Synchrotron radiation effects on Beam Polarization

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Precession in magnetic field

The magnetic moment interacts with the magnetic field in the magnets and give raise to an hamiltonian of interaction:

$$H_{int} = -\vec{\mu}\vec{B} = -\frac{g}{2}\frac{e\hbar}{mc}\vec{s}\vec{B}$$

Which results in the equation of motion (in spin frame)

$$\frac{d\vec{s}}{dt} = \frac{i}{\hbar}[H_{int}, \vec{s}] = \left(\frac{g}{2}\right)\frac{e}{mc}\vec{s} \times \vec{B}$$

However, this is not valid for a relativistic system..

Relativistic Effects:Thomas BMT

Previous semi-classical approach does not take into account relativistic effects. If one does so, one gets the Thomas BMT equation in the lab frame:

$$\frac{d\vec{s}}{dx} = \frac{e}{mc}\left(a + \frac{1}{\gamma}\right)\vec{s} \times \vec{B} - \frac{a\gamma}{\gamma + 1}\vec{\beta}(\vec{\beta}\vec{B}) - \left(a + \frac{1}{\gamma + 1}\right)\vec{\beta} \times \vec{E}$$

Where $a = \frac{g-2}{2}$ is the anomalous magnetic moment.

Its essentially an extra term to the hamiltonian:

$$H_{int} = \frac{1}{\gamma}\mu_0\vec{B} + \hbar\frac{\gamma^2}{\gamma + 1}\left(\frac{d\vec{\beta}}{dt} \times \vec{\beta}\right)$$

Spin tune and CEBAF spin precession

- From Inspection of the Thomas BMT equation, one can define a angular rotation $\Omega = (1 + a\gamma)$ and we call (in accelerator physics) the term $\upsilon_s = a\gamma$ the spin tune.
- Relative to the electron trajectory in the lab frame, the precession of the electron around the magnetic field is $\theta_s = \frac{\upsilon_s}{2\pi}\theta_b$ where θ_b is the bending angle through the magnet.

Wien Filter, What does this do?

- If one has has electric and magnetic fields that are orthogonal and set them to satisfy $q\vec{E} + \vec{\beta} \times \vec{B} = 0$ then we made a Wien filter. No Lorentz force but spin rotates.   Pluggin that condition into Thomas BLT gives us $\theta = \frac{(a+1)}{\gamma^2 mc} \int B_{\parallel}$. This rotates inversely proportional to the square of momentum. Only work well at low energy.

Precession through CEBAF

- See JLAB TN-04-042 . Formula assumes gains for NL, SL and injector and gives the precession. Gains are constant, and no energy is lost. (on pathlength and no synchrotron radiation)

$$\Psi_n^h = \left(\frac{g-2}{2m_e}\right)\left[(n\theta_1 + (n-1)\theta_2)E_0 + \frac{n}{2}((n+1)\theta_1 + (n-1)\theta_2)E_1 + \frac{n(n-1)}{2}(\theta_1+\theta_2)E_2 + (E_0 + n(E_1+E_2)\theta_h)\right],$$

What should we do in the presence of synchrotron radiation?

Spin flip synchrotron radiation

- When synchrotron photons are emitted, the electron may or may not spin flip. Most of the transitions are non spin flip. The spin flip favor the direction opposite (for electrons) to the magnetic field of the magnets.
- Hence, synchrotron radiation can build up over time and polarize the beam. Sokolov and Ternov first pointed this out. It was later observed at various machines

$$P = 0.924\left(1 - e^{-\frac{t}{\tau}}\right)$$

Sokolov Ternov time for CEBAF

- ${}^{1}/_{\tau} = \frac{5\sqrt{3}}{8}\frac{e^2\hbar\gamma^5}{m^2c^2\rho^3}$ or in practical units $\tau = 98.66\frac{\rho^3}{E^5}\frac{R}{\rho}$

Where R is ring radius, ρ magnet bending radius, E in GeV.

For CEBAF, assuming we have 3 meter dipoles everywhere (not true at pass 1 and 2), R=200m, we get at 11 GeV, $\tau = 113\ sec$. It takes $20\mu S$ to go from injector to Hall A at five pass. This would generate a 10^{-7} transverse component. If we flip helicities, that component will end up averaging to zero. So no asymmetries can be created that way.

So, what is an issue ?

- Pretty much nothing in terms of producing false asymmetries. What has to be accounted is simply a dilution from pass to pass due to the non-spin flip radiation. The usual formula for spin precession can simply be modified to take that into account per pass instead of adding the gains.

- Even easier, we can use the model of the machine and calculate precession by summing over all bends:

$$\bar{\psi}^h = \frac{g-2}{2m_e}\sum p_i \Delta\Theta_i$$