\chi_c^2}{\chi_a^2} \quad (8)$$

$$\chi_c^2 = 0.157z \left( \frac{Z(Z+1)}{A} \right) \frac{x}{p^2 \beta^2} \quad (9)$$

$$\chi_a^2 = 2.007 * 10^{-5} Z^{2/3} \frac{1 + 3.34 \frac{Zz\alpha^2}{\beta}}{p} \quad (10)$$

In the above equations;  $Z$  is the atomic number,  $A$  is the atomic weight,  $z$  is the electric charge,  $p$  is the momentum,  $\alpha$  is the fine structure constant,  $\beta$  is  $v/c$ ,  $x$  is path length in  $gm/cm^3$ , and  $F$  is the central fraction of the distribution taken into account [5]. Taking the square root of equation (6) yields the root mean square scattering angle. When determining the scattering angle with this method it is important to note that when  $10 < \Omega < 10^8$  then an  $F$  between 90 – 99.5 percent yields a result within 2 percent. The value of  $F$  cannot be 100 percent because that would give an infinite number of angles as

the solution [4]. The *G4Beamline* and *Mathematica* predictions for scattering angles were then compared to simulations prepared using *Geant4*. This allowed for testing the physics behind *G4Beamline* and validation of its results as well as the results of the analytical values.

### 3.3 Beam Power Distribution

In order to determine the power distribution on the dump plate the multiple scattering distribution attained using *G4Beamline* was assumed.

$$P(\theta) = P_0 \exp\left(-\frac{\theta^2}{2\sigma^2}\right) \quad (11)$$

In Equation[11], P is the power as a function of angle in Watts,  $P_0$  is a constant,  $\sigma$  is the scattering angle as determined previously using *G4Beamline*, and  $\theta$  is the angle of interest. The power of the beam is also equal to  $I * E$  were I is the current and E is the energy of the beam. Using that it was possible to determine the constant  $P_0$  by the equation below;

$$P_0 = \frac{I * E}{\int_0^\pi \exp\left(-\frac{\theta^2}{2\sigma^2}\right) d\theta} \quad (12)$$

The constant  $P_0$  changes with target thickness, since  $\sigma$  changes. Once the constants were found at various target thicknesses it was possible to find the power distributed on the dump plate. Using basic geometry, it was determined that the dump plate subtends an angle of 67.8 milliradians with respect to the target. To find the distribution on the whole plate;

$$P(\theta) = P_0 \int_0^{.0678} \exp\left(-\frac{\theta^2}{2\sigma^2}\right) d\theta \quad (13)$$

However it is more beneficial to know the power per unit area on the plate. That was accomplished by finding the area of a ring on the plate. The power per unit area was found for rings starting at 0 to 1 cm in radius all the way out to 9 to 10 cm in radius.

## 4 Simulations

### 4.1 Beam Optimization

As stated previously, directing the beam down the correct line was a simple procedure which involved redefining the bend angle in the lattice file. The difficult part was focusing the beam to a certain size. This required using two quadrupole magnets, MQJ0102 and MQJ0L02A, to focus the beam down to a specific size. The need for two magnets arises from the fact that due to the Lorentz force, a quadrupole magnet focuses in one plane while it defocuses in another, so by having two magnets this is corrected. The original goal of this

process was to be able to focus the beam down to about 1 mm in both the x and y planes at the target, achieving a circular beam. In order to get the beam to do this an optimization command needed to be implemented in the elegant code and the optimal K1 values for the magnets needed to be determined to achieve the specific spots sizes at the target. Once the optimization code was written the next step was to find the smallest possible circular beam sizes at the target. It was found that for all five beam momentum used in the Mott (3.2 MeV, 4.2 MeV, 5.5 MeV, 6.3 MeV, 8.3 MeV) a 1 mm circular spot size was not achievable.

Table 1: Optimized K1 Values and Smallest Achievable Circular Beam Sizes

p (MeV/c)	L02 K1	L02A K1	Beam Size
8.3	-13.443	4.821	1.5 mm
6.3	4.501	-10.729	1.25 mm
5.5	-10.696	4.997	1.5 mm
4.2	-9.999	4.864	2 mm
3.2	-9.891	4.963	2 mm

As seen in Table [1], all the different beam momentum were able to achieve different beam sizes. Something that initially may seem odd is the smallest circular beam sizes that beam momenta of 3.2 and 4.2 produce. One would think that it may be easier to focus lower beams with lower beam momenta down to smaller sizes, however, due to the initial conditions of the beam coming out of a cryomodule before the Mott the lower momenta beams have larger beam sizes when they emerge from the cryomodule and therefore cannot be focused down as much in the same distance.

A plot similar to Figure [2] was created for each beam momentum to verify that Elegant was actually optimizing to the beam size desired. Figure [2] shows that round 4.1 meters the quadrupoles start affecting beam and changing it's size. Subtle effects from the dipole used to steer the beam can also be seen around 5.5 meters. The target is located where the two lines intersect and the dump is at the end of the two lines at about 10 meters.

## 4.2 Multiple Scattering

After looking at how to manage and optimize the beam at the target, it was important to look at how the beam interacted with the target foil. The most important thing at this stage was to get a handle on how the beam was scattering through the foil as foil thickness increased and to find a way to validate the results that *G4Beamline* provided. To do this, simulations were run at thicknesses of .01, .1, 1, 10, and 100 micrometers for gold and at three kinetic energies of 3, 5, and 8 MeV. At first the scattering angles were taken by the method described in section 3.2. However, it was later determined that this method was incorrect. While looking at the script for how *Geant4* handles multiple scattering, it was found that it looks at the histogram of  $\arctan(P_x/P_z)$  and the RMS value is the



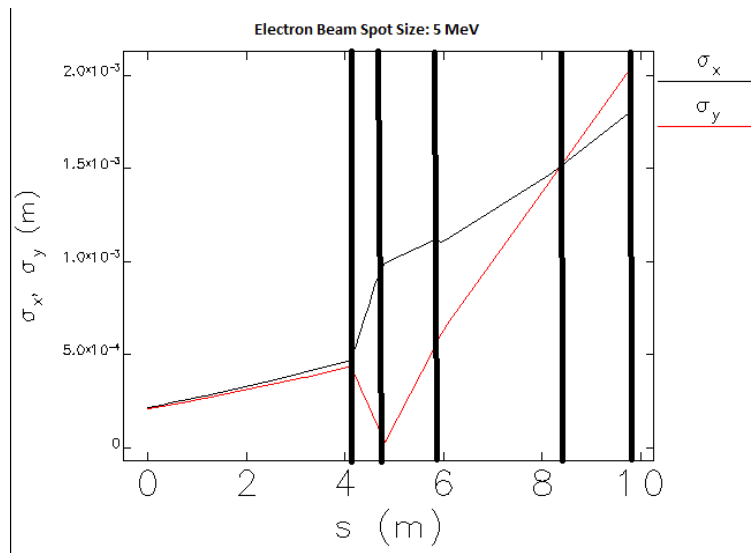


Figure 2: Plot of beam size versus position. This particular plot is for a beam momentum of 5.5 MeV/c which is the beam momentum used most often within the Mott polarimeter. The black lines mark Quad 1, Quad 2, the Dipole, the Target, and the dump in that order.

root mean square of the scattering angle in the 2-D plane. Instead of attempting to alter the *Geant4* script, the other two methods were altered. Finding the angle using *G4Beamline* was changed to look at the same histogram and the analytical method was changed using a relation described in Leo's book [4]. That relation is;

$$\langle \theta_x^2 \rangle = \frac{\langle \theta^2 \rangle}{2} \quad (14)$$

Using that equation it was possible to compare angles correctly. In the figure below the scattering angles generated by the three methods are plotted as a function of thickness. While runs were conducted for the thicknesses stated above, it was more appropriate to run at lower thicknesses to better represent the types of foils that are available at the lab. There are data points for .01, .1 , 1, 2, 6, and 10 micrometers. For both 3 MeV and 8 MeV the comparison graphs look almost identical to Figure (3) and are therefore left out.

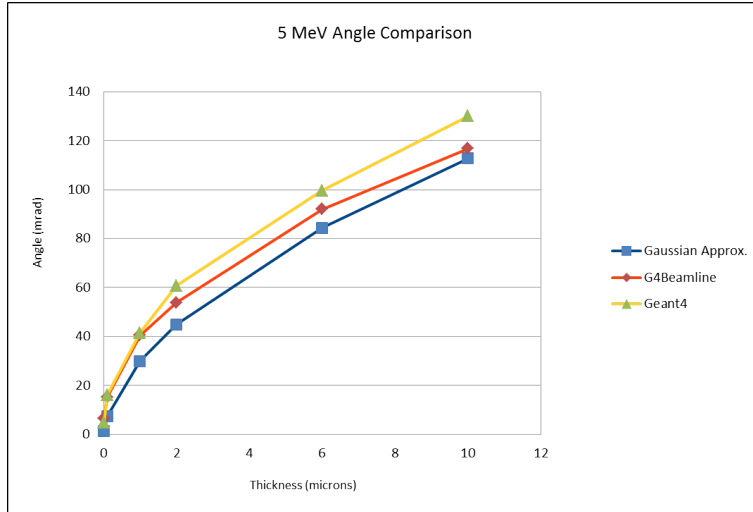


Figure 3: Plot of angle as a function of thickness.

As shown in Figure[3], all of the methods are fairly close to each other. Due to time constraints *Geant4* simulations were done for 10,000 runs instead of 100,000 runs like *G4Beamline*. If the full 100,000 runs were conducted it is believed that the lines for the two simulations would converge onto each other. The line generated by the Gaussian approximation will never match the simulations completely due to the fact that most approximations for multiple scattering are off by a fair amount. The method used was constructed to have lower error than the older ways of approximating scattering angle and was prepared by Gerald Lynch and Orin Dahl [5].

Table [2] displays the numerical values for the scattering angles as computed by the simulations and the analytical calculation. Of note is that below .8

Table 2: Scattering angles computed using three different methods.

Thickness ( $\mu m$ )	Approx. (mrad)	<i>G4Beam</i> (mrad)	<i>Geant4</i> (mrad)
.01	1.46	6.64	4.9
.1	7.27	15.04	16.03
1	29.93	40.5	41.4
2	44.9	53.9	60.7
6	84.4	91.7	99.6
10	112.8	116.8	130

$\mu m$  the analytical calculation becomes invalid as stated in section 3.2. This is because it is believed that below that threshold there are less than 20 scatters occurring for each electron as it passes through the medium. Multiple scattering is described as being scattering above 20 scatters per electron, while below that number it is called plural scattering. So it makes sense that thicker targets produce more scattering and thinner targets enter the realm of plural scattering. The results from *Geant4* and *G4Beamline* may also be invalid at that threshold as well.

### 4.3 Beam Power Distribution

The power distributed on the plate was determined for an 8 MeV beam running in the Mott polarimeter. It was important to look at the distribution of power per unit area in small rings out from the center of the dump plate. For targets that are especially thin, the power per unit area concentrated in a small area near the center of the plate can be much greater than for beams scattered through thick foils.

Table 3: Power per Area for target of thickness .01  $\mu m$  at 8 MeV and 10 micro Amps of current. Power per area was computed in rings with steps of .1 cm, (example: entry 2 is the power per area of a ring with outer radius .2 and inner radius .1.)

Radius (cm)	P/A ( $\frac{W}{cm^2}$ )
.1	276.09
.2	90.3375
.3	52.2283
.4	35.2865
.5	25.4823
.6	112.8

Table [3] shows that for a high energy beam and a thin foil the power per area near the center of the plate is very high but quickly dies off as the distance from the center increases. However, this result is only the power illuminating the plate not how much power is deposited onto the plate which would be the

next thing to be determined.

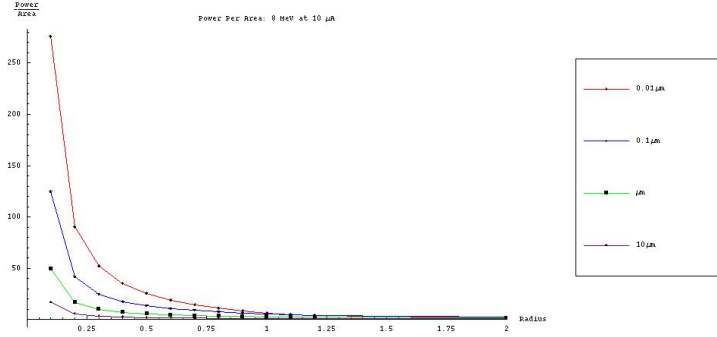


Figure 4: Plot of Power/Area as a function of thickness with Power/Area in  $\frac{W}{cm^2}$  and radius in cm.

Figure [4] shows that as the thickness of the target increases the power on the plate decreases. This is due to the fact that thicker targets produce bigger scattering angle distributions so that less of the beam energy is ending up on the dump, and some of it is ending up on the beam pipe. However, the beam hitting the pipe at that point is not as dangerous as one would think because the area over which the power is distributed increases and the power/area decreases.

## 5 Conclusion

As the work above shows, the main concern for the development of a new dump plate is whether the new plate can withstand the power deposition from the beam striking it. Due to the high amount of power illuminating the plate at the thinnest foils, it is important to consider different cooling methods that can help carry heat away and reduce the risk of the beam boring through the plate.

Table 4: Cooling Methods [7]

Type	T (Celsius)	q ( $\frac{W}{cm^2}$ )
Radiation	400-1000	4-10
Water	30-150	30-110
Forced Air	150-200	0.5-1
Evaporative	100-120	80-125
Conduction	100-250	5-30

Table [4] shows appropriate cooling methods for various levels of power deposition. While the power illuminating the plate may be higher than all of these cooling methods can handle, the deposition of power may still be low enough

for these methods. Future work will look at the power deposition and then determine which cooling methods would better suit the needs of the Mott group at Jefferson Lab.

## 6 Appendix

### 6.1 *Mathematica* Files

1. Allowable Thickness Determines thinnest foil that can be used in the Gaussian Approximation based on momentum and target material. Changeable Parameters:  $Z$ ,  $p$ ,  $A$ ,  $h$  where  $Z$  is the atomic number,  $p$  is the momentum,  $A$  is the atomic mass, and  $h$  is the smallest thickness where  $\Omega$  is equal to 10. 2. Multiple Gives the RMS scattering angle by taking the equation found in Leo, dividing by 2 and taking the square root. All the parameters are listed at the top of the notebook and can be changed. 3. PowerOnPlate The top portion of this notebook finds the normalization constant and total power illuminating the plate. The next portion finds the power/area and you can find the power/area for any portion of the plate you want by changing the variables alpha and beta. Alpha always needs to be less than beta. The bottom of this notebook includes how to plot the lists of data generated for each thickness. The lists of power/area are named P1-P5 and at the end of the lists the command to plot multiple lists on one plot is issued. If you try to do MultipleListPlot before hitting enter on that command, it won't like you. There's also examples of the export command, so that you can export any picture or data you'd like. I think Mathematica 5.2 can export data to excel and things like that.

### 6.2 *elegant*

This directory contains all the necessary .ele and .lte files to optimize the quads. In order to do so, edit the edited.ele file to optimize to whichever value you'd like to set and make sure the initial conditions you want are uncommented. Then run the file. It will generate another .lte file with the optimized K values. Then make sure the same initial conditions are in the edited2.ele file and run that file. That file will run reading the .lte file that edited.ele produced. Also in this directory are the optimized .lte and .sig files for all beam momenta. You can just change the lattice file that edited2 reads to any of these and be good to go. These files are in the format of fivemev.lte and fivemev.sig, so you know exactly which beam momentum each file is for.

### 6.3 Workbooks

This directory contains four .ods files. 1. Sigma Values.ods this file contains all of the sigma values at the target and dump for each beam momentum. 2. PowerPerArea.ods this file contains the total power in each ring and the power/area for 3 MeV 3. 5MeVAngleComparison.ods this file has the angles calculated using the Gaussian Approximation, G4Beamline, and Geant4 for 5

MeV and thickness from .01 micron to 10 micron. 4. Eight.ods this contains the power/are in slices of .1 for 8 MeV.

## References

- [1] Joe Grames
- [2] Jay Benesch
- [3] Michael Borland *User's Manual for elegant* Program Version 19.0 Advanced Photon Source June 6, 2008
- [4] W. R. Leo, *Techniques for Nuclear and Particle Physics Experiments: A How to Approach*. Springer-Verlag, New York, 2nd Revised Edition, 1994.
- [5] Gerald R. Lynch, Orin I. Dahl, *Approximations to multiple Coulomb scattering*. Nuclear Instruments and Methods in Physics Research, B58, pgs 6-10, 1991
- [6] S. Agostinelli *et. al*, *Geant4 - a simulation toolkit*. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, Volume 506, Issue 3, pgs 250-303, 2003
- [7] Reference Data for Radio Engineers, H.W. Sams Pub. 1970