

Chicane optimization

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Plan

- 1 Analytics optimization
- 2 ELEGANT's Simulations
- 3 Conclusion & Questions

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1 Analytics optimization

2 ELEGANT's Simulations

3 Conclusion & Questions

Rectangular dipole matrix

- The rectangular dipole matrix is defined as :

$$M_{dipole}(\rho\theta) = \begin{bmatrix} \cos\theta & \rho\sin\theta & 0 & \rho(1 - \cos\theta) \\ -\frac{1}{\rho} & \cos\theta & 0 & \sin\theta \\ -\sin\theta & -\rho(1 - \cos\theta) & 1 & \left(\frac{\rho\theta}{\gamma^2}\right) - \rho(\theta - \sin\theta) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- $L_{dipole} = \rho\theta$
- ρ is the bend radius.
- θ is the bend angle.
- [y, y'] and [x, x', z, δ] elements are decoupled.

Sector dipole matrix

- The sector dipole matrix is defined as :

$$M_{dipole}(\rho\theta) = \begin{bmatrix} \cos\theta & \rho \sin\theta & 0 & 0 \\ -\frac{1}{\rho} & \cos\theta & 0 & 0 \\ ? & ? & 1 & (\frac{\rho\theta}{\gamma^2}) - \rho(\theta - \sin\theta) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- $L_{dipole} = \rho\theta$
- ρ is the bend radius.
- θ is the bend angle.
- [y, y'] and [x, x', z, δ] elements are decoupled.

Achromaticity condition

To simplify the mathematics we apply:

Achromaticity criterion : $D = \begin{bmatrix} \eta_x \text{exit} \\ \eta'_x \text{exit} \end{bmatrix} = \begin{bmatrix} \eta_x \text{entrance} \\ \eta'_x \text{entrance} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$D_{\text{exit}} = \begin{bmatrix} M_x & 0 \end{bmatrix} \times D_{\text{entrance}} + \begin{bmatrix} R_{16} \\ R_{26} \end{bmatrix}$$

$$R_{16} = R_{26} = 0$$

Achromaticity condition

$$M_{chicane} = \begin{bmatrix} 1 & R_{12} & R_{13} & R_{14} & R_{15} & 0 \\ R_{21} & 1 & R_{23} & R_{24} & R_{25} & 0 \\ R_{31} & R_{32} & 1 & R_{24} & R_{25} & 0 \\ R_{41} & R_{42} & R_{43} & 1 & R_{25} & 0 \\ 0 & 0 & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z_{exit\,chicane} = R_{55}z_0 + R_{56}\delta_0$$

$$\Delta z = R_{56}\delta_0$$

Longitudinal beam chirp

- Using z & $\frac{\Delta P}{P}$ space, we have:

$$\kappa = \frac{d\delta_p}{dz} = \frac{-keV_0}{E_0 + eV_0 \cos \phi} \sin \phi$$

- $k = 2\pi \frac{f}{c}$ [m^{-1}]
- f is the cavity frequency
- eV_0 Cavity acceleration [MeV]
- E_0 Central energy [MeV]
- ϕ Cavity phase advance.

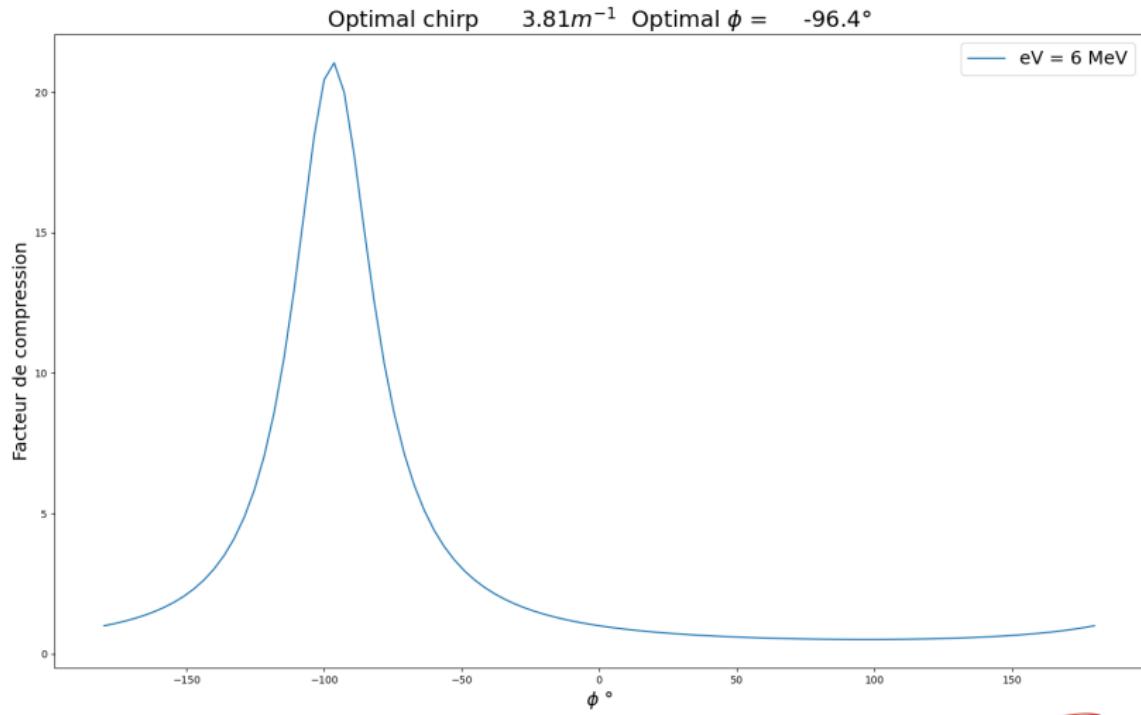
- Compression factor

$$C = \frac{1}{1 + [R_{56} \times \kappa]}$$

$$C = \frac{1}{1 + \left[R_{56} \times \frac{-keV_0}{E_0 + eV_0 \cos \phi} \sin \phi \right]}$$

Compression factor

- $R_{56} = -0.25 \text{ m}$



Beam size along the chicane

- How to reduce the beam size along the chicane?
- Answer : FODO
- Motivation: $\frac{\Delta P}{P_0} = \pm 10\%$
- Focusing quadrupole =

$$\begin{bmatrix} \cos \sqrt{K}L_q & \frac{1}{\sqrt{K}} \sin \sqrt{K}L_q \\ -\sqrt{K} \sin \sqrt{K}L_q & \cos \sqrt{K}L_q \end{bmatrix}$$

- Defocusing quadrupole =

$$\begin{bmatrix} \cosh \sqrt{K}L_q & \frac{1}{\sqrt{K}} \sinh \sqrt{K}L_q \\ -\sqrt{K} \sinh \sqrt{K}L_q & \cosh \sqrt{K}L_q \end{bmatrix}$$

- FODO

$$M_{FODO} =$$

$$M_{HALF\ QF} \ M_{DRIFT} \ M_{QD} \ M_{DRIFT} \ M_{HALF\ QF}$$

Linear beam optics

- **Initial FODO parameters**

- Focusing Quadrupole strength $K_{QF} = 0.6 \text{ m}^{-2}$
- Quadrupole length $L_Q = 0.2 \text{ m}$
- Defocusing quadrupole strength $K_{QDF} = ?$

- **Drift parameter:**

- Drift length $L_{drift} = 5.6 \text{ m}$

- **Motivation** Apply the periodicity condition on the FODO lattice to

$$\text{get : } \begin{bmatrix} \beta_{\text{exit}} \\ \alpha_{\text{exit}} \\ \gamma_{\text{exit}} \end{bmatrix} = \begin{bmatrix} \beta_{\text{entrance}} \\ \alpha_{\text{entrance}} \\ \gamma_{\text{entrance}} \end{bmatrix}$$

- β , α and γ are the twiss parameters of the beam which describes the behaviour of the optics along the lattice.
- In periodic system, for stability of the equation of the motion we have :

$$|trace(M)| < 2$$

Linear beam optics

- If the FODO matrix is given by :

$$M(s_1 s_2) = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix}$$

- The transformation matrix from point s_1 to s_2 in the lattice is given by :

$$\begin{bmatrix} \beta_{s2} \\ \alpha_{s2} \\ \gamma_{s2} \end{bmatrix} = \begin{bmatrix} C^2 & -2SC & S^2 \\ -CC' & SC' + S'C & -SS' \\ C'^2 & -2S'C' & S'^2 \end{bmatrix} \begin{bmatrix} \beta_{s1} \\ \alpha_{s1} \\ \gamma_{s1} \end{bmatrix}$$

- From the stability condition:

$$|trace M(s_1 s_2)| = C + S' < 2$$

We get :

$$K_{QDF} = -1.096 \text{ } m^{-2}$$

Linear beam optics

- The FODO matrix become :

$$M_{FODO} = \begin{bmatrix} 0.95 & 6.59 \\ -0.014 & 0.95 \end{bmatrix}$$

- With $\alpha = 0$ then we have $\beta = \beta_0$ and $\gamma = \frac{1}{\beta_0}$, then Using the transformation matrix:

$$\beta_0 = 11.6 \text{ m}$$

- We define the phase advance matrix per cell:

$$\begin{bmatrix} \cos \phi + \alpha \sin \phi & \beta \sin \phi \\ -\gamma \sin \phi & \cos \phi - \alpha \sin \phi \end{bmatrix}$$

- We can immediately get the phase advance :

$$\cos \phi = 0.95 \quad (1)$$

$$\phi = \arccos 0.95$$

Plan

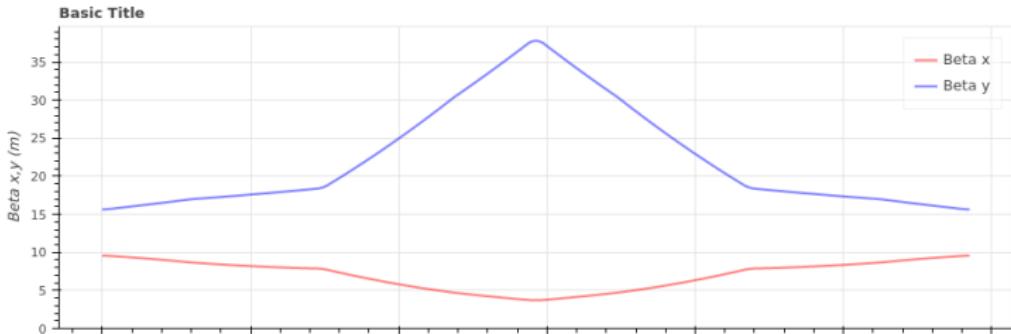
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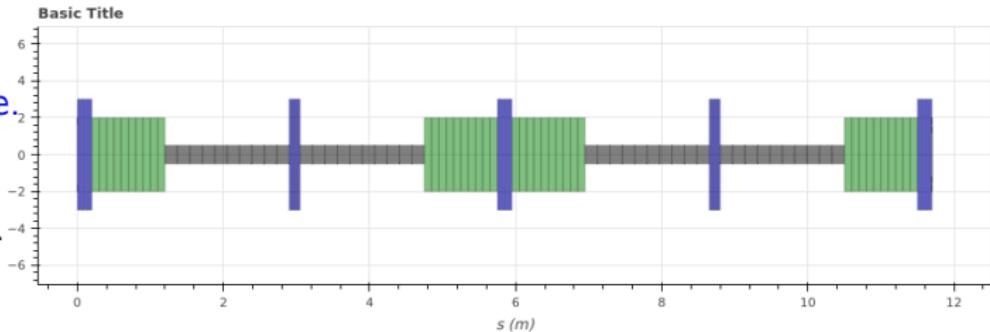
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ELEGANT Results

- Layout :

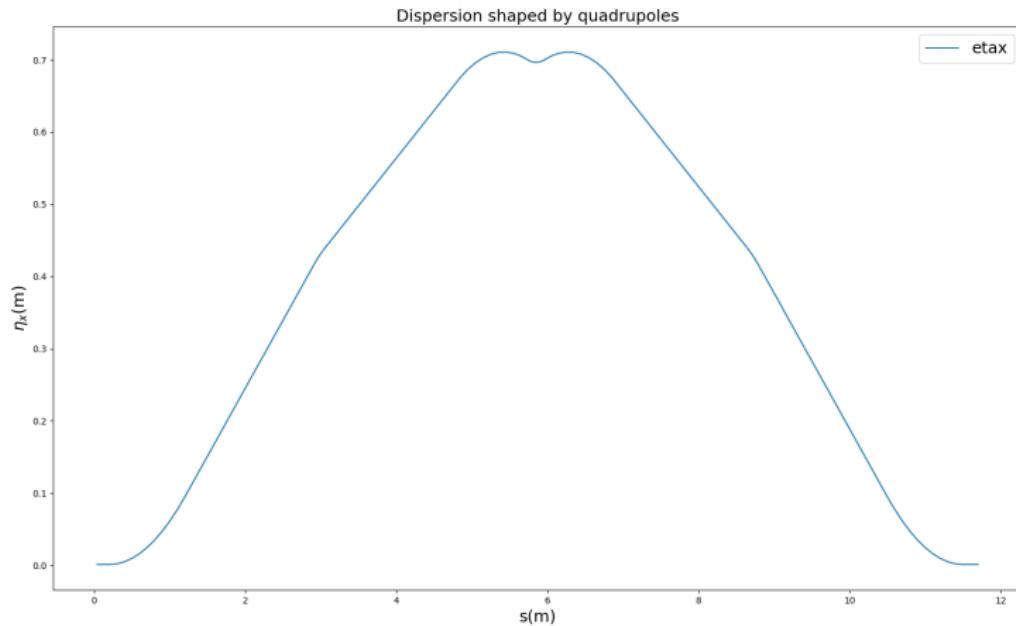


- Quadrupole.
- Dipole.
- Beam pipe.



Dispersion

- Dispersion peak at the middle of the chicane.



Beam size [x,x'] plane

- The beam size at the middle of the chicane is given by :

$$\sigma = \sqrt{\epsilon \times \beta_{min}}$$

- From the positron distribution $\epsilon = 0.039 \text{ mm rad}$, and from the β function, we get β_{min} at the middle of the chicane:

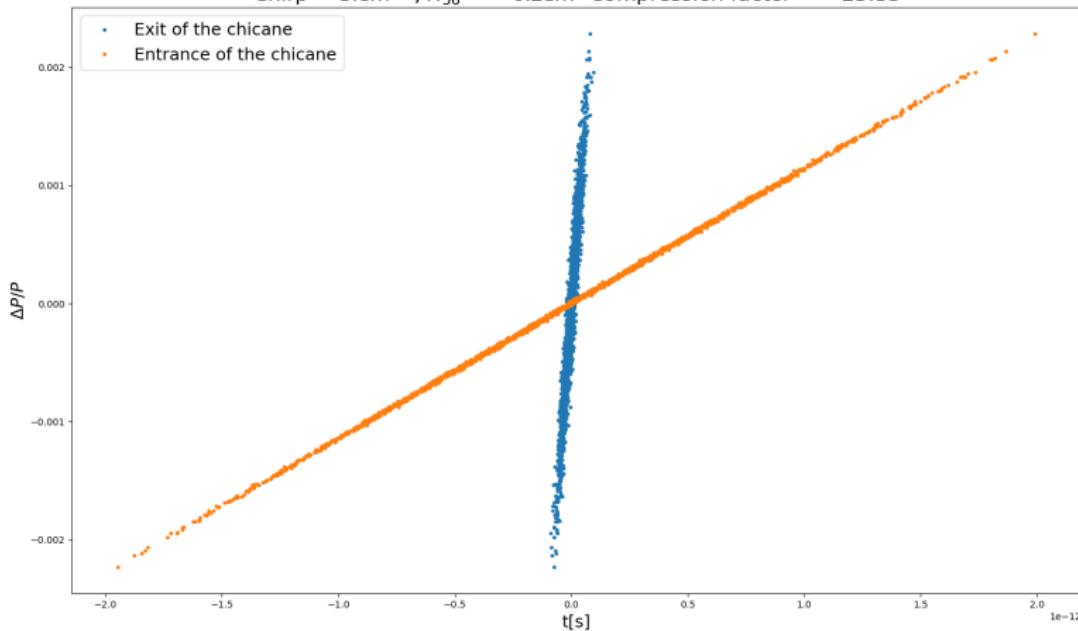
$$\beta_{min} = 3.7 \text{ m}$$

- The beam size at the middle of the chicane is:

$$\sigma = 0.012 \text{ m}$$

Chicane exit

Chirp = $3.8m^{-1}$, $R_{56} = -0.25m$ Compression factor = 23.33



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Conclusion

- Fodo lattice allow us to control the beam size along the chicane.
- Optimized cavity to chirp the beam.
- Need to increase the dispersion at the middle of the chicane.
- Need an optimized matching section (quadrupoles) before the FODO lattice to match the twiss parameters.
- Mathematic calculations helps a lot for the software optimization.
- To be continued ...