Nuclear Inst. and Methods in Physics Research, A 875 (2017) 141–145

Contents lists available at ScienceDirect

Nuclear Inst. and Methods in Physics Research, A

journal homepage: www.elsevier.com/locate/nima

A critical analysis of the technique of spin tune mapping in storage rings

S.R. Mane

Convergent Computing Inc., P. O. Box 561, Shoreham, NY 11786, USA

A R T I C L E I N F O

Keywords:
Polarized beams
Storage rings
Polarimetry
Electric and magnetic moments

A B S T R A C T

A paper has recently been published which describes the technique of so-called 'spin tune mapping' to measure the 'stable spin axis' (spin closed orbit) of a spin polarized beam circulating in a storage ring. This paper presents an independent analysis of the technique, and significantly different findings are reported below. In particular, it is derived that there are several unquantified systematic errors which are not treated in the previous analysis.

1. Introduction

A paper has recently been published [1] which describes the technique of so-called 'spin tune mapping' to measure the 'stable spin axis' of a spin polarized beam circulating in a storage ring. The authors employ nonstandard notation and terminology in [1]: the 'stable spin axis' is the spin closed orbit or the rotation axis of the one-turn spin map on the closed orbit of the ring. A review of spin dynamics in accelerators can be found in [2]. It is claimed in [1] that the spin tune mapping technique can determine the orientation of the stable spin axis to microradian accuracy. Note, however, that the components of the polarization vector were not measured directly in [1]. Instead, the direction of the stable spin axis was deduced via measurements of the spin tune and the application of a theoretical model.

I have independently examined the analysis in [1] and my findings differ from the claims made in [1]. In addition to identifying various errors of algebra, I found there are several unquantified systematic errors which are not treated in the analysis in [1]. Numerous priority claims are also made in [1]. I comment on some of those priority claims and supply references to prior work in the literature [3–9].

This paper is organized as follows. Section 2 presents the basic notation and definitions. The spin maps for relevant beamlines and ring elements, which are pertinent to the analysis, are shown in Section 3. The solution for the spin tune is derived in Section 4. Differences with the formulas in [1] are pointed out. Section 5 presents the exact solution of an idealized model. It is shown that the solution derived in Section 4 agrees with the exact solution, up to terms of the first order in small quantities. However, the formulas derived in [1] do not agree with the exact model, even for the first order terms. In particular, for the scenario studied in [1], the determination of the radial component of the stable spin axis is subject to large uncertainties, which are not accounted for in the analysis in [1]. Section 6 comments on some of the priority claims made in [1] and describes prior work on the subject. Section 7 concludes.

2. Basic notation and definitions

We refer the reader to [2] for a review of spin dynamics in accelerators, including the electric dipole moment (EDM). We treat a particle of mass m and charge e, with velocity $\vec{v} = \vec{\beta}c$ and Lorentz factor $\gamma = 1/\sqrt{1-\beta^2}$. The canonical particle coordinate and conjugate momentum are denoted by $\vec{r}$ and $\vec{p}$, respectively. Most of the analysis in this paper employs coordinate-free notation. Where explicit vector components are required, we follow [1] and employ the (right-handed orthonormal) basis vectors $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$, respectively radial (outward), vertical (up) and longitudinal (along the ring reference axis). We denote the spin vector by $\vec{s}$, treated as a semiclassical unit vector, with magnetic moment anomaly $G = (g-2)/2$. We treat the vector polarization only, and denote the polarization vector by $\vec{P}$. Neglecting the EDM,[1] the spin precession equation of motion in the externally prescribed electric and magnetic fields of the accelerator ($\vec{E}$ and $\vec{B}$, respectively) is given by the Thomas–BMT (Bargmann–Michel–Telegdi) equation [11,12]

$$\frac{d\vec{s}}{dt} = -\frac{e}{mc}\left[\left(G+\frac{1}{\gamma}\right)\vec{B} - \frac{G\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(G+\frac{1}{\gamma+1}\right)\vec{\beta}\times\vec{E}\right]\times\vec{s}\,. \tag{2.1}$$

Radiation fields are ignored and we treat nonradiatively polarized beams only. In this paper, the spin state of a particle is parameterized by a two-component spinor and the spin map through a beamline is

[1] The semiclassical relativistic spin precession equation including EDM terms is given in [10]. See also the review [2].

E-mail address: srmane001@gmail.com.

http://dx.doi.org/10.1016/j.nima.2017.09.026
Received 5 August 2017; Accepted 13 September 2017
Available online 20 September 2017
0168-9002/

parameterized using a 2 × 2 SU(2) matrix. The one-turn spin map on the closed orbit is conventionally written as follows[2]

$$M_{\rm OTM} = \exp\{-i\pi\nu\vec{\sigma}\cdot\vec{n}_0\}\,. \tag{2.2}$$

Here ν is the spin tune and $\vec{\sigma}$ is a vector of Pauli matrices. The rotation axis of the one-turn spin map on the closed orbit is denoted by the unit vector $\vec{n}_0$ (this vector is also known as the 'spin closed orbit'). For off-axis motion, the quantization axis of the spin eigenstates is denoted by $\vec{n}(\vec{r},\vec{p})$, which is a function of the orbital phase space [13]. Throughout most of this paper, we shall restrict attention only to motion on the closed orbit. For a steady state spin polarized beam circulating in a storage ring, i.e. after transients have decohered, the polarization vector $\vec{P}$ is parallel to the average $\vec{P}\parallel\langle\vec{n}\rangle$, where the average is taken over the orbital phase space. In general, this average is almost parallel to $\vec{n}_0$, and this approximation will be employed below. (See [14, Sec. (3.3)] for a model example where $\langle\vec{n}\rangle$ is not parallel to $\vec{n}_0$.)

The model treated in [1] was a racetrack ring, with a solenoid in each of the diametrically opposed straight sections. Optically, the two arcs (also the straight sections) were identical except for lattice imperfections. The two solenoids were treated as localized zero-length perturbations. We shall treat the above model in this paper. The spin tune in the unperturbed ring (no solenoids) was denoted by ν_s^0 and in the full ring by ν_s. The authors called ν_s^0 the 'unperturbed spin tune' [1]. The authors also employed the notation $\vec{c}$ instead of $\vec{n}_0$ for the spin closed orbit of the unperturbed ring and called it the 'stable spin axis.' It was assumed in [1] that the polarization vector in the unperturbed ring (after transients had decohered) points along $\vec{c}$.

Note that the authors in [1] claimed to measure the direction of the stable spin axis but they did *not* measure the components of the polarization vector directly. Instead they determined the values of two parameters a_+ and a_- [1, eq. (31)] where it was stated 'Consequently, the determination of $a_\pm$ amounts to the determination of the projections of the stable spin axis $\vec{c}$ onto a plane spanned by the vectors $\vec{n}_1$ and $\vec{n}_2{}^{\rm r}$.' (The vectors $\vec{n}_1$ and $\vec{n}_2{}^{\rm r}$ will be defined below.) A beam dynamics study using stored polarized deuterons was performed at COSY using two electron cooler solenoids in diametrically opposed straight sections. The authors generated artificial longitudinal 'imperfection fields' using the electron cooler solenoids. The formalism in [1] presents an analysis of the data from that experiment. The experimentally measured quantity was the spin tune. The authors employed the term 'spin tune jump' to refer to the change in the spin tune $\Delta\nu_s = \nu_s - \nu_s^0$, with the solenoids on and off. The direction of the stable spin axis was therefore *deduced* via measurements of the spin tune jump and a theoretical model.

3. Spin maps

The analysis below treats only rings where the spin closed orbit is vertical everywhere, in the ideal design, and the ring has no Siberian Snakes or spin rotators. See [15] for a review of Siberian Snakes and spin rotators in storage rings. COSY is an example of such a ring. We shall mostly employ the notation in [1] for ease of reference to make contact with their analysis. Note, however, that their notation does not follow the standard practice in the field. The origin was placed just before the first solenoid. The one-turn spin map is, with an obvious notation [1, eq. (21)]

$$M_{\rm OTM} = M_{\rm A_2}M_{\rm S_2}M_{\rm A_1}M_{\rm S_1}\,. \tag{3.1}$$

Here the term 'arc' includes the straight sections (lattice imperfections in the straight sections can tilt the spin closed orbit away from the vertical). The one-turn spin map of the unperturbed ring (i.e. without solenoids) is parameterized via

$$M_{\rm R} = M_{\rm A_2}M_{\rm A_1} = \exp\{-i\pi\nu_s^0\vec{\sigma}\cdot\vec{c}\}\,. \tag{3.2}$$

See [1, eq. (17)] and Eq. (2.2) above. The spin map of each arc is parameterized via [1, eq. (24)]

$$M_{\rm A_j} = \exp\Big\{-\frac{i}{2}\,\theta_j(\vec{\sigma}\cdot\vec{m}_j)\Big\} \qquad (j=1,2)\,. \tag{3.3}$$

Here θ_j is the spin rotation angle and $\vec{m}_j$ is the spin rotation axis of the spin map for each arc. It is assumed that $\vec{c}$ is almost but not exactly vertical. (It would be exactly vertical in the absence of lattice imperfections). The arcs are almost but not exactly identical (i.e. they would be exactly identical in the absence of lattice imperfections). Hence $\vec{m}_1$ and $\vec{m}_2$ are both nearly vertical (but they are not assumed to be equal). Also $\theta_1 \simeq \pi\nu_s^0$ and $\theta_2 \simeq \pi\nu_s^0$ (but they are not assumed to be equal). The spin map of each solenoid is parameterized via [1, eq. (25)]

$$M_{\rm S_j} = \exp\Big\{-\frac{i}{2}\,\chi_j(\vec{\sigma}\cdot\vec{n}_j)\Big\} \qquad (j=1,2)\,. \tag{3.4}$$

Here χ_j is the spin rotation angle and $\vec{n}_j$ is the spin rotation axis of each solenoid.[3] The solenoids are treated as zero length elements. The vectors $\vec{n}_1$ and $\vec{n}_2$ are nearly longitudinal (along the reference axis of the ideal ring) but they are not assumed to be exactly equal. The angles χ_1 and χ_2 were variable parameters in the analysis in [1]. In addition let us define [1, eq. (31)]

$$\chi_\pm = \frac{\chi_1 \pm \chi_2}{2}\,. \tag{3.5}$$

In addition to $\vec{n}_1$ and $\vec{n}_2$, the authors also employed a vector $\vec{n}_2{}^{\rm r}$ defined via [1, eq. (26)]

$$M_{\rm A_1}^{-1}M_{\rm S_2}M_{\rm A_1} \equiv \exp\Big\{-\frac{i}{2}\,\chi_2(\vec{\sigma}\cdot\vec{n}_2{}^{\rm r})\Big\}\,. \tag{3.6}$$

It is given by [1, eq. (27)]

$$\vec{n}_2{}^{\rm r} = \cos\theta_1\,\vec{n}_2 + \sin\theta_1\,(\vec{n}_2\times\vec{m}_1) + (1-\cos\theta_1)(\vec{m}_1\cdot\vec{n}_2)\vec{m}_1\,. \tag{3.7}$$

The authors then defined the spin map of the 'combined artificial imperfection' via $M_{\rm AI} = M_{\rm A_1}^{-1}M_{\rm S_2}M_{\rm A_1}M_{\rm S_1}$ [1, eqs. (23) and (28)]. The full one-turn spin map is then given by $M_{\rm OTM} = M_{\rm R}M_{\rm AI}$. The authors also defined the two variables [1, eq. (31)]

$$a_\pm = \vec{c}\cdot\vec{n}_2{}^{\rm r} \pm \vec{c}\cdot\vec{n}_1\,. \tag{3.8}$$

The above expressions are all in coordinate-free notation. The authors then made various approximations, using a coordinate basis, to derive the approximate expressions up to the first order in small quantities [1, eq. (78)]

$$a_\pm \simeq \cos(\pi\nu_s^0)c_z - \sin(\pi\nu_s^0)c_x \pm c_z\,. \tag{3.9}$$

For later use, I shall define the two parameters

$$\alpha_\pm = \vec{c}\cdot\vec{n}_2{}^{\rm r} \pm \vec{c}\cdot\vec{n}_1\,. \tag{3.10}$$

In coordinate-free notation, these are the same as $a_\pm$ in Eq. (3.8). However, when expanded in components, I shall show their values are different from those in Eq. (3.9). The matter will be treated below.

4. Spin tune

The spin tune of the full ring (with solenoids) is obtained from the parameterizations of the spin maps above and is obtained via $\cos(\pi\nu_s) =$

[2] The closed orbit includes the effects of lattice imperfections and in general is not equal to the ideal design orbit of the ring.

[3] The vectors $\vec{n}_1$ and $\vec{n}_2$ should not be confused with the quantization axis of the spin eigenstates, which is conventionally denoted by $\vec{n}$ by workers in the field, see, e.g. [2,13].

$\frac{1}{2}\,\mathrm{Tr}(M_{\mathrm{R}} M_{\mathrm{AI}})$. Then, recalling $\nu_s = \nu_s^0 + \Delta\nu_s$,

$$\begin{aligned}\cos(\pi[\nu_s^0 + \Delta\nu_s]) &= \cos(\pi\nu_s^0)\cos\frac{\chi_1}{2}\cos\frac{\chi_2}{2} \\ &\quad - \sin(\pi\nu_s^0)\left[\sin\frac{\chi_1}{2}\cos\frac{\chi_2}{2}\,(\vec{c}\cdot\vec{n}_1)\right. \\ &\quad \left. + \cos\frac{\chi_1}{2}\sin\frac{\chi_2}{2}\,(\vec{c}\cdot\vec{n}_2^{\,\mathrm{r}})\right] \\ &\quad - \left[\cos(\pi\nu_s^0)\,(\vec{n}_2^{\,\mathrm{r}}\cdot\vec{n}_1) + \sin(\pi\nu_s^0)\,\vec{c}\cdot(\vec{n}_2^{\,\mathrm{r}}\times\vec{n}_1)\right] \\ &\quad \times \sin\frac{\chi_1}{2}\sin\frac{\chi_2}{2}\,.\end{aligned} \tag{4.1}$$

The above expression corrects some errors of algebra in [1, eq. (C1)]. The above expression is an exact formula, and is in coordinate-free notation. I now expanded the above expression in vector components. I retained all the components of the various vectors in all the vector dot and cross products. The zeroth order (or 'large') components are $c_y \simeq 1$, $m_{1y} \simeq 1$, $m_{2y} \simeq 1$, $n_{1z} \simeq 1$ and $n_{2z} \simeq 1$. All the other vector components are of the first order in small quantities. I also set $\theta_j = \pi\nu_s^0 + \Delta\theta_j$, where $|\Delta\theta_j|$ is a small angle. Then the term in the square brackets in the last line of Eq. (4.1) evaluated to

$$\begin{aligned}&\cos(\pi\nu_s^0)\,(\vec{n}_2^{\,\mathrm{r}}\cdot\vec{n}_1) + \sin(\pi\nu_s^0)\,\vec{c}\cdot(\vec{n}_2^{\,\mathrm{r}}\times\vec{n}_1) \\ &\quad \simeq \cos(\Delta\theta_1) - \sin(\Delta\theta_1)(n_{1x} - n_{2x}) \\ &\quad \simeq 1\,.\end{aligned} \tag{4.2}$$

The expression equals unity up to corrections of the second order in small quantities, which we neglect. Then, approximating it to unity in Eq. (4.1) yields

$$\begin{aligned}\cos(\pi\nu_s^0) - \cos(\pi[\nu_s^0 + \Delta\nu_s]) \simeq\ & [1 + \cos(\pi\nu_s^0)]\sin^2\frac{\chi_+}{2} \\ & - [1 - \cos(\pi\nu_s^0)]\sin^2\frac{\chi_-}{2} \\ & + \frac{\alpha_+}{2}\,\sin(\pi\nu_s^0)\sin\chi_+ \\ & - \frac{\alpha_-}{2}\,\sin(\pi\nu_s^0)\sin\chi_-\,.\end{aligned} \tag{4.3}$$

This has a similar form to [1, eq. (30)] but corrects some errors of algebra in the latter. Retaining only terms of the first order in small quantities, I obtained the following expressions for $\alpha\pm$ (see Eq. (3.10))

$$\begin{aligned}\alpha_\pm &\simeq \cos(\pi\nu_s^0)\,c_z + n_{2y} + [1 - \cos(\pi\nu_s^0)]\,m_{1z} \\ &\quad - \sin(\pi\nu_s^0)\,(c_x - m_{1x}) \\ &\quad \pm (c_z + n_{1y})\,.\end{aligned} \tag{4.4}$$

- There are several additional terms not present in the analysis in [1] (see Eq. (3.9) and [1, eq. (78)]).
- The additional terms are of the same order as c_x and c_z and cannot *a priori* be neglected.
- In particular, c_x always appears in the combination $c_x - m_{1x}$. Since the stable spin axis $\vec{c}$ is non-vertical because of lattice imperfections, and the vectors $\vec{m}_1$ and $\vec{m}_2$ are also non-vertical for the same reason, the value of m_{1x} will in general be comparable to c_x. The values of m_{1x} and m_{1z} yield unquantified systematic errors which are not treated in the analysis in [1].

5. Exact solution of idealized model

It is possible to derive the exact solution for the spin tune for a model with (a) two identical arcs, and (b) parallel spin rotation axes for the two 'perturbation fields.' The 'perturbations' are treated as zero length elements. The one-turn spin map of the unperturbed ring (i.e. only the two arcs) is

$$M_{\mathrm{unpert}} = \exp\{-i\pi\nu_s^0\,\vec{\sigma}\cdot\vec{n}_0\}\,. \tag{5.1}$$

See Section 2 for details of notation and definitions. Hence the spin map of each arc is

$$M_{\mathrm{arc}} = \exp\left\{-\frac{i}{2}\,\pi\nu_s^0\,\vec{\sigma}\cdot\vec{n}_0\right\}. \tag{5.2}$$

The spin maps of the 'perturbations' are

$$M_j = \exp\left\{-\frac{i}{2}\,\chi_j\,\vec{\sigma}\cdot\vec{\zeta}\right\} \qquad (j = 1, 2)\,. \tag{5.3}$$

The spin rotation axis $\vec{\zeta}$ is the same in both elements. The full one turn spin map is then $M_{\mathrm{OTM}} = M_{\mathrm{arc}} M_2 M_{\mathrm{arc}} M_1$. Let the spin tune of the full ring be $\nu_s = \nu_s^0 + \Delta\nu_s$. Then $\cos(\pi\nu_s) = \frac{1}{2}\,\mathrm{Tr}(M_{\mathrm{OTM}})$ and

$$\begin{aligned}\cos(\pi\nu_s^0) - \cos(\pi[\nu_s^0 + \Delta\nu_s]) =\ & [1 + \cos(\pi\nu_s^0)]\sin^2\frac{\chi_+}{2} \\ & - [1 - \cos(\pi\nu_s^0)]\sin^2\frac{\chi_-}{2} \\ & + (\vec{n}_0\cdot\vec{\zeta})\sin(\pi\nu_s^0)\,\sin\chi_+ \\ & - (\vec{n}_0\cdot\vec{\zeta})^2[1 - \cos(\pi\nu_s^0)] \\ & \times \left(\sin^2\frac{\chi_+}{2} - \sin^2\frac{\chi_-}{2}\right).\end{aligned} \tag{5.4}$$

Let us compare this with the approximate expressions derived above. We set $\vec{c} = \vec{m}_1 = \vec{m}_2 = \vec{n}_0$. Let us also set $\vec{\zeta}$ to be longitudinal, i.e. $\vec{\zeta} = \vec{e}_z$. Then $\vec{n}_1 = \vec{n}_2 = \vec{\zeta} = \vec{e}_z$. Then $\vec{n}_0\cdot\vec{\zeta} = \vec{c}\cdot\vec{e}_z = c_z$. Although the exact solution of the idealized model is valid for arbitrary $\vec{c}$, for comparison with the previous results we say the term in the last line in Eq. (5.4) is of second order $O(c_z^2)$ and we neglect it. Substituting in Eq. (5.4) then yields, to the first order in small quantities,

$$\begin{aligned}\cos(\pi\nu_s^0) - \cos(\pi[\nu_s^0 + \Delta\nu_s]) \simeq\ & [1 + \cos(\pi\nu_s^0)]\sin^2\frac{\chi_+}{2} \\ & - [1 - \cos(\pi\nu_s^0)]\sin^2\frac{\chi_-}{2} \\ & + c_z\,\sin(\pi\nu_s^0)\,\sin\chi_+\,.\end{aligned} \tag{5.5}$$

The answer (even the exact result in Eq. (5.4)) depends only on c_z and *not* on the radial component c_x.

- Let us compare Eq. (5.5) with the expression derived in Eqs. (4.3) and (4.4). From Eq. (4.4),

  $$\begin{aligned}\alpha_+ &= \cos(\pi\nu_s^0)\,c_z + [1 - \cos(\pi\nu_s^0)]\,c_z - \sin(\pi\nu_s^0)\,(c_x - c_x) + c_z \\ &= 2c_z\,.\end{aligned} \tag{5.6}$$

  Next

  $$\begin{aligned}\alpha_- &= \cos(\pi\nu_s^0)\,c_z + [1 - \cos(\pi\nu_s^0)]\,c_z - \sin(\pi\nu_s^0)\,(c_x - c_x) - c_z \\ &= 0\,.\end{aligned} \tag{5.7}$$

  Substituting into Eq. (4.3) yields

  $$\begin{aligned}\cos(\pi\nu_s^0) - \cos(\pi[\nu_s^0 + \Delta\nu_s]) \simeq\ & [1 + \cos(\pi\nu_s^0)]\sin^2\frac{\chi_+}{2} \\ & - [1 - \cos(\pi\nu_s^0)]\sin^2\frac{\chi_-}{2} \\ & + c_z\,\sin(\pi\nu_s^0)\sin\chi_+\,.\end{aligned} \tag{5.8}$$

  This agrees with Eq. (5.5). The term in c_x cancels out.
- The analysis in [1] yields the following. First, from [1, eq. (78)] (see also Eq. (3.9) above)

  $$a_+ \simeq [\cos(\pi\nu_s^0) + 1]c_z - \sin(\pi\nu_s^0)c_x\,, \tag{5.9a}$$

  $$a_- \simeq [\cos(\pi\nu_s^0) - 1]c_z - \sin(\pi\nu_s^0)c_x\,. \tag{5.9b}$$

  Now [1, eq. (30)] states

  $$\begin{aligned}\cos(\pi\nu_s^0) - \cos(\pi[\nu_s^0 + \Delta\nu_s]) =\ & [1 + \cos(\pi\nu_s^0)]\sin^2\frac{\chi_+}{2} \\ & - [1 - \cos(\pi\nu_s^0)]\sin^2\frac{\chi_-}{2} \\ & - \frac{a_+}{2}\,\sin(\pi\nu_s^0)\sin\chi_+ \\ & + \frac{a_-}{2}\,\sin(\pi\nu_s^0)\sin\chi_-\,.\end{aligned} \tag{5.10}$$

Substitution using Eq. (5.9) yields

$$\begin{aligned}\cos(\pi\nu_s^0) - \cos(\pi[\nu_s^0 + \Delta\nu_s]) = {} & [1+\cos(\pi\nu_s^0)]\sin^2\frac{\chi_+}{2} \\ & - [1-\cos(\pi\nu_s^0)]\sin^2\frac{\chi_-}{2} \\ & - \frac{[\cos(\pi\nu_s^0)+1]c_z - \sin(\pi\nu_s^0)c_x}{2} \\ & \times \sin(\pi\nu_s^0)\sin\chi_+ \\ & + \frac{[\cos(\pi\nu_s^0)-1]c_z - \sin(\pi\nu_s^0)c_x}{2} \\ & \times \sin(\pi\nu_s^0)\sin\chi_- .\end{aligned} \tag{5.11}$$

This does *not* agree with Eq. (5.5), which was derived from the exact solution of the idealized model. In particular, the coefficient of the term in c_x is $\sin^2(\pi\nu_s^0)(\sin\chi_+ - \sin\chi_-)/2$, and is not zero.

- The exact solution of the idealized model shows that, if the two solenoid axes are exactly parallel, and if the two arcs are exactly identical, then measurements of the spin tune alone will not yield information about the radial component of the stable spin axis. In practice, the two solenoid axes are nearly parallel and the spin rotations in the two arcs of COSY are nearly identical (they differ only because of lattice imperfections). This indicates that for the scenario studied in [1], the determination of the radial component of the stable spin axis is subject to large uncertainties.

6. Comments on priority claims

The analysis in [1] makes numerous priority claims. I comment here on some of those claims and supply references to prior work in the literature.

6.1. Stable spin axis

- Abstract of [1]: "Up to now, the stable spin axis has never been determined experimentally, . . . "
- Introduction of [1]: ". . . and we report here about the first ever direct measurement of the stable spin axis in a storage ring".
- Section VI ("Summary and Outlook") of [1] (italics in original): "We reported about the first ever attempt for the *in situ* determination of the spin stable axis of polarized particles in a storage ring".

The stable spin axis was determined in experiments using spin polarized proton beams at the IUCF Cooler Ring in 1989 [3]. Vertically polarized protons were injected into the IUCF Cooler Ring. The integrated magnetic field of the electron cooler solenoids of the ring were varied, so as to generate an artificial 'imperfection field' to tilt the direction of the stable spin axis away from the vertical. The polarimeter detected the vertical and radial components of the steady state polarization vector. A graph of the data is displayed in [3, Fig. 2]. (Note that the Siberian Snake in the IUCF Cooler Ring was switched off in these measurements.) The dashed curves indicate the theoretical calculations, for the parameters of the experiment, and agree well with the data.

6.2. Measurement of stable spin axis at two locations in the ring

- Abstract of [1]: ". . . and for the first time, the angular orientation of the stable spin axis at two different locations in the ring has been determined . . . ".

The HERA lepton ring was equipped with spin rotators, to deliver longitudinally polarized electron and positron beams to experiments. (HERA is the only high energy storage ring to attain longitudinal positron polarization.) The HERA lepton ring was equipped with a transverse laser Compton backscattering polarimeter, which measured the vertical polarization in the arcs [4] and a longitudinal laser Compton backscattering polarimeter [5], which measured the longitudinal polarization in the east straight section (the HERMES internal atomic gas jet experiment). The two polarization values were equal, which confirmed that the stable spin axis had been correctly rotated from vertical in the arcs to longitudinal in the HERMES straight section. If the polarization had not been vertical in the arcs and/or if the polarization had not been longitudinal in the straight section, the two polarization values would not have been equal.

In particular, to overcome the depolarizing effects on the radiative polarization due to so-called 'spin diffusion' in high energy electron rings, special care was taken to ensure that the equilibrium polarization direction was very close to the vertical in the HERA arcs [6]. See in particular [6, Secs. (2.2) and (2.4)] for a description of the optimization procedures. Quoting from [6, Sec. (2.4)], 'Before correction, the rms tilt $|\delta \boldsymbol{n}_0|$ is 19.4 mrad, and the equilibrium polarization is 27.1%; with each of the 8 harmonic components set to its corresponding D_{opt}, the rms tilt is reduced to 12.5 mrad and $P_{\max}$ is 81.0%.' Here the vector $\boldsymbol{n}_0$ denotes the spin closed orbit (stable spin axis) and $P_{\max}$ denotes the asymptotic degree of the radiative polarization. See [2] for an overview of radiative polarization in high energy storage rings.

HERA underwent an energy and luminosity upgrade, to HERA-II. Spin rotators were installed in the H1 and ZEUS interaction regions. The detector solenoids were no longer compensated by anti-solenoids, and combined function magnets were introduced to reduce the beta functions at the interaction points. In addition the H1 detector solenoid was longitudinally off center with respect to the interaction point. This required careful mapping of the magnetic fields, for the passage of polarized beams through the interaction regions. See [7] for details of the project. HERA-II successfully delivered longitudinally polarized lepton beams at all the three interaction points of the ring. See [8] for an overview of operations with longitudinally polarized beams in HERA-II.

The RHIC storage ring partly operates as a polarized proton collider. The polarization is vertical in the arcs and spin rotators are employed to deliver longitudinal/radial/vertical polarization to the STAR and PHENIX detectors. See [9] for details of the commissioning of the RHIC spin rotators. It is stated in [9] that 'In order to have a verification of polarization direction at the experiments, both PHENIX and STAR developed local polarimeters for measuring transverse components of the polarization.'

7. Conclusion

The technique of so-called 'spin tune mapping' was described in [1], as a means to determine the spin closed orbit (or 'stable spin axis') for a spin polarized beam circulating in a storage ring. Two electron cooler solenoids were employed to generate artificial imperfection fields in an otherwise planar ring. Due to lattice imperfections, the spin closed orbit is almost but not exactly vertical everywhere. The components of the spin closed orbit were not determined by measuring the polarization vector directly; instead they were deduced via measurements of the spin tune and the application of a theoretical model.

I have independently examined the analysis in [1]. In addition to various errors of algebra, there are several unquantified systematic errors which are not treated in the analysis in [1]. The formulas in [1] also do not agree with the solution derived from an exact model, in the special case where the ring arcs are identical and the solenoid axes are parallel. In particular, for the scenario studied in [1], the determination of the radial component of the spin closed orbit is subject to large uncertainties. I also commented on various priority claims in [1] and supplied references to prior work in the literature [3–9].

References

[1] A. Saleev, et al., JEDI collaboration, Phys. Rev. Accel. Beams 20 (2017) 072801.
[2] S.R. Mane, Yu.M. Shatunov, K. Yokoya, Rep. Progr. Phys. 68 (2005) 1997–2265.
[3] A.D. Krisch, et al., Phys. Rev. Lett. 63 (1989) 1137–1140.
[4] D.P. Barber, et al., Phys. Lett. B 343 (1995) 436–443.
[5] M. Beckmann, et al., Nucl. Instrum. Methods A 479 (2002) 334–348.
[6] D.P. Barber, et al., Nucl. Instrum. Methods A 338 (1994) 166–184.
[7] M. Berglund, D.P. Barber, E. Gianfelice, Proc. 5th European Particle Accelerator Conf., Geneva: EPAC, 1998, pp. 1253–1255.
[8] D.P. Barber (for the HERA team), in: Proc. 9th European Particle Accelerator Conf., Geneva: EPAC, 2004, pp. 644–646.
[9] W.W. MacKay, et al., Proc. 2003 Particle Accelerator Conf., IEEE, New York, 2003, pp. 1697–1699.
[10] D.F. Nelson, A.A. Schupp, R.W. Pidd, H.R. Crane, Phys. Rev. Lett. 2 (1959) 492–495.
[11] L.H. Thomas, Phil. Mag. 3 (1927) 1–21.
[12] V. Bargmann, L. Michel, V.L. Telegdi, Phys. Rev. Lett. 2 (1959) 435–436.
[13] Ya. S. Derbenev, A.M. Kondratenko, Sov. Phys.–JETP 37 (1973) 968–973.
[14] S.R. Mane, Nucl. Instrum. Methods A 485 (2002) 277–297.
[15] S.R. Mane, Yu. M. Shatunov, K. Yokoya, J. Phys. G 31 (2005) R151–R209.