Bubble Chamber Systematics
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1 Energy Parameters

Energy profiles were fit to simulation using a softened Shiff formula, which better accounts for effects of losses before the hard production of a photon with the employed X_0 =40%, 6 mm Cu radiator. The photon flux N_γ which is a function of photon energy k, electron kinetic energy T_e, and radiator atomic number Z

$$N_\gamma(k, T_e, Z) \propto \int_k^{T_e} dk' N_{\gamma,0}(k', T_e, Z) e^{-\left(\frac{T_e+m_e}{k'+m_e}\right)^2 / \left(75\left(T_e - k'\right)\right)} \quad (1)$$

where m_e is the electron mass and $N_{\gamma,0}$ is the Schiff 3BSe formula. The factor 75 is was empirically determined for our thick radiator matching to data, Fig. 1. This produces an effective energy shift of about 0.1 MeV and energy spread fo 10-20 keV.

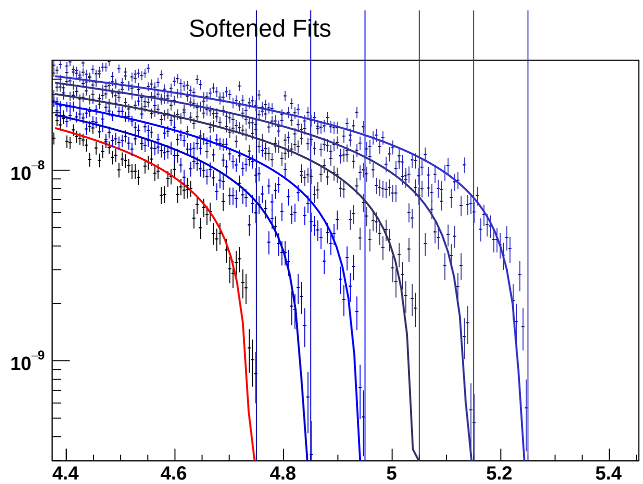

Figure 1: Brehmstrahlung spectrum from Geant4 for several electron beam energies.

1.1 Energy Width

An additional Gaussian was convoluted to Eq. 1 represent energy resolution δE. The relative difference of reconstructed cross section $\sigma(k)$ compared to a resolution of 3 keV is shown in Fig. 2. For 10 keV, the difference was about 2%. For δE <20 keV, the difference was approximately

$$\frac{\delta\sigma}{\sigma} \approx (-3 \times 10^{-4}/\text{keV})\,(\delta E - 3\text{ keV}) + (4 \times 10^{-4}/\text{keV}^2)\,(\delta E - 3\text{ keV})^2 .$$

An energy resolution determined to 50% of itself (~ 1.5 keV) contributes $< 0.5\%$ to the resolution, mainly due to the noise floor from energy losses in the radiator.

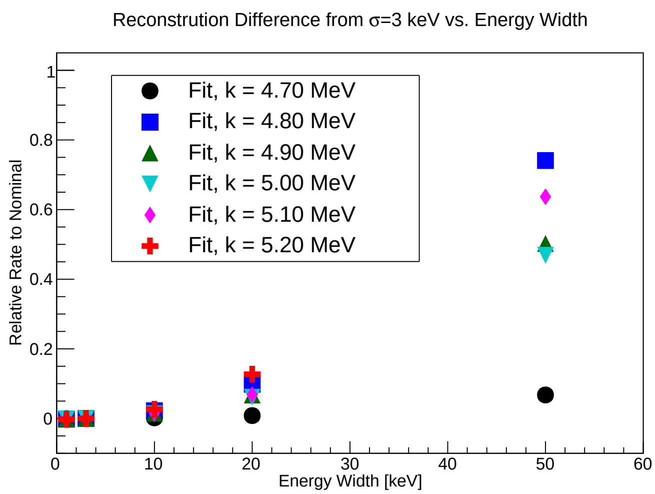

Figure 2: The effect of resolution relative to 3 keV on reconstructed cross section $\sigma(k)$.

1.2 Relative Energy

A full reconstruction was performed given a change in energy for a single kinematic setting. The maximum change from all the reconstructed cross sections is shown in Table 1.2. A 1 keV individual shift introduces a $\sim$3% uncertainty in the reconstruction for the five highest points. This represents a major systematic. If all energies are shifted together by 5 keV, this represents a 10% change in the overall reconstruction.

2 Position Parameters

2.1 Position Shift

The position of the beam was shifted in one direction and the change in the reconstructed cross section relative to no shift was considered. The results are

T MeV	Max σ_{recon} % change
4.75	1.7
4.85	6.3
4.95	5.0
5.05	4.9
5.15	5.3
5.25	6.7

Table 1: Maximum change in reconstructed $\sigma(k)$ for a single relative beam energy shift of 4×10^{-4} (approximately 2 keV).

shown in Fig. 3. The deviations were fit to the form

$$\frac{\delta\sigma}{\sigma} \approx (1.5 \times 10^{-3}/\text{mm})\delta x - (0.014/\text{mm}^2)\delta x^2.$$

A 100 μm position resolution contributes a $< 0.5\%$ deviation.

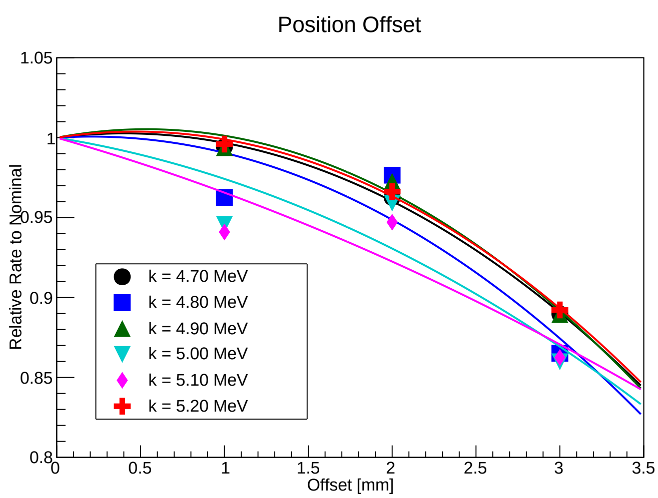

Figure 3: The effect of shifts on reconstructed cross section $\sigma(k)$.

2.2 Spot Width

The size of the beam was symmetrically change and the reconstructed cross section relative to a 1 mm width was considered. The results are shown in Fig. 4. The deviations were fit to the form

$$\frac{\delta\sigma}{\sigma} \approx (-0.07/\text{mm})(\delta x - 1\text{ mm}) - (0.036/\text{mm}^2)(\delta x - 1\text{ mm})^2.$$

A 50 μm width resolution contributes a $< 0.5\%$ deviation.

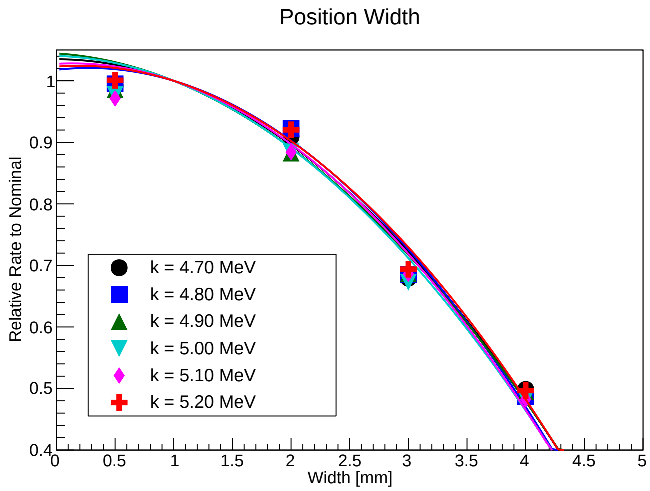

Figure 4: The effect of spot width on reconstructed cross section $\sigma(k)$.

2.3 Divergence

Maximum tolerable divergence can be estimated from the sensitivity to position. For a 6 in long collimator with a 4 mm radius, a shift in position of 150 μm is similar to a 1 mrad change from nominal incident angle. An angular spread of 150 μm would contribute $< 0.5\%$ uncertainty.

3 Background Rates

Background rates will contribute significantly for the lowest energy points. It will make approximately 0.25-0.5 of the counts.

$$Y = Y_{\text{total}} - Y_{\text{back}} \tag{2}$$

$$Y = Rt \tag{3}$$

$$\tag{4}$$

so

$$\frac{\delta Y}{Y} = \frac{\sqrt{Y_{\text{total}} + t^2 \delta R_{\text{back}}^2}}{Y_{\text{total}} - Y_{\text{back}}}. \tag{5}$$

If we establish $Y_{\text{total}} > 4t^2 \delta R_{\text{back}}^2$ or that the relative increase in statistical uncertainty for a point is less than 10% then

$$\delta R = \frac{1}{t}\sqrt{\frac{Y}{4}} = \sqrt{\frac{R}{t_{\text{back}}}}. \tag{6}$$

where t_{back} is the amount of time dedicated to background measurements.

T (MeV)	t_{back} (hours)
4.75	56
4.85	44
4.95	10
5.05	2
5.15	0.4
5.25	0.06

Table 2: Amount of time required to establish background to level described in the text.

4 Deadtime Uncertainty

Uncertainty in the deadtime contributes to the relative uncertainty as

$$\frac{\delta Y}{Y}|_{\text{dead}} = \sigma L \delta t_{\text{dead}}. \tag{7}$$

For a 1% uncertainty, $\delta t_{\text{dead}} = 0.1$ s requires rates no higher than 0.1 Hz.

5 Uncertainty Summary

Parameter	Nominal	Δ	$\delta\sigma/\sigma$
Abs. Energy	4.75-5.25 MeV	5 keV	10%
Energy Step	0.1 MeV	1 keV	3%
Energy Width	3 keV	1.5 keV	<0.5%
Position		100μm	<0.5%
Width	1 mm	50μm	<0.5%
Divergence		1 mrad	<0.5%