

Solenoid

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Magnetic profile along the solenoid

$$B_z = \frac{1}{2} \mu_0 \left[\frac{z + \frac{1}{2}L}{[(z + \frac{1}{2}L)^2 + R^2]^{\frac{1}{2}}} \frac{z - \frac{1}{2}L}{[(z - \frac{1}{2}L)^2 + R^2]^{\frac{1}{2}}} \right] nI$$

$$B_r = \frac{dB_z}{dz}$$

$$B_r = -\frac{R}{4} \mu_0 \left[\frac{1}{[(z + \frac{1}{2}L)^2 + R^2]^{\frac{3}{2}}} - \frac{1}{[(z - \frac{1}{2}L)^2 + R^2]^{\frac{3}{2}}} \right] nIR^2$$

- L : solenoid length
- $B_0 = \mu_0 nI$: Magnetic field peak
- nl : Ampere turn per meter
- R : Solenoid radius

At the entrance of the solenoid

- We assume a particle with :
 - $P_x = 0$ and $P_y = 0$
 - $P = P_z$
- The radial coordinate : $r = r_0 \cos \frac{\omega_L z}{v_z}$
- The azimuthal coordinate : $\theta = \theta_0 + \frac{\omega_L z}{v_z}$
- Where :
 - θ_0 Initial azimuthal coordinate.
 - r_0 Initial radial coordinate
 - $\omega_L = \frac{eB_0}{2\gamma m}$ is the Larmour frequency
- Lorentz force $\vec{F}_{Lorentz} = q\vec{v} \times \vec{B}$
- $v_r = \frac{dr}{dt}$ and using $\dot{v} = \frac{d\vec{r}}{dt} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta + \dot{z}\vec{e}_z$ in cylindrical coordinates we get:

$$v_r = -r\omega_L \tan\left(\frac{\omega_L z}{v_z}\right)$$

$$v_\theta = r\omega_L$$

At the exit of the solenoid

- At the exit edge of the solenoid $B_r \neq 0 \longrightarrow \vec{F}_\theta$
- $\Delta v_\theta = -r_1 \omega_L$
 - r_1 is the radial coordinate at the exit of the solenoid.

$$\Delta v_\theta = -r_1 \omega_L$$

$$\Delta v_\theta = v_{\theta_{final}} - v_{\theta_{initial}}$$

$$-r_1 \omega_L = v_{\theta_{final}} - r_1 \omega_L$$

$$v_{\theta_{final}} = 0$$

- The only component left is the radial velocity : $v_r = -r_1 \omega_L \tan\left(\frac{\omega_L z}{v_z}\right)$

Thin lens approximation

- $L \leq \frac{v_z}{\omega_L}$ we end up with :

$$v_r \cong -r_1 \omega_L \frac{\frac{\omega_L L}{v_z}}{1 - \frac{\omega_L^2 L^2}{v_z^2}}$$

- We simplify the previous equation we get :

$$v_r = -\frac{r_0 e^2}{4\gamma^2 m^2 v_z} B_0^2 L$$

- For a random variation of B_z we can write:

$$v_r = -\frac{re^2}{4\gamma^2 m^2 v_z} \int B^2 dz$$

Focal length

- The focal length is expressed by this relation :

$$\frac{1}{f} = -\frac{r'}{r}$$

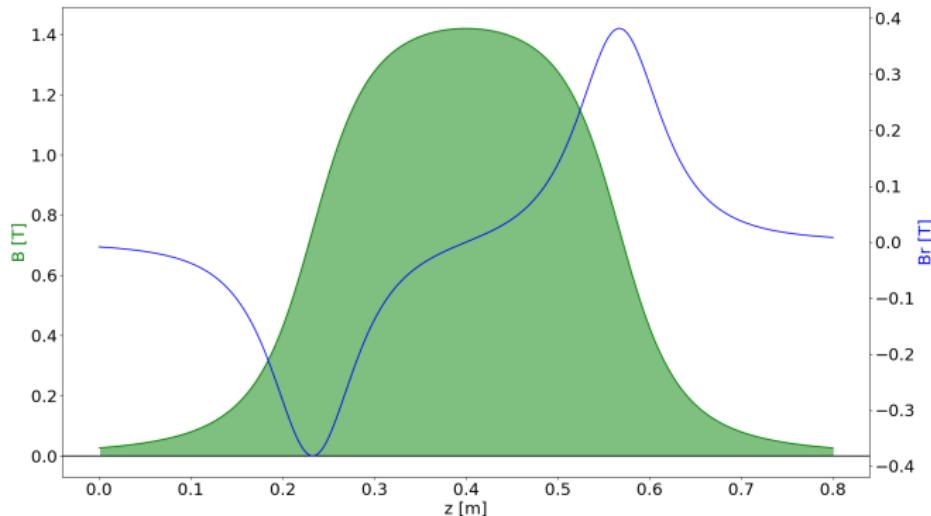
- From the previous formula, we can re-write:

$$\frac{1}{f} = \frac{e^2}{4\gamma^2 m^2 v_z^2} \int B^2 dz$$

Magnetic profile

- Using the following solenoid parameters:

- $L = 0.334 \text{ m}$
- $R = 0.07 \text{ m}$
- $B_0 = 1.54 \text{ T}$

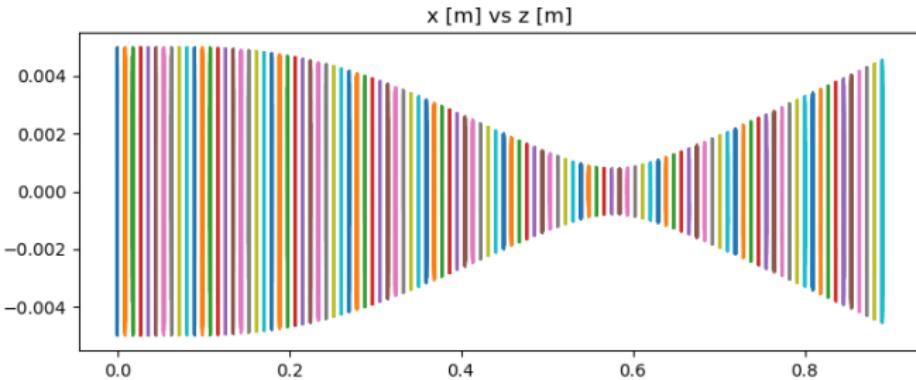


Solenoid simulation

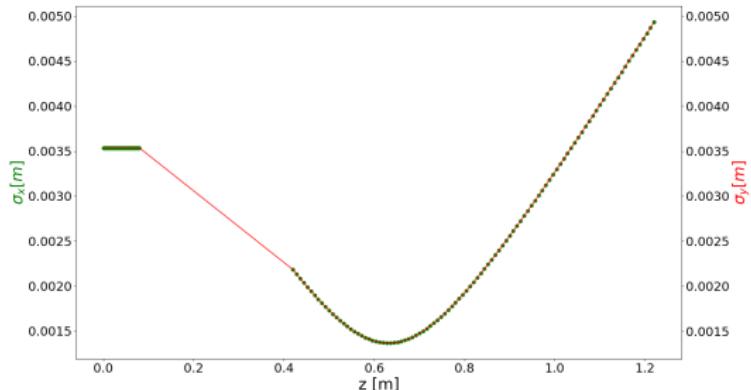
- The following formula describes the focal length:
$$\frac{1}{f} = \left[\frac{e}{2\gamma\beta mc} \right]^2 \times \int_0^L B^2 dz$$
 which equivalent to $\frac{1}{f} = \left[\frac{e}{2P} \right]^2 \times \int_0^L B^2 dz$
- Where P is the particle momentum, which is set at $P = 60$ [MeV/c], we get $f = 0.29$ m.
- center of lens: 0.245 m
- distance source-solenoid 0.08 m
- The beam should be focalized at $z_f = 0.08 + 0.245 + 0.29 = 0.61$ m

GPT VS ELEGANT

GPT



ELEGANT



Spin Tracking

- Let's consider a particle with the following spin coordinates:
 - $S_x = 0$
 - $S_y = 1$
 - $S_z = 0$
- Using the same solenoid, the angle of rotation of the spin vector is given by:

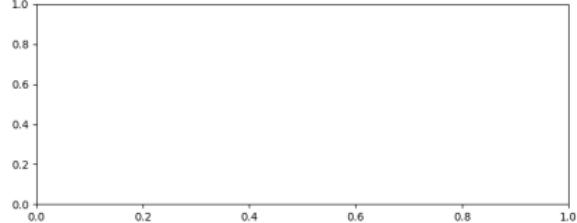
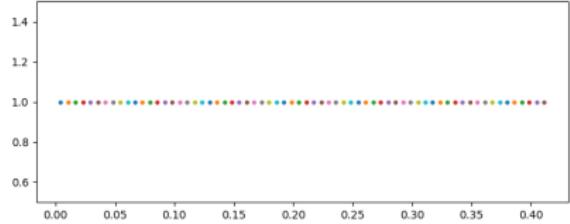
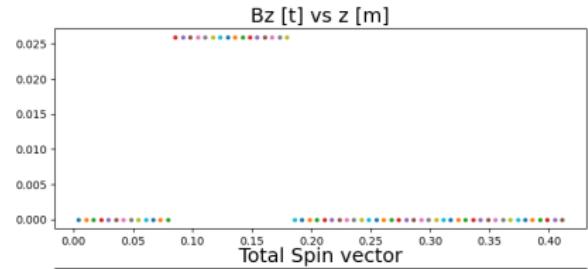
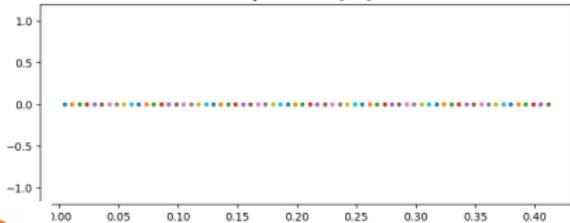
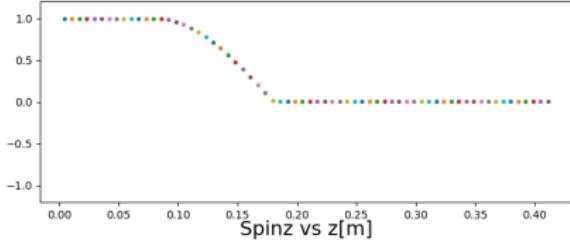
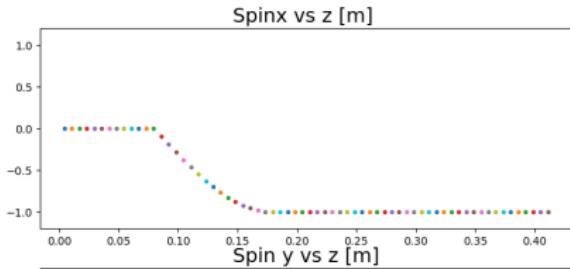
$$\theta = \frac{180}{\pi} \frac{e}{m} \frac{L_{eff}}{\beta c} \left[\frac{1}{\gamma(1 + \alpha)} B \right]$$

- Where :
 - L_{eff} is the solenoid length.
 - $\beta = \frac{v}{c}$
 - γ is the Lorentz factor
 - B is the parallel component of the magnetic field to the direction of the spin.
 - $\alpha = \frac{g}{2} - 1$ is the magnetic moment.

Spin Tracking

- For $L_{eff} = 10cm$ and $B = 0.02T$, $\beta = 0.7$, and $\gamma = 1.39$, we get $\theta = 90$ degree precession of spin.
- Using an hard edge model to check the GPT's results :

Spin Tracking



Conclusion

- Prediction of the focal length distance along a solenoid
- The radial magnetic field has a strong effect on the rotation of the particle in the azimuthal direction
- Results obtained from GPT and ELEGANT are similar
- The spin tracking in GPT match with the analytical predictions