Josh Yoskowitz
Ionization Rate vs. Voltage

1 Purpose

Calculating the ion production rate (IPR) vs. gun voltage for the different gas species found in the "After 2 Days" RGA spectrum below:

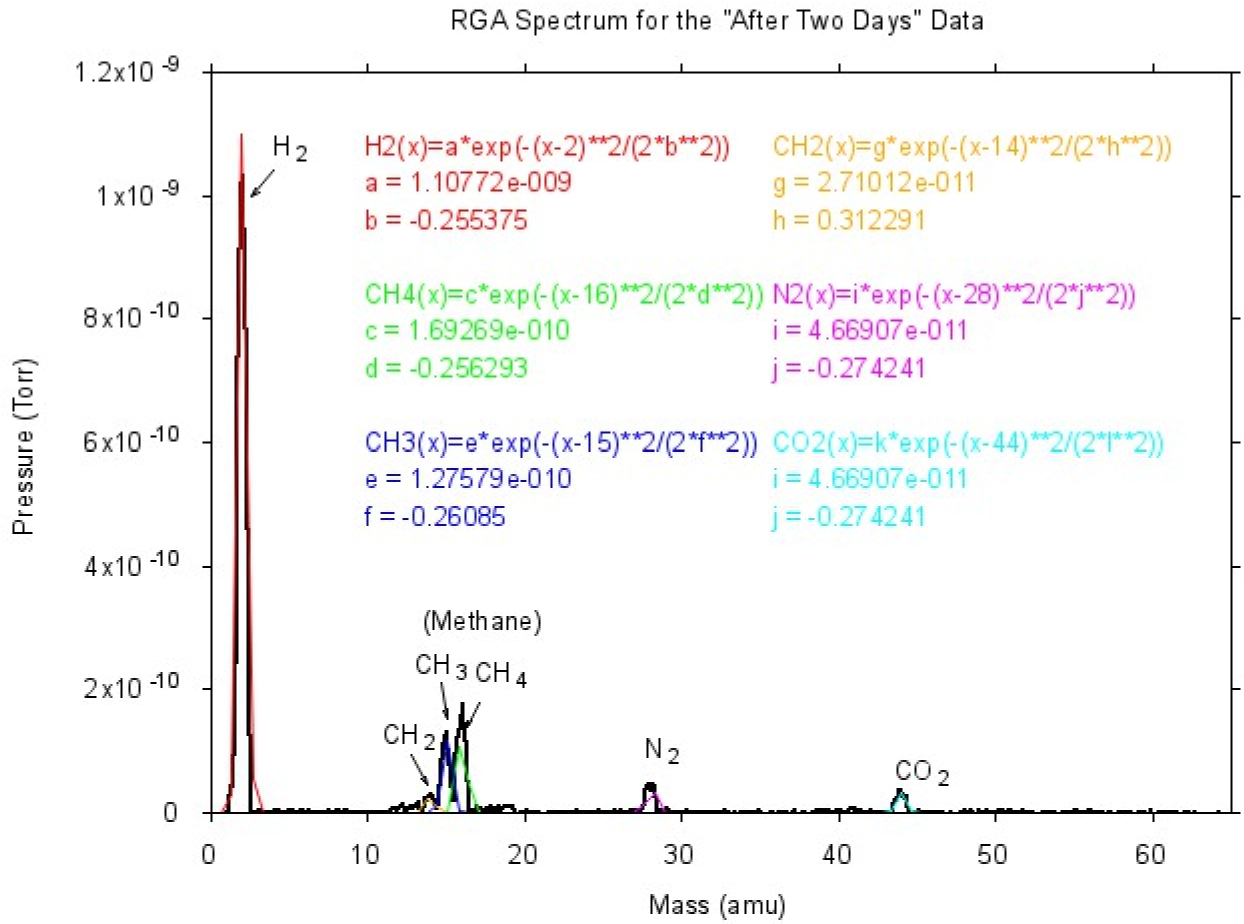

Figure 1: Analysis of the RGA spectrum for the "After 2 Days" data (before correction factor)

2 IPR vs. Gun Voltage

The ion production rate (i.e. change in number density over time) is given by Reiser[1]

$$\frac{dn}{dt} = n_b n_g \sigma_i v = n_b n_g \sigma_i \beta_e c \tag{1}$$

We would like to rewrite the IPR as a function of voltage (or really in terms of beam energy T_e). We can start by rewriting β_e in terms of T_e:

$$
\begin{aligned}
T_e &= (\gamma - 1)\, m_e c^2 \\
\gamma &= 1 + \frac{T_e}{m_e c^2} \\
\frac{1}{\sqrt{1-\beta_e^2}} &= 1 + \frac{T_e}{m_e c^2} \\
1 - \beta_e^2 &= \left(\frac{1}{1+\frac{T_e}{m_e c^2}}\right)^2 = \left(\frac{m_e c^2}{m_e c^2 + T_e}\right)^2 \\
\beta_e^2 &= 1 - \left(\frac{m_e c^2}{m_e c^2 + T_e}\right)^2 \qquad (2)
\end{aligned}
$$

The numerical equation for the ionization cross section given by Reiser [1]

$$
\begin{aligned}
\sigma_{i[\mathrm{m}^2]} &= \frac{1.872 \times 10^{-24} A_1}{\beta_e^2} f\left(T_e\right)\left[\ln\left(7.515 \times 10^4 A_2 \beta_e^2 \gamma^2\right) - \beta_e^2\right] \qquad (3) \\
f\left(T_e\right) &= \frac{I_i}{T_e}\left(\frac{T_e}{I_i} - 1\right) \\
A_1 &= M^2 \\
A_2 &= \frac{e^{\frac{C}{M^2}}}{7.515 \times 10^4}
\end{aligned}
$$

Plugging in equation (2) into (3) yields:

$$
\begin{aligned}
\sigma_i\left(T_e\right) = &\frac{1.872 \times 10^{-24} A_1}{1 - \left(\frac{m_e c^2}{m_e c^2 + T_e}\right)^2} \frac{I_i}{T_e}\left(\frac{T_e}{I_i} - 1\right) \times \\
&\ln\left[7.515 \times 10^4 A_2 \left(1 - \left(\frac{m_e c^2}{m_e c^2 + T_e}\right)^2\right)\left(1 + \frac{T_e}{m_e c^2}\right)^2\right] - \left(1 - \left(\frac{m_e c^2}{m_e c^2 + T_e}\right)^2\right) \qquad (4)
\end{aligned}
$$

We can then plug (2) and (4) into (1) to yield a relationship between $\frac{dn}{dt}$ and T_e. Using Mathematica, this relationship can be shown graphically below for each gas species. Here, we assume that we have a 1mA, uniform, cylindrical electron beam 1mm in radius. We see that for each gas species, the graph has a zero at the ionization energy and is dominated by the ionization cross section σ_i. A table of IPR values for selected beam energies is also shown below.

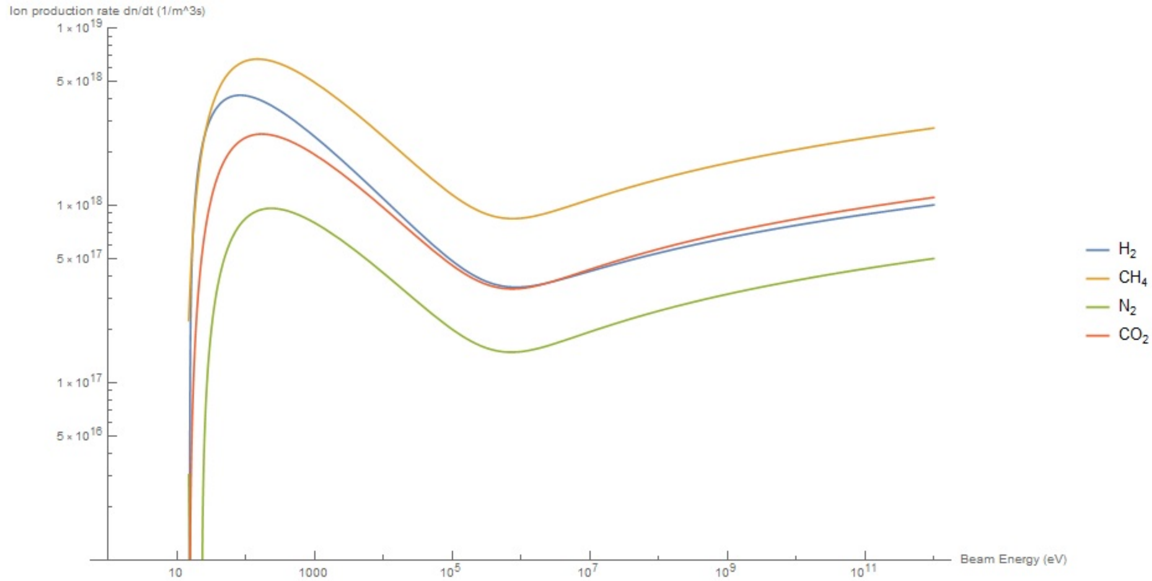

Figure 2: Log-log plot of the ion production rate as a function of beam energy for each gas species found in the RGA spectrum excluding CH_3 and CH_2.

Gas Species	IPR at 50eV $(m^{-3}\,s^{-1})$	IPR at 100eV $(m^{-3}\,s^{-1})$	IPR at 500eV $(m^{-3}\,s^{-1})$	IPR at 1keV $(m^{-3}\,s^{-1})$
H_2	3.93×10^{18}	4.15×10^{18}	3.03×10^{18}	2.45×10^{18}
CH_4	5.22×10^{18}	6.52×10^{18}	5.81×10^{18}	4.95×10^{18}
N_2	5.22×10^{17}	8.43×10^{17}	9.07×10^{17}	7.99×10^{17}
CO_2	1.80×10^{18}	2.41×10^{18}	2.26×10^{18}	1.94×10^{18}
Gas Species	IPR at 100keV $(m^{-3}\,s^{-1})$	IPR at 130keV $(m^{-3}\,s^{-1})$	IPR at 180keV $(m^{-3}\,s^{-1})$	IPR at 1MeV $(m^{-3}\,s^{-1})$
H_2	4.88×10^{17}	4.52×10^{17}	4.16×10^{17}	3.46×10^{17}
CH_4	1.15×10^{18}	‘1.07×10^{18}	9.91×10^{17}	8.46×10^{17}
N_2	2.00×10^{17}	1.87×10^{17}	1.73×10^{17}	1.50×10^{17}
CO_2	4.60×10^{17}	4.29×10^{17}	3.97×10^{17}	3.40×10^{17}

Table 1: Ion Production Rates (IPR) of each gas species for selected beam energies.

3 Normalizing IPR to Electron Beam Current

In the previous section, we computed IPR values for a 1mA electron beam. We can rewrite equation 1 and normalize to beam current:

$$\frac{dn}{dt} = n_g \sigma_i \left(n_b v\right)$$

The quantity in parentheses can be thought of as the volume *number* density of electrons moving at velocity v. We can relate this to conventional current I via the definition of the volume *current* density J for a volume *charge* density ρ:

$$J = \rho v \equiv e n_b v = \frac{dI}{da_\perp}$$

where e is the elementary charge and $a_\perp$ is the geometric transverse area of the electron beam. We can integrate through the entire electron beam and rewrite this as:

$$J = \frac{I}{a_\perp} = e n_b v \rightarrow n_b v = \frac{I}{e a_\perp}$$

Thus,

$$\frac{dn}{dt} = n_g \sigma_i \frac{I}{ea_\perp}$$
$$\frac{dn/dt}{I} = \frac{n_g \sigma_i}{ea_\perp} \tag{5}$$

The above equation yields the number of ions produced per second per ampere per cubic meter. Note that, in most cases, $a_\perp$ is *not* constant and depends on distance along the beam. n_g does not have to be constant/uniform either and may also have distance dependence. Below is a Mathematica log-log plot of normalized IPR vs electron beam energy where we have assumed a 1mm^2 beam.

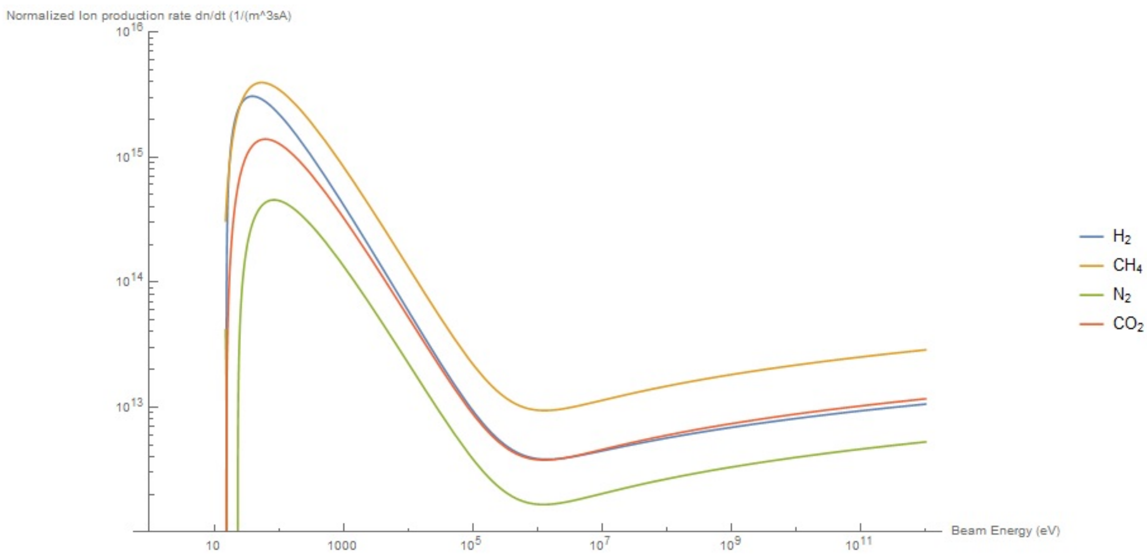

Figure 3: Log-log plot of normalized IPR for a 1mm^2 beam as a function of beam energy for the gas species found in the RGA spectrum excluding CH_3 and CH_2.

References

[1] Martin Reiser. *Theory and Design of Charged Particle Beams.* Wiley VCH Verlag GmbH, 2008.