# Simplistic QCM energy gain model

Max Bruker

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We are trying to accurately calibrate the GSET of the QCM cavities to be able to operate them at the design parameters. How much this matters in practice is not clear, but we will hopefully learn something about that as we go.

On Feb 1 and 2, Yan measured the energy gain of both QCM cavities individually for a range of GSET values while leaving the GSET of the other cavity fixed. The beam energy was determined using the 601 dipole magnet as a spectrometer. Matt extrapolated these curves to the hypothetical case of the 2-cell cavity being at zero field (log entry 02/10/2021). He concludes that the current field setting of the 2-cell is considerably above the design value and should be decreased.

To get a feeling for the interactions and numbers, let us revisit a simulation model I made a few months ago. I'm going to explain what I think I understand so you can tell me how it actually is :)

## 1 General

I assume the GSET value to be proportional to the field amplitude. In the nonrelativistic case, because $\beta$ changes in flight as a function of phases, amplitudes, and initial conditions, the field amplitude is not necessarily proportional to the energy gain, and the concept of average energy gain per unit length is therefore nonsensical. To some extent, the TOF changes can be compensated for by adjusting the phases, but we have more $\beta < 1$ cells than we do phase knobs.

Because cavities do not care about momentum and I share their ignorance, I will convert all momenta (e.g., those inferred from the spectrometer setting) to kinetic energy according to

$$E_\mathrm{k} = \sqrt{p^2c^2 + m^2c^4} - mc^2. \tag{1}$$

In the context of this study, "optimum" phases refer to those that maximize the total energy gain because I assume that was the objective of optimization during the measurement. Energy spread and bunch length should most likely factor into the figure of merit and are easy to add at a later time, see below.

## 2 Simulation method

The beam is modeled as a single particle with a starting energy of 200 keV and at a starting coordinate on the cavity axis, considerably before the onset of the field. This particle is tracked onedimensionally on a discrete time axis, applying energy gain accordingly. The cavities are modeled using Haipeng's normalized on-axis field maps. Spline interpolation allows for the computation of $E_z(z)$ for any value of $z$. The time dependency of the total field is:

$$\begin{aligned} E_z(z,t) &= \cos(2\pi f t - \phi_2) A_2 E_2(z) \\ &\quad + \cos(2\pi f t - \phi_7) A_7 E_7(z) \end{aligned} \tag{2}$$

with $f = 1497\,\mathrm{MHz}$[1] and the spline functions $E_i(z)$ for Haipeng's field maps. These functions are shown in Fig. 1. The phases $\phi_i$ have an arbitrary offset depending on the starting coordinate of the particle.

*Note that the field amplitudes $A_i$ represent the peak values and are therefore much higher than the GSET numbers. Once we understand what we are doing, we may be able to infer the constant of proportionality from the simulation, keeping in mind that the numbers are not strictly proportional in every case.*

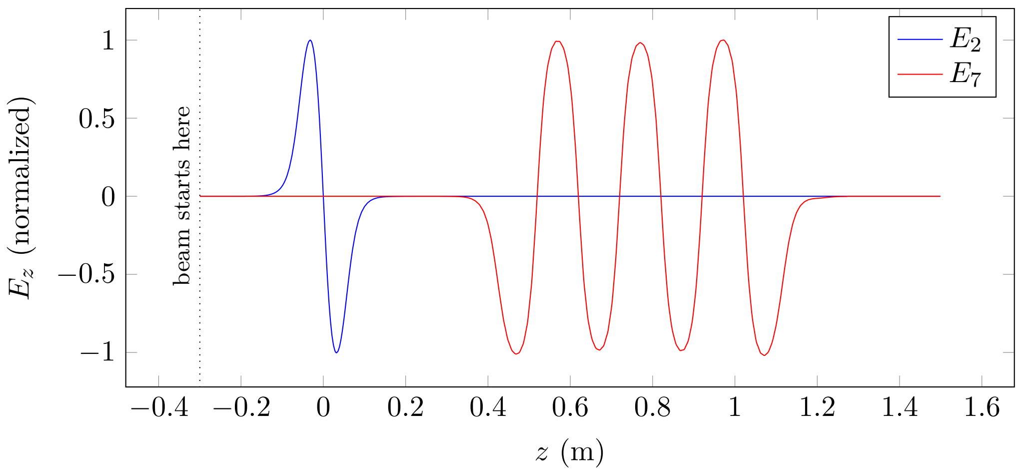


Figure 1: Normalized on-axis field maps.

This simulation is trivially extensible to include bunch length and energy spread by applying a Monte-Carlo beam instead of a single particle. My last model included that, but I decided against it for this particular case for the sake of computation time. It's not what we're here to learn today.

[1] Is this correct? Let me know if it isn't.

## 3 First tests

First, let us look at the $[\phi_2, \phi_7]$ parameter space for a given tuple of amplitudes to see if the result makes sense. Such a scan is shown in Fig. 2. Unfortunately, the low number of paper/screen dimensions makes the complete four-dimensional parameter space difficult to visualize at once.

Unsurprisingly, there is a contour inside of which the beam does what it is supposed to. The shape of the contour and the position of the maximum are not trivial because $\phi_2$ affects both the transit time through the first cavity and the TOF to the 7-cell, where $\beta$ is still less than 1. If you look at the kinetic energy as a function of $t$ or $z$, you will find that at the maximum-energy-gain point, the 7-cell actually decelerates the beam a little at first, which I guess is OK. This is shown in Fig. 3.

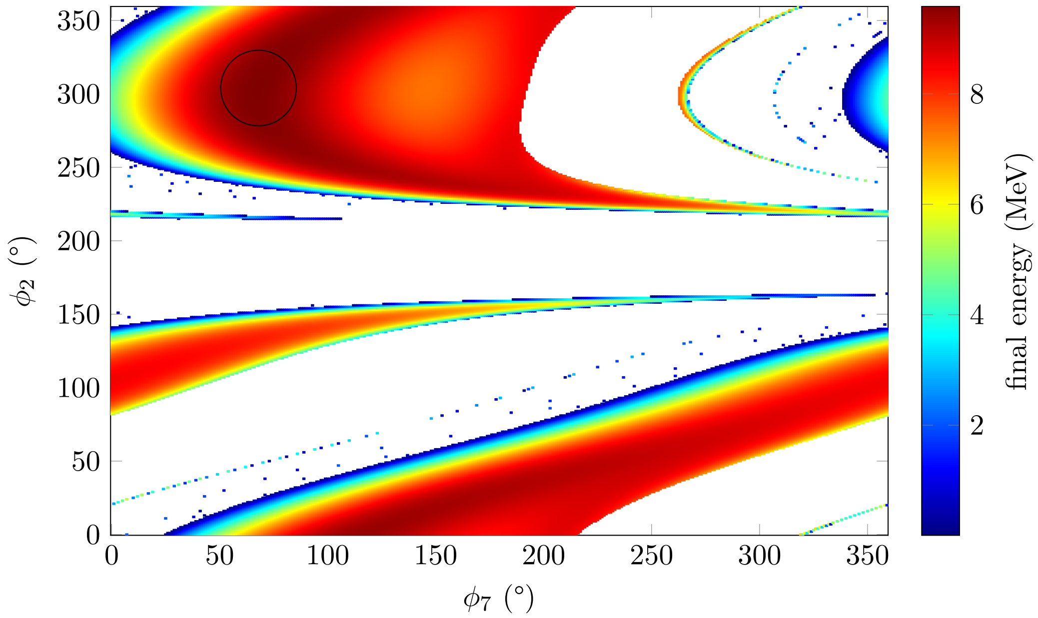


Figure 2: Final beam energy in the $[\phi_2, \phi_7]$ parameter space. $A_2 = 7.47\,\mathrm{MV\,m^{-1}}$, $A_7 = 24.0\,\mathrm{MV\,m^{-1}}$. The amplitudes are chosen to match the experiment as explained in Sec. 4.1. Do not be alarmed by the high numbers; I believe they are correct and our view is distorted by the inflationary use of effective average fields. White areas in this plot correspond to unphysical cases where the kinetic energy goes below zero. The maximum energy is 9.58 MeV at $\phi_2 = 304°$, $\phi_7 = 68°$. This is the same energy as the highest value from Yan's measurement ($G_{\mathrm{set}} = 12$).

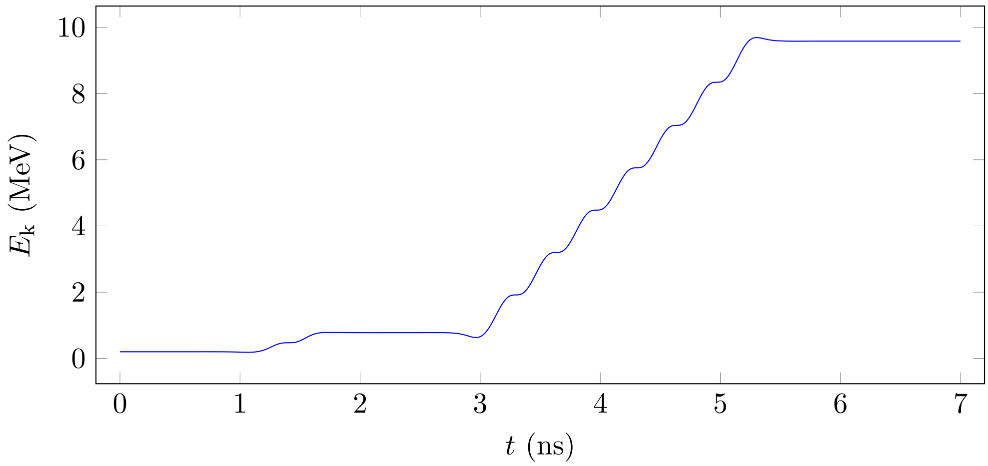


Figure 3: Kinetic energy as a function of time (maximum-energy conditions). $\phi_2 = 304°$, $\phi_7 = 68°$, $A_2 = 7.47\,\mathrm{MV\,m^{-1}}$, $A_7 = 24.0\,\mathrm{MV\,m^{-1}}$.

# 4 Comparison with measurements

## 4.1 7-cell amplitude scan

Let us try to reproduce Yan's measurement where he varied $A_7$, keeping $\phi_2$ and $A_2$ constant. I assume $\phi_2$ was kept at the value that gave the maximum energy gain at nominal settings, which is 304° in our simulation. For the full range of $A_7$ settings, we scan the $\phi_7$ space for the $E_\mathrm{k}$ maximum, choosing $A_2$ such that the $y$-axis intercept of the resulting $E_\mathrm{k}(A_7)$ straight line fit is at about 0.643 MeV as in Yan's data ($p = 1.10\,\mathrm{MeV/c}$). Finding the $A_2$ value involves a bit of ~~toying~~ iterative optimization because the points are not on a straight line in the vicinity of $A_2 = 0$. This is why I'm saying what meaning this intercept holds, if any, is a matter of definition. Open for debate, of course! The result of this simulation is shown in Fig. 4. It qualitatively resembles Yan's data.

Let us compare how the 7-cell phase needs to be adjusted for maximum gain. Since we're not confident in our calibration factor yet, we will compare the phase setting as a function of total energy gain from measurement and simulation; we pretend to know the measured beam energy accurately. This comparison is shown in Fig. 5. The curves agree fairly well.

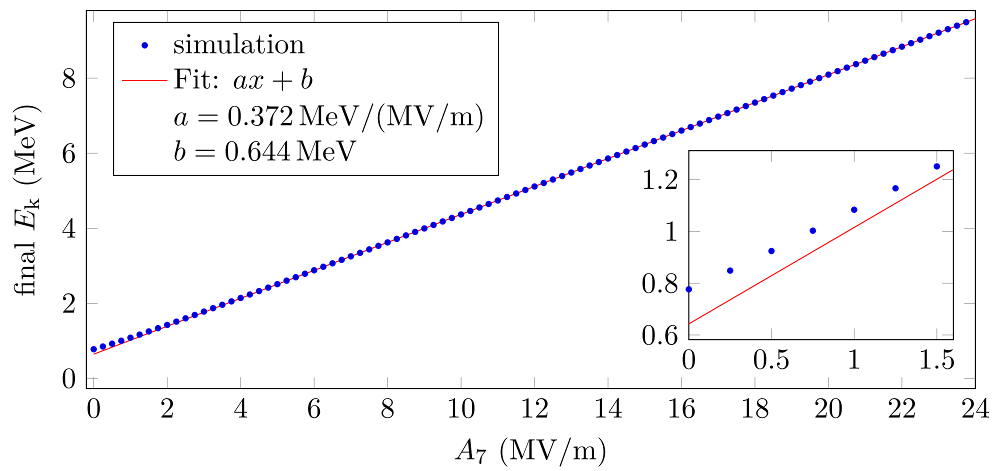


Figure 4: Maximum total energy gain for the whole range of 7-cell amplitudes at the respective optimum value of $\phi_7$. $\phi_2 = 300°$, $A_2 = 7.47\,\mathrm{MV\,m^{-1}}$. The fit only uses data with $A_7 > 5\,\mathrm{MV\,m^{-1}}$.

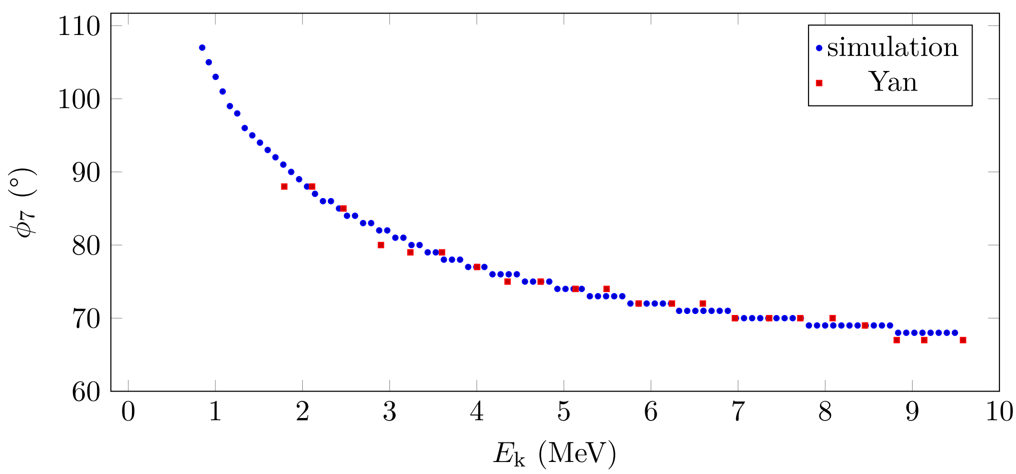


Figure 5: Optimum 7-cell phase as a function of final beam energy. $\phi_2 = 304°$, $A_2 = 7.47\,\mathrm{MV\,m^{-1}}$. As the absolute phase is arbitrary, the measured values are shifted by 55° to make the curves overlap. You might want to get excited now, but read on.

### 4.2 2-cell amplitude scan

Yan's second measurement where he varied the 2-cell amplitude can be simulated in a similar fashion, though the parameter space is much larger because both phases are adjusted. The result is shown in Fig. 6. After all the good agreement we've seen up to this point, the discrepancy here is egregious. The simulated beam energy at $A_2 = 0$ is much too low (6.1 MeV vs. 7.8 MeV), and the nonlinear slope at high amplitudes is not reproduced. I have no explanation yet. Until we know what causes this curvature in the first place, fitting lines to these data and using them as a basis for extrapolation seems ill-advised.

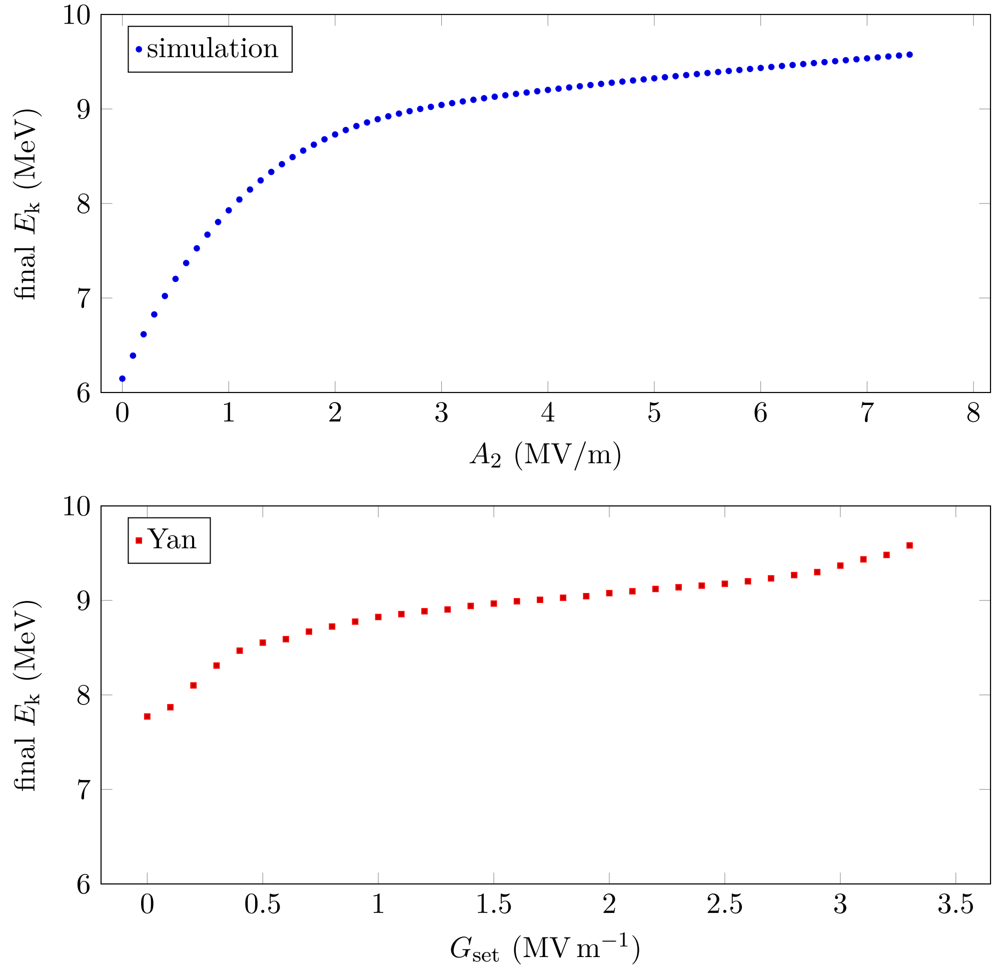


Figure 6: Maximum total energy gain for the whole range of 2-cell amplitudes at the respective optimum values of both $\phi_2$ and $\phi_7$. $A_7 = 24.0\,\mathrm{MV\,m^{-1}}$. Yan's measurement is shown in the second plot for comparison.

The attempt to compare the optimum phases from this measurement with the simulated result also ends up in spectacular failure, see Fig. 7.

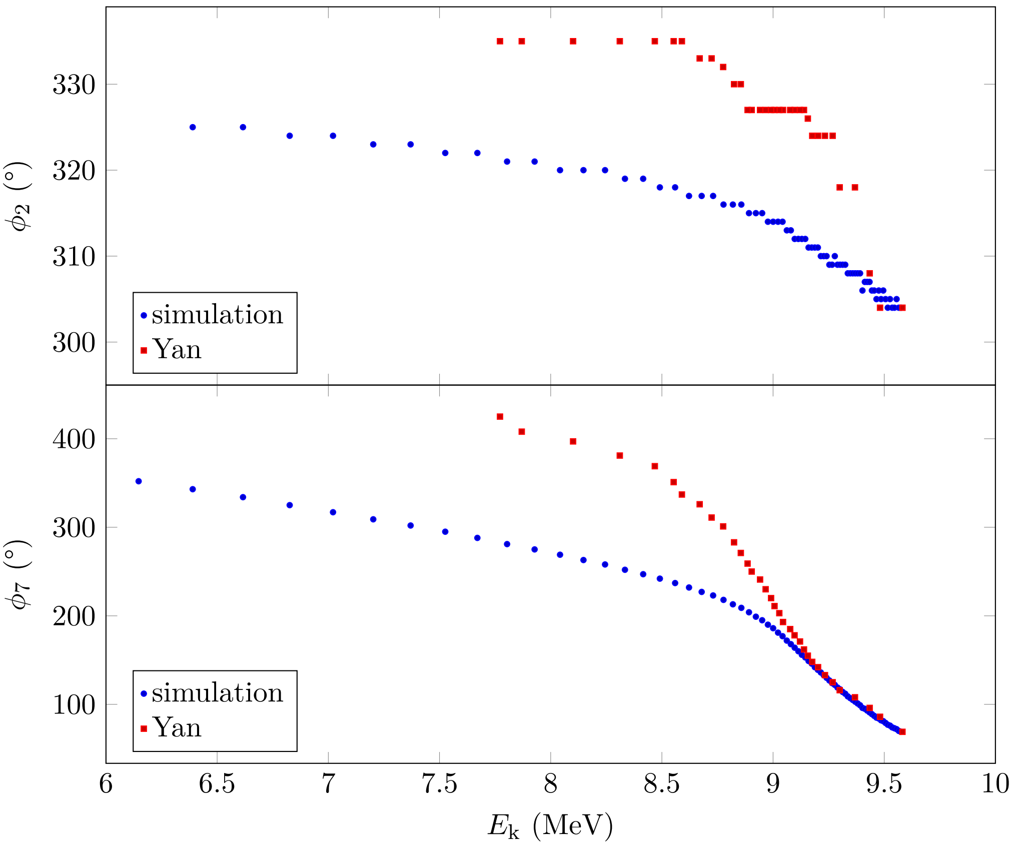


Figure 7: Optimum values of $\phi_2$ and $\phi_7$ for the whole range of 2-cell amplitudes as a function of final beam energy. $A_7 = 24.0\,\mathrm{MV\,m^{-1}}$. Yan's data has been shifted up by 479° ($\phi_2$) and 57° ($\phi_7$), respectively, but there is no way to line up the data.

The main reason why I wanted to write this was to convince you that you cannot blindly add or subtract the energy gains from the individual cavities. Calibrating the field amplitude of a single cavity in isolation based on its energy gain in the system is not trivial, and the result of it, even if correct, does not have a well-defined meaning. In any case, now it would be interesting to find the reason why the results from this section don't match.

## 5 Potential error sources

- Calibration of dipole magnet?
- Cavity frequency correct?
- I wasn't there when the measurement was done. Is there something I'm missing as to the procedure? Dipole magnet on loop?
- I measured the distance between the two cavities in a drawing with a ruler. It may be off by a percent or two. My intuition says a few mm should not be the end of the world, but I can check if it makes a difference.
- The discretization in the simulation (time, phase angles) has been checked for convergence and presents no significant error source. See Fig. 8.

The potential impact of these errors is quantifiable with relative ease. Before I waste time on something stupid, though, feel free to let me know if there is anything superobvious I have overlooked.

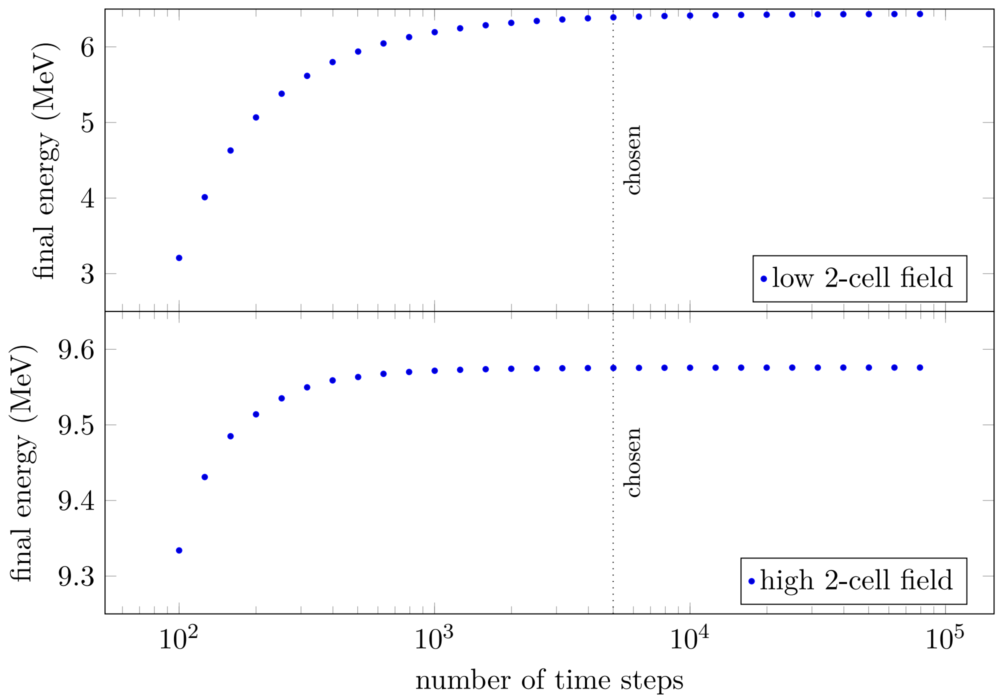


Figure 8: Asymptotic behavior of algorithm with increasing number of time steps. The total simulated time is 7 ns in every case. This is enough to always leave the field region.