Analysis summary for 3/15/2016

Things to consider before final text

1. What functional form should I be using to transform from x uncertainties to y uncertainties? Is using the same function throughout ok?
2. Do I get the same answers plotting thickness vs. asym as I do with asym vs. thickness? Is it worth the bother with the difficult error propogation necessary for solving to find the x intercept?
3. Do I have significant differences in the data with and without the 2% uncertainty?
4. Does plotting confidence error ellipses rather than standard error help in the cases that it can be done?

dThickness -> dAsym

Root:

- **TGraphErrors fit:**
- In case of a TGraphErrors object, when x errors are present, the error along x, is projected along the y-direction by calculating the function at the points x-exlow and x+exhigh. The chisquare is then computed as the sum of the quantity below at each point:
- $$\frac{(y - f(x))^2}{ey^2 + (\frac{1}{2}(exl + exh)f'(x))^2}$$
- where x and y are the point coordinates, and f'(x) is the derivative of the function f(x).
- In case the function lies below (above) the data point, ey is ey_low (ey_high).
- thanks to Andy Haas (haas@yahoo.com) for adding the case with TGraphAsymmErrors University of Washington
- The approach used to approximate the uncertainty in y because of the errors in x is to make it equal the error in x times the slope of the line. The improvement, compared to the first method (f(x+ exhigh) - f(x-exlow))/2 is of (error of x)**2 order. This approach is called "effective variance method". This improvement has been made in version 4.00/08 by Anna Kreshuk. The implementation is provided in the function FitUtil::EvaluateChi2Effective

- Conclusion: Root is doing this very well

Best try at transforming, propogating errors Run 1 data, full errors

	Pade(n,m) Run 1	intercept	dA	red. $\chi 2$	dof	Ftest
A vs. T	(1,0)	43.85	0.145	2.24	9	n/a
T vs. A	T=a+bA	43.99	0.0039	3.39	9	n/a
A vs. T	(2,0)	44.12	.121	0.96	8	12.95
T vs. A	T=a+bSqrt(A)	44.0205	0.017	2.80	9	1.86
A vs. T	(0,1)	44.086	0.0974	.980	8	12.6
T vs. A	T=a+b/A	44.1208	0.0458	1.93	8	7.80
A vs. T	(1,1)	44.17	0.128	1.14	7	11.7
T vs. A	T=(a+bA)/ (1+dA)	44.17	0.0013	2.06	7	7.7

Conclusion: results similar, uncertainty propagation either faulty, or uncertainties smaller when using dT on the y axis

Run 1 with and w/o 2% uncertainty

Pade (n,m) Run 1	intercept	dA	red. χ2	dof	Ftest
(1,0)	43.85	0.145	2.24	9	n/a
(2,0)	44.12	0.121	0.96	8	12.95
(0,1)	44.086	0.097	.980	8	12.6
(1,1)	44.17	0.128	1.14	7	11.7
A=a+b* e^(cT)	44.0635	8.58	1.43	9	4.89 vs. linear

Pade (n,m) Run 1 without 2%	intercept	dA	red. χ2	dof	Ftest
(1,0)	43.8208	.127	2.99	9	n/a
(2,0)	44.0407	.1187	1.06	8	8.9
(0,1)	44.055	0.092	1.003	8	9.9
(1,1)	44.0768	0.125	1.33	7	7.53
A=a+b* e^(cT)	44.0647	8.52	1.54	9	5.83

Considering if there is enough change when we take out the potentially redundant 2% uncertainty across the film to put this change into the “official” data set instead of that taking into account all uncertainties

Run 2 data with and w/o 2%

Pade (n,m) Run 2	intercept	dA	red. $\chi 2$	dof	Ftest
(1,0)	43.87	.140	2.27	9	n/a
(2,0)	44.13	.117	.972	8	13.0 (vs Pade 1,0)
(0,1)	44.09	.0970	1.04	8	11.7 (vs Pade 1,0)
(1,1)	44.20	0.124	1.12	7	12.2 (vs Pade 1,0)
A=a+b* e^(cT)	44.1533	5.43	1.43	9	4.39 (vs. lin)

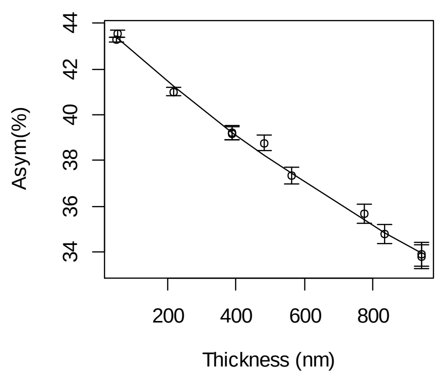
Pade (n,m) w/o 2%	intercept	dA	red. $\chi 2$	dof	Ftest
(1,0)	43.8335	0.149	3.65	9	n/a
(2,0)	44.1134	.125	1.05	8	12.723
(0,1)	44.0803	0.101	1.07	8	12.41
(1,1)	44.17	.132	1.25	7	11.5
A=a+b* e^(cT)	44.1497	5.479	1.56	9	5.51

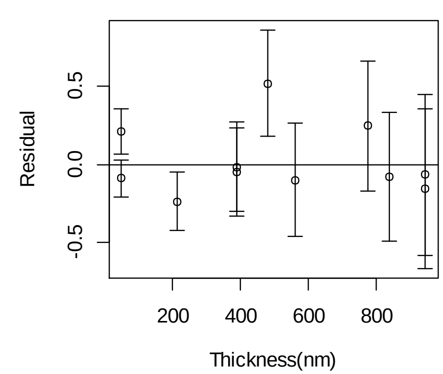
Error table for measured thickness

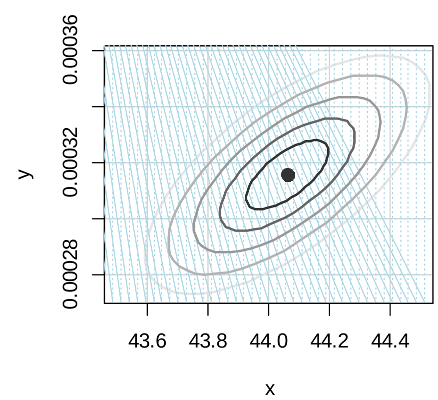
		Au_5385_B	Au_3057_C	Au_5134_B	Au_7028_B	Au_5275_C	Au_5613_D	Au_7029_B	Au_6809_B
	nominal thickness (nm)	1000	870	750	625	500	355	225	50
	mean thickness (all data, nm)	943.7	836.8	774.6	561.2	482.0	389.4	215.2	52.0
Stat.	Stdev, nom. identical data (nm)	29.0	7.1	9.1	8.0	9.7	4.5	1.9	2.3
	stdev image reanalysis (nm)	22.5	7.7	9.4	7.5	4.0	2.7	1.8	2.1
Syst.	Image analysis: ± 4 Pixel	20.0	8.0	10.0	8.0	8.0	8.0	2.6	2.6
	Resolution (1.2 nm inherent)	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
	Tilt (0.4%)	4.6	4.2	3.9	2.8	2.5	1.9	1.1	0.3
	Focus (1%)	9.4	8.4	7.7	5.6	4.8	3.9	2.2	0.5
	Different spots (Lebow: 2%)	18.9	16.7	15.5	11.2	9.6	7.8	4.3	1.0
	Sibling difference (Lebow:5%)	47.2	41.8	38.7	28.1	24.1	19.5	10.8	2.6
Totals									
	stat uncertainty (nm)	36.7	10.5	13.1	11.0	10.5	5.2	2.6	3.1
	syst uncertainty (nm)	55.6	46.7	43.8	31.9	27.7	22.9	12.2	4.1
	total uncertainty (nm)	66.6	47.9	45.7	33.7	29.6	23.5	12.5	5.1

R language: can develop error ellipses rather than relying on standard errors

Pade(0,1)







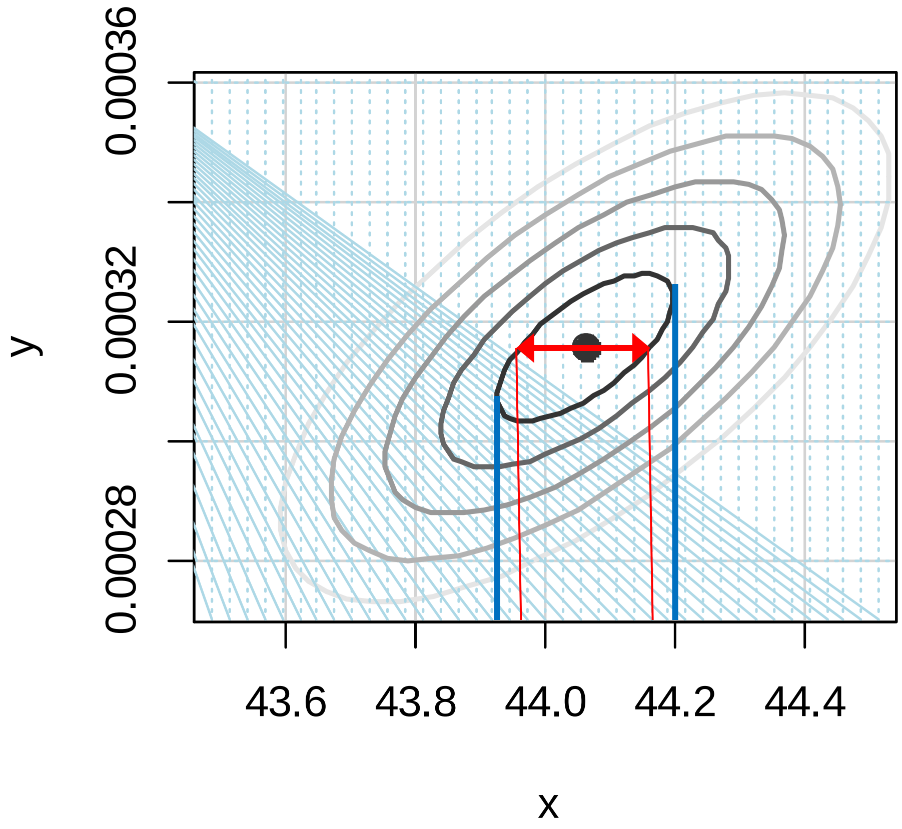

```
Formula: y ~ (a0)/(1 + b1 * x)

Parameters:
    Estimate Std. Error t value Pr(>|t|)
a0 4.406e+01  8.915e-02  494.20  < 2e-16 ***
b1 3.157e-04  8.087e-06   39.04 2.36e-11 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.

Residual standard error: 0.887 on 9 degrees of freedom

Number of iterations to convergence: 5
Achieved convergence tolerance: 2.158e-06
```

Correlation ellipse



- **Standard error** returned by all analysis programs doesn't take into account correlation ellipse
- **Correlation ellipses** can show errors due to both slope and intercept uncertainties
- Only works for linear or pseudo-linear functions (quadratic, Pade(11), exponential can't find this)

Conclusions?

1. What functional form should I be using to transform from x uncertainties to y uncertainties? Is using the same function throughout ok?

- Root is doing a nice job on this – I am just driving which functions to fit, Daniel can do the final fitting

2. Do I get the same answers plotting thickness vs. asym as I do with asym vs. thickness? Is it worth the bother with the difficult error propogation necessary for solving to find the x intercept?

- Answers similar or the same, error propagation is difficult when finding the x intercept value

3. Do I have significant differences in the data with and without the 2% uncertainty?

- Not really? Leave it as is with all the error sources included?

4. Does plotting confidence error ellipses rather than standard error help in the cases that it can be done?

- Open to suggestions here. It is indeed an issue, but not necessarily one that dominates here.

What do I need to give Charlie?

- Thickness document sent (posted here too)
- Leave all with Asym vs. thickness
- Leave all with 2% foil uncertainty included
- Show how the Pade method limits the potential fitting functions
  - Can we eliminate exponential or linear fits?
- Determine the uncertainty from the uncertainty if the model were not able to be predicted with Geant or if the Geant simulation has incorrect assumptions