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## Optical particle trapping with higher-order doughnut beams produced using high efficiency computer generated holograms

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**Abstract.** Laser beams containing higher-order phase singularities can be produced with high efficiency computer generated holograms made with very simple equipment. Using such holograms in an optical tweezers experiment we have successfully trapped reflective and absorptive particles in the dark central spot of a focused charge 3 singularity beam. Angular momentum absorbed from the beam can set particles into rotation.

### 1. Introduction

A phase singularity is a point in an optical field around which the phase of the field changes by an integer multiple of  $2\pi$  (this integer  $q$  is called the topological charge of the phase singularity). Consequently, the phase at this point is undefined and it appears as an isolated dark spot against a bright background [1]. In recent years, phase singularities in optical fields have drawn great interest because they are of importance for understanding fundamental physics and have many important applications. There are several methods for generating phase singularities. They can be generated by transformation of Hermite–Gaussian modes [2] or using the process of cooperative mode locking [3] in a properly designed laser such as He–Ne [4] or Na<sub>2</sub> [5]. We have previously shown that beams containing phase singularities can conveniently be produced using computer generated holograms [1], but the amplitude holograms we used were of low efficiency (below 5%) and designed to produce charge one singularities. These holograms also are unsuitable for use with high power lasers because of their high absorption. However, higher order phase singularities have more interesting behaviour than charge one singularities and many applications rely on high intensity light fields. Hence, higher order and higher efficiency holograms are needed.

Angular momentum is associated with the helical structure of the wave surrounding a singularity so that a linearly polarized beam with a singularity carries angular momentum  $\hbar q$  [2]. Hence, higher order holograms are more suitable for observing photon angular momenta.

In response to these needs, we have successfully developed a simple method requiring only limited equipment to produce computer generated holograms which have very high efficiencies. By using these holograms in a laser tweezers apparatus, we have successfully trapped reflective and absorptive particles with a very low laser power. Although doughnut trapping has been performed before, we believe that our

work gives the first evidence of trapping using higher charge phase singularities. Using a properly focused laser beam with a charge 3 phase singularity, we have been able to set black particles into rotation. We have also been able to demonstrate dark spatial soliton like behaviour [6] of a high power singularity beam propagating in an optically nonlinear material.

## 2. Production of computer generated holograms

The simplest mathematical form of a phase singularity can be written as [7]

$$E(r, \theta, z) = E_0 \exp(iq\theta) \exp(-ikz), \quad (1)$$

where  $q$  is the topological charge of the singularity and  $\theta$  is an angle measured in the plane transverse to the direction of propagation.

In essence, a hologram is just a recording of the interference pattern between two light beams. Since the mathematical form of a phase singularity is known, the interference pattern between it and a plane wave beam can be easily worked out.

Consider the interference pattern between the field described by equation (1) and a uniform plane wave  $u$ , propagating at an angle to the axis

$$u = \exp(-ik_x x - ik_z z). \quad (2)$$

For simplicity we assume the recording plane at  $z = 0$ , and we can express the interference pattern as

$$I = 1 + E_0^2 + 2E_0 \cos(k_x x - q\theta). \quad (3)$$

Phase singularity holograms are thus similar to gratings except that there is a defect in the centre. Hence, we can treat them as simple gratings to find their efficiencies. One type of hologram of proven high efficiency is the thick Bragg type [8]. A hologram is termed thick or thin according to its spacing period  $\Lambda = 2\pi/k_x$  and the emulsion thickness  $d$  of the holographic plate by using a parameter  $D$  where

$$D = 2\pi\lambda d/n_0\Lambda^2, \quad (4)$$

where  $n_0$  is the mean refractive index and  $\lambda$  is the wavelength of the incident light.

Unfortunately, the thick Bragg type hologram involves a three dimensional structure which cannot be produced synthetically.

If  $D < 1$  holograms are thin. For computer generated holograms, their spacing periods are quite long. The typical values of the parameters in equation (4) for computer generated holograms are  $\Lambda \approx 10^2 \mu\text{m}$  and  $d \approx 10 \mu\text{m}$ , so for the wavelength of around  $1 \mu\text{m}$ , the parameter  $D$  will be of the order of  $10^{-3}$ .

The Fourier transform provides a fairly good understanding of Fraunhofer diffraction for thin holograms. The periodical transmittance function  $T(x)$  of the hologram can be expressed by the Fourier series as

$$T(x) = \sum_{n=-\infty}^{n=+\infty} A_n \exp(i2\pi nx/\Lambda), \quad (5)$$

where

$$A_n = \frac{1}{\Lambda} \int_{x'}^{x'+\Lambda} T(x) \exp(-i2\pi nx/\Lambda) dx, \quad (6)$$

and  $x'$  is an arbitrary point in a spacing period.

Table. Maximum theoretical efficiencies for different types of holograms.

Hologram type	Maximum theoretical efficiency
Amplitude, binary, 100% modulation	10.1%
Amplitude, sinusoidal, 100% modulation	6.25%
Phase, binary, $\pi$ modulation	40.4%
Phase, sinusoidal, 1.841 modulation	33.9%

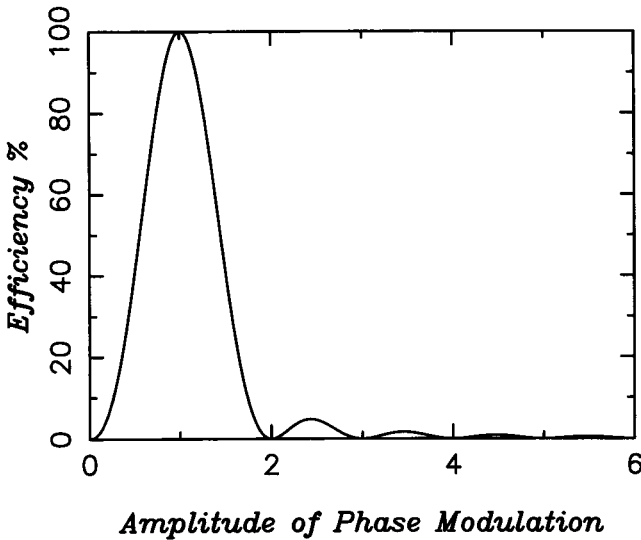


Figure 1. Efficiency curve for blazed holograms.

The Fourier component amplitude,  $A_n$ , also represents the amplitude of the associated diffraction order when a wave with unit amplitude is normally incident on the hologram.

Thus,  $|A_{+1}|^2$  is the diffraction efficiency of a hologram defined as the ratio of the intensity of the +1 diffraction order to the intensity of the light incident on the hologram.

Using equations (5) and (6) it is possible to calculate the maximum efficiency of the various types of holograms, binary or continuous tone, amplitude or phase, and results are shown in the table.

We see from this table that even though binary holograms apparently waste light by producing higher orders, the actual amount of light going into the first order is slightly greater than for sinusoidal modulation. Actual efficiencies of amplitude holograms we have produced previously [1] are a little lower but consistent with these values. In all the cases shown in the table positive and negative orders are produced, effectively halving the power available in the desired beam.

It is well known that a blazed grating can have 100% diffraction efficiency. The transmittance function of a blazed phase hologram can be given as

$$T(x) = \exp[im \text{Mod}(x, A)], \quad (7)$$

where  $m$  is the amplitude of phase modulation and  $\text{Mod}(a, b) = a - b \text{Int}(a/b)$ .

Its Fourier transform gives

$$A_n = \frac{1}{A} \int_{-A/2}^{A/2} \exp \left[ -i \left( \frac{2n\pi}{A} - m \right) x \right] dx. \quad (8)$$

The efficiency curve for this type of hologram is shown as figure 1. If  $mA/2 = \pi$  then  $|A_1|^2 = 100\%$ . It diffracts all the light into one order. Therefore, it is possible to make a hologram with 100% efficiency using only masks and printing techniques.

In producing such holograms, the blazed interference patterns can be written as

$$I = \text{Mod}(q\theta - kx, 2\pi), \quad (9)$$

where we neglect the constant term.

Hence, by using computer graphic techniques, a blazed interference pattern can be generated according to equation (9). By photo reducing these patterns onto 35 mm film, we obtain amplitude holograms. Typical blazed computer generated hologram patterns are shown in figure 2. However, the efficiencies of amplitude holograms are very low. If the phase changes according to equation (9), we can get a transmittance function similar to that of equation (7). To do so, we can bleach the amplitude holograms to get phase holograms. We expect that the efficiency of the bleached holograms will behave as equation (8). Experiments confirm this.

To obtain high efficiency, a high amplitude of phase modulation is required. Several bleach agents have been tried and among them the mercuric chloride agent achieves the highest efficiency after bleaching. The mercuric chloride bleach solution is made by mixing about 10 g mercuric chloride into 1000 ml distilled water [9]. Bleached holograms were produced by making contact prints of photo reduced patterns onto holographic plates and then bleaching these developed holographic plates. The amplitude of phase modulation can be adjusted by varying the exposure time. Very high efficiency holograms can be obtained by optimising the exposure time. In this way, holograms with better than 50% efficiencies can be easily obtained for use with the 633 nm He-Ne laser. The efficiencies are limited by nonlinearity (usually, the phase modulation doesn't increase linearly with the exposure, which results in higher diffraction orders) and residual absorption and scattering in the bleached emulsion. Even where not all the light is diffracted into the desired order, the relative transparency of a phase hologram allows the incident power to be increased without damage occurring.

We also found a new bleach which is suitable for bleaching films directly. The bleach solution is made by mixing 9 g  $\text{Fe}(\text{NO}_3)_3 \cdot 9\text{H}_2\text{O}$ , 5 g ammonium dichromate and 6 ml concentrated sulphuric acid into 1 litre distilled water. Films bleached by this recipe have near ideal linearity and very high transmittance but the range of amplitude of phase modulation is smaller compared with mercuric chloride bleach. However, the amplitude of phase modulation is wavelength dependent. Therefore, for shorter wavelength laser light, very high efficiencies ( $> 90\%$ ) may be obtained.

### 3. Doughnut trapping

Transparent particles with refractive index higher than their surroundings can be trapped in the middle of a highly focused Gaussian beam [10]. Alternatively, the  $\text{TEM}_{01}^*$  doughnut mode can be used in a conventional trap. This trapping has increased trapping efficiency over the Gaussian spot trap [9, 11]. It has been proven

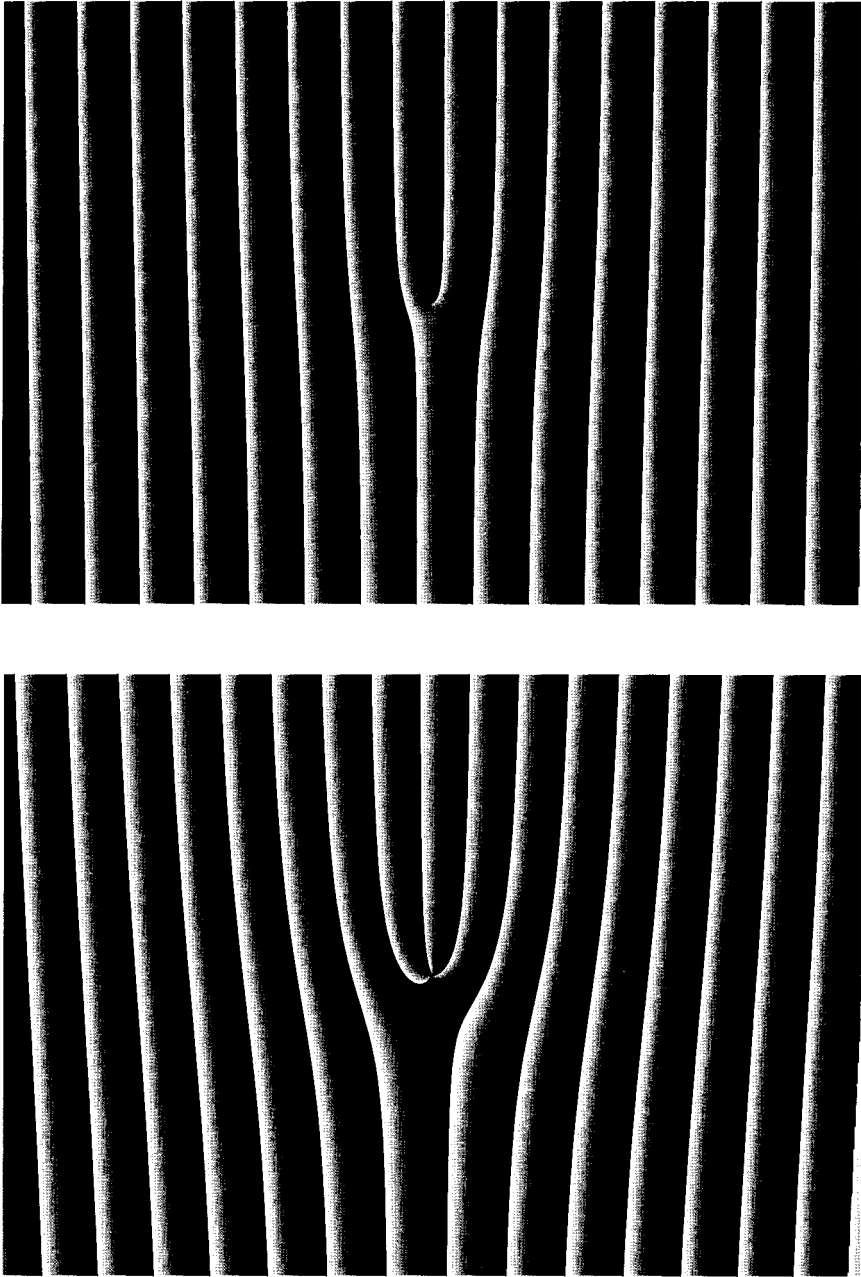


Figure 2. Blazed computer generated hologram patterns. Upper: charge 1, lower: charge 3.

that the far field beam produced by binary computer generated holograms closely resembles a TEM<sub>01</sub>\* doughnut of slightly increased spot size [1]. The same conclusion can also be applied for the blazed holograms.

The experimental set-up is similar to that used in our earlier experiments [12] except that a charge 3 blazed hologram produces a 3 mW doughnut equivalent to a Gauss-Laguerre LG<sub>03</sub> when the 7 mW He-Ne laser is turned on. Since the laser beam covers only part of the hologram, by moving the hologram sideways, we can switch the diffracted beam between doughnut mode and Gaussian mode. The beam enters the vertical illumination port of the microscope and is brought to a tight focus with a high numerical aperture oil immersion objective.

Practical observation has proven that transparent high refractive index particles of 1–3 μm sizes can be trapped easily along the ring of the doughnut with a laser power of around 3 mW.

Reflective and absorptive particles cannot be trapped by using a Gaussian spot as strong repulsive forces act on these particles. However, such particles can be trapped in the dark central spot of a doughnut [13]. For our test, we used aluminium particles as the reflective particles and a black high  $T_c$  superconductor ceramic powder as the absorptive particles, dispersed in water and kerosene, respectively. Experimentally, particle sizes around 1–2 μm were successfully trapped in the middle of a hologram produced doughnut. We use the charge 3 doughnut because it has a wider central dark spot, allowing more effective trapping than in a charge 1 or 2. Once a particle is trapped, it can be moved around easily. A trapped particle can be kicked out when we switch from the doughnut mode to the Gaussian mode. It has also been observed that such a particle can be attracted from well outside the doughnut, right into the centre where it will remain trapped, rather than being repelled by the outer wings of the doughnut. We are studying this unexpected effect further.

When the sizes of particles (10–30 μm) are much larger than the beam size (3–4 μm), they cannot be trapped by the focused doughnut. However, a doughnut, the size of which has been increased by defocusing, may set such a particle into rotation which in most cases can be reversed by changing the sign of the singularity. This can be ascribed to absorption of angular momentum from the singularity beam itself [2]. Of course asymmetric particles will tend to rotate as a result of unbalanced radiometric and radiation pressure forces and it appears that in a minority of cases this effect dominates leading to a rotation direction independent of charge. The fact that we do not see any rotations when we use non-helical beams for trapping strongly suggests that the rotation is a direct result of the angular momentum carried by the beam which we emphasize is linearly polarized. The small particles which we can trap inside the doughnut cannot be resolved well enough to verify or rule out rotation. Further experimental work on the rotation of the particles is now in progress in our laboratory.

#### 4. Other applications of holographic doughnuts

A holographic doughnut can also be used to form a dark spatial soliton in a nonlinear medium. In a self-defocusing medium, as a singularity beam propagates, the central dark spot will tend to collapse but cannot because of the presence of the singularity, so that a dark spot of constant size and shape develops. The associated refractive index structure can act as a wave guide, so that another laser beam can propagate along the middle of the doughnut. This could also be used as an

optical switch which may find use in optical computers. As the efficiencies are high, low power diode lasers could be used. Hence, an integrated optical circuit is also possible.

In this connection, we have been able to confirm the observations of Swartzlander and Law of dark spatial soliton like behaviour [6] using a 500 mW argon ion laser beam in a weak solution of chlorophyll in ethylene glycol in a 40 mm diameter, 22 cm long glass cell. Like them, we were able to observe the expected contraction of the central spot as the power was increased.

## 5. Conclusions

We have developed a very easy way to produce high efficiency singularity holograms. The TEM<sub>01</sub>\* and higher order doughnuts can be produced by illuminating these holograms with Gaussian laser beams. Our experiments have shown that with further improvement in bleaching techniques, near 100% conversion of a Gaussian laser beam to a higher order doughnut mode will be possible. Using high efficiency holograms is a simple and efficient way of achieving doughnut particle trapping with a very low power laser source. Particles can also be set into rotation by the helical waves. The high efficiency holograms may also help to realize optical switches.

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