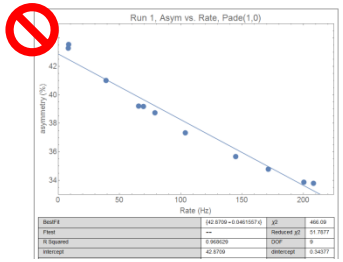
# Summary of Ao extrapolation

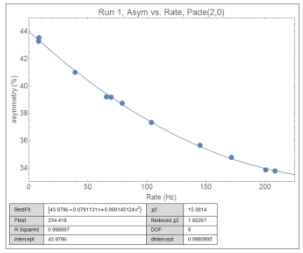
Feb 3 2017

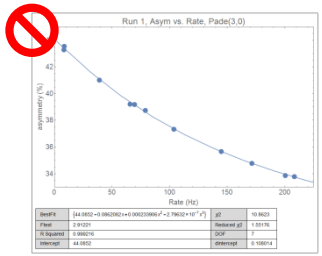
# Pade

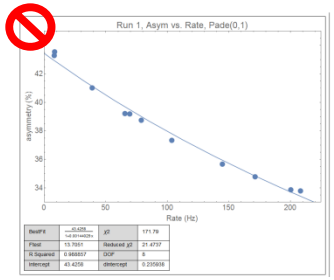
- First use Pade to determine the Pade orders that are not rejected by the F test – put in as many orders as makes sense for A(t), A(R) and R(t)
- Details in file in Jan 17 2017 meeting wiki

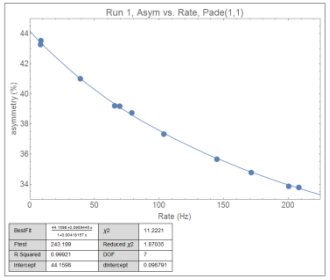
# Old data presentation:
# Pade orders, run 1: Asym vs. Rate

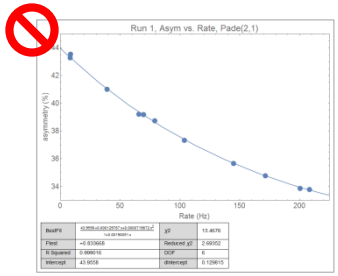


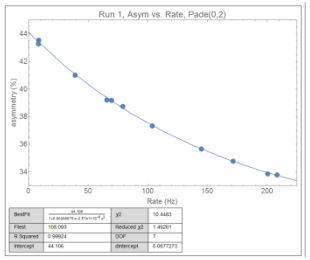



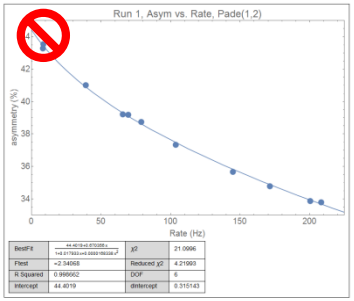



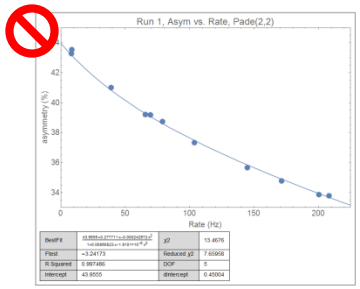



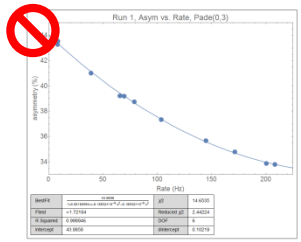



# Same data, new look

# Pade Analysis Run 1

| Pade(x,y) | (0,y) | (1,y) | (2,y) | (3,y) |
|---|---|---|---|---|
| (x,0) | | 43.82(14),n/a,2.6 | **44.05(13),7.7,1.4** | 44.26(17),2.8,1.1 |
| (x,1) | **44.04(10),8,1.3** | **44.09(14),6.3,1.8** | 44.35(38),-2.2,5.4 | |
| (x,2) | 44.09(14),0.3,1.5 | 43.98(33),-0.41,2.5 | | |

A(t) Run 1

Ao, Ftest, reducedχ2

| Pade(x,y) | (0,y) | (1,y) | (2,y) | (3,y) |
|---|---|---|---|---|
| (x,0) | | 43.42(24),n/a,15.2 | 43.98(10),84, 1.33 | 44.08(11),2.4, 1.11 |
| (x,1) | 43.77(15),16.8, 5.1 | **44.08(09),90,1.28** | 44.15(15),0.32,1.48 | |
| (x,2) | **44.05(09),25, 1.16** | 44.04(12),-0.49,1.83 | 44.00(35),-0.92,1.93 | |
| (x,3) | 43.98(10),-1.23, 1.84 | | | |

A(R) Run 1

Ao, Ftest, reducedχ2

| Pade(x,y) | (0,y) | (1,y) | (2,y) | (3,y) |
|---|---|---|---|---|
| (x,0) | | n/a,2.7 | **17.2,0.89** | 1.77, 0.71 |
| (x,1) | | **14.0,1.13** | 1.08,1.11 | |
| (x,2) | | 0.76,1.18 | | |

R(t) Run 1

Ftest, reducedχ2

# Pade Analysis Run 2

| Pade(x,y) | (0,y) | (1,y) | (2,y) | (3,y) |
|---|---|---|---|---|
| (x,0) | | 43.84(16),n/a,2.7 | **44.12(14),10,1.3** | 44.35(17),3.4,0.96 |
| (x,1) | **44.08(11),9.4,1.3** | **44.18(15),8.5,1.6** | 44.43(39),-2.0,4.6 | |
| (x,2) | 44.17(15),0.84,1.4 | 44.14(65),-0.04,2.0 | | |

A(t) Run 2

Ao, Ftest, reducedχ2

| Pade(x,y) | (0,y) | (1,y) | (2,y) | (3,y) |
|---|---|---|---|---|
| (x,0) | | 43.48(28),n/a,12.6 | **44.06(12),65,1.40** | 44.20(12),3.6, 1.02 |
| (x,1) | 43.81(19),12,5.3 | **44.20(11),76, 1.25** | 44.00(17),-1.27, 2.28 | |
| (x,2) | **44.15(11),25,1.19** | 44.35(24),0.54, 1.38 | 43.96(84),-1.16, 3.22 | |
| (x,3) | 44.06(13),-1.4,1.99 | | | |

A(R) Run 2

Ao, Ftest, reducedχ2

| Pade(x,y) | (0,y) | (1,y) | (2,y) | (3,y) |
|---|---|---|---|---|
| (x,0) | | n/a,3.4 | **21,0.98** | 2.9,0.69 |
| (x,1) | | **18,1.19** | 1.18,1.15 | |
| (x,2) | | 1.07,1.17 | | |

R(t) Run 2

Ftest, reducedχ2

# Summary: Pade orders not rejected

For Asy. vs. Thickness:

(2,0), (0,1), (1,1) *(1,0) bad by eye, χ2*

For Asy. vs. Rate:

(2,0), (1,1), (0,2)

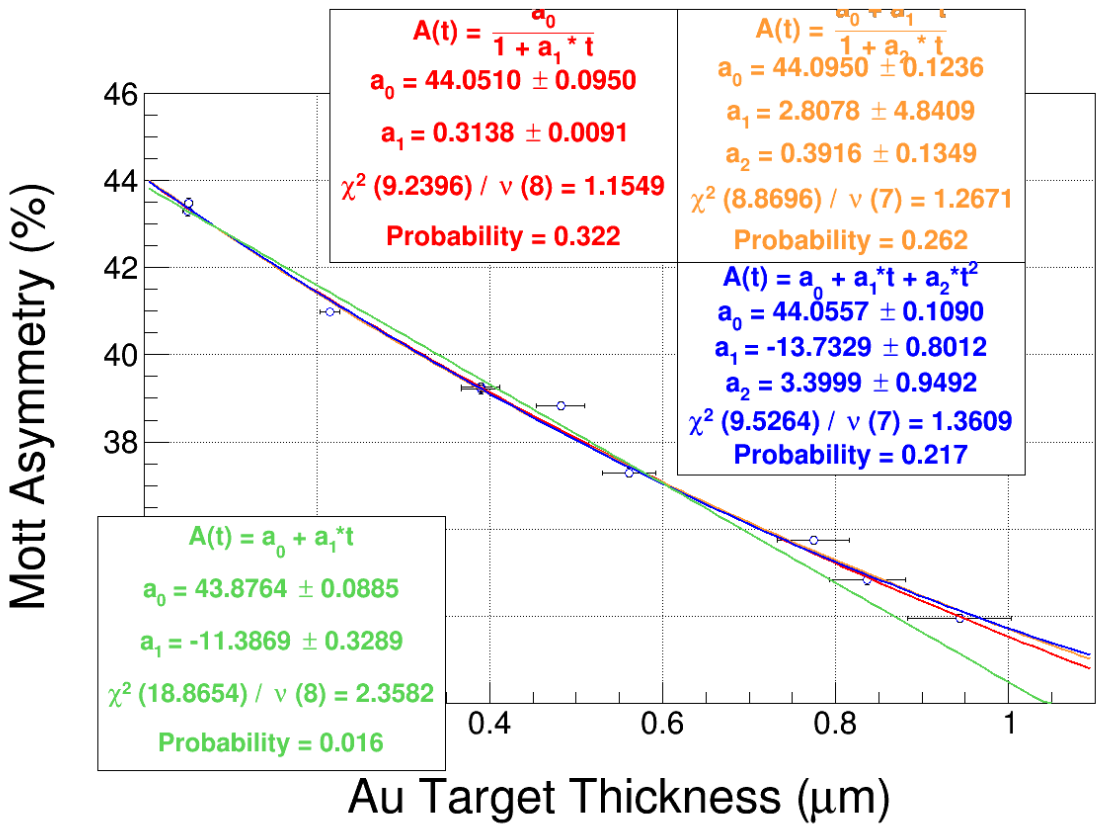
For Rate vs. Thickness (*=forced zero)

(2,0)*, (1,1)* *(1,0) bad by eye, χ2*

Theory suggested fits: in red

# Next: Use allowed Pade orders to fit

- Check Root with mathematica to verify we understand what Root is doing with dx, fitting
- Use Root values, microns for thickness
- Eliminate Pade(1,0), poor fit by eye.
- Eliminate Pade (2,0) – wrong curvature with increasing thickness



# Root vs. Mathematica Analysis A(t) Run 1&2

| A vs. T | ao | Δao | a1 | a2 | b1 | Chi sq | |
|---|---|---|---|---|---|---|---|
| Pade (2,0) | 44.0557 | .1090 | -13.76 | 3.4 | | 1.36 | Run 1 |
| Pade(1,1) | 44.095 | .124 | 2.8 | | 0.39 | 1.27 | Run 1 |
| Pade (0,1) | 44.05 | 0.095 | | | 0.31 | 1.15 | Run 1 |
| Pade (2,0) | 44.1216 | .1222 | -13.99 | 3.74 | | 1.29 | Run 2 |
| Pade(1,1) | 44.175 | .1393 | 4.8 | | 0.45 | 1.17 | Run 2 |
| Pade (0,1) | 44.09 | 0.1006 | | | 0.3136 | 1.14 | Run 2 |

Root

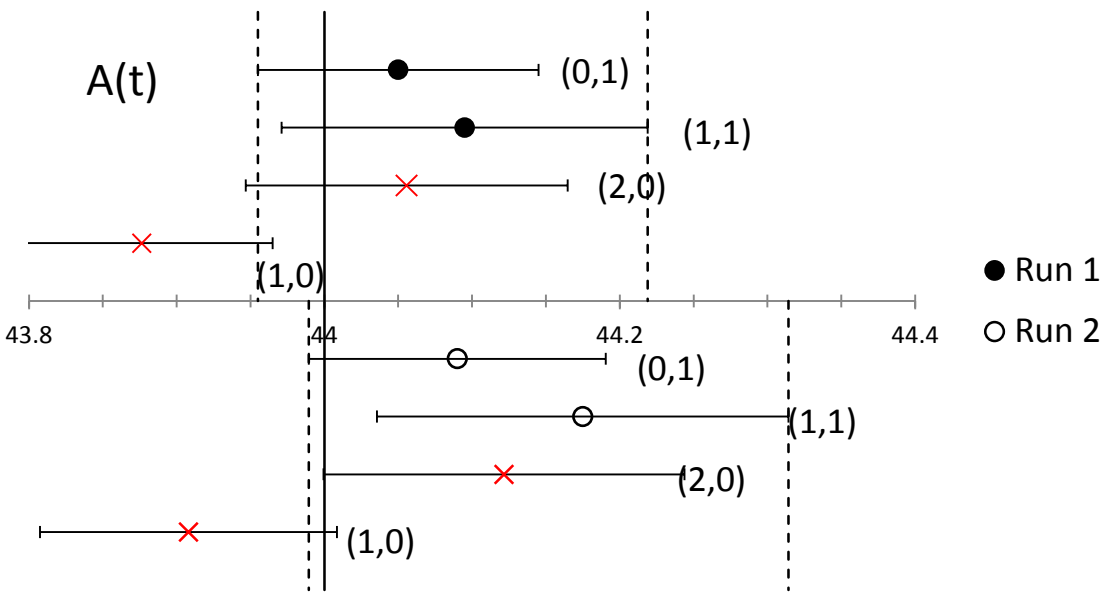
| A vs. T | ao | Δao | a1 | a2 | b1 | Chi sq | |
|---|---|---|---|---|---|---|---|
| Pade (2,0) | 44.05 | .13 | -13.7 | 3.45 | | 1.37 | Run 1 |
| Pade(1,1) | 44.09 | .14 | 3.36 | | .406 | 1.27 | Run 1 |
| Pade (0,1) | 44.04 | .101 | 0.31 | | | 1.15 | Run 1 |
| Pade (2,0) | 44.12 | .14 | -14.0 | 3.85 | | 1.29 | Run 2 |
| Pade(1,1) | 44.18 | .152 | 5.6 | | 0.47 | 1.18 | Run 2 |
| Pade (0,1) | 44.08 | .113 | | | 0.31 | 1.14 | Run 2 |

Mathematica

**Extremely good agreement on parameters and chi squared**

# Root Analysis A(t) Run 1&2

| A vs. T | ao | Δao | a1 | a2 | b1 | Chi sq | |
|---|---|---|---|---|---|---|---|
| ~~Pade (2,0)~~ | ~~44.0557~~ | ~~.1090~~ | ~~-13.76~~ | ~~3.4~~ | | ~~1.36~~ | ~~Root~~ |
| Pade(1,1) | 44.095 | .124 | 2.8 | | 0.39 | 1.27 | Root |
| Pade (0,1) | 44.05 | 0.095 | | | 0.31 | 1.15 | Root |
| ~~Pade (2,0)~~ | ~~44.1216~~ | ~~.1222~~ | ~~-13.99~~ | ~~3.74~~ | | ~~1.29~~ | ~~Rt run2~~ |
| Pade(1,1) | 44.175 | .1393 | 4.8 | | 0.45 | 1.17 | Rt run2 |
| Pade (0,1) | 44.09 | 0.1006 | -11.36 | | 0.3136 | 1.14 | Rt run2 |




Run 1 Ao(t)= 44.07(13)
Run 2 Ao(t)= 44.13(16)

Open question:
Δao for average?

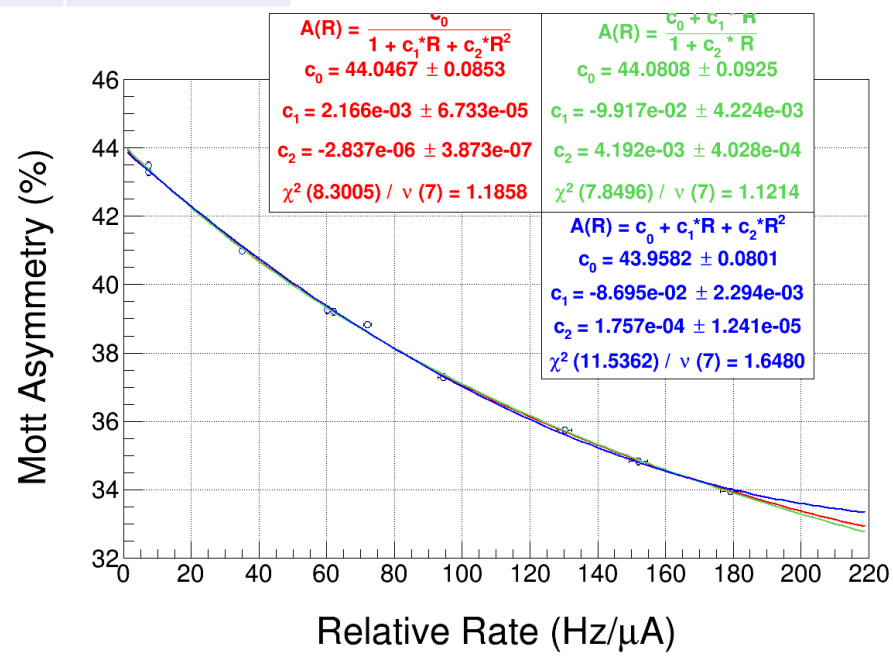
Pade (1,0) shown but not used to find mean

# Root A(R) Run 1&2

| A vs. R | Ao | dAo | Chi squared | Run |
|---|---|---|---|---|
| ~~Pade (2,0)~~ | ~~43.96~~ | ~~0.08~~ | ~~1.65??~~ | ~~Run 1~~ |
| Pade (1,1) | 44.08 | 0.09 | 1.12 | Run 1 |
| Pade (0,2) | 44.05 | 0.09 | 1.19 | Run 1 |
| ~~Pade (2,0)~~ | ~~43.99~~ | ~~0.093~~ | ~~3.00??~~ | ~~Run 2~~ |
| Pade (1,1) | 44.18 | 0.11 | **1.71** | Run 2 |
| Pade (0,2) | 44.11 | 0.100 | **2.00** | Run 2 |

Root

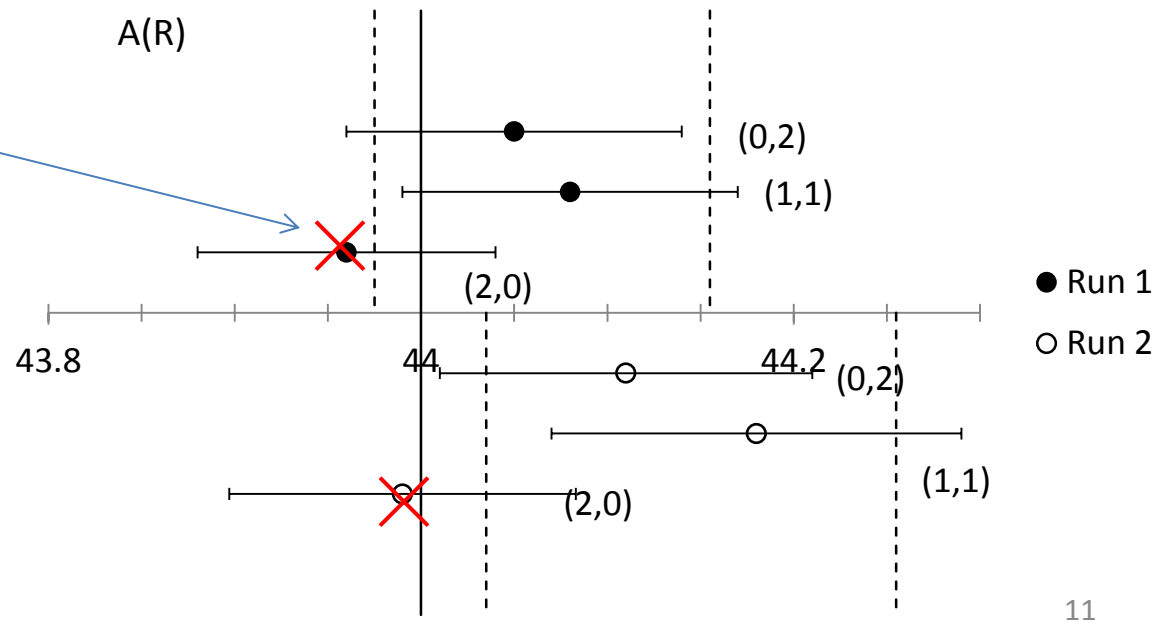
Pade(2,0) wrong curvature as t increases



# Root Analysis A(R) Run 1&2

| A vs. R | Ao | dAo | Chi squared | Run |
|---|---|---|---|---|
| ~~Pade (2,0)~~ | ~~43.96~~ | ~~0.08~~ | ~~1.65~~ | ~~Root, 1~~ |
| Pade (1,1) | 44.08 | 0.09 | 1.12 | Root, 1 |
| Pade (0,2) | 44.05 | 0.09 | 1.19 | Root, 1 |
| ~~Pade (2,0)~~ | ~~43.99~~ | ~~0.093~~ | ~~3.00~~ | ~~Root, 2~~ |
| Pade (1,1) | 44.18 | 0.11 | **1.71** | Root, 2 |
| Pade (0,2) | 44.11 | 0.100 | **2.00** | Root, 2 |

Questions:
should we eliminate Pade (2,0)?
(again, ΔAo =?? Max? Range?)

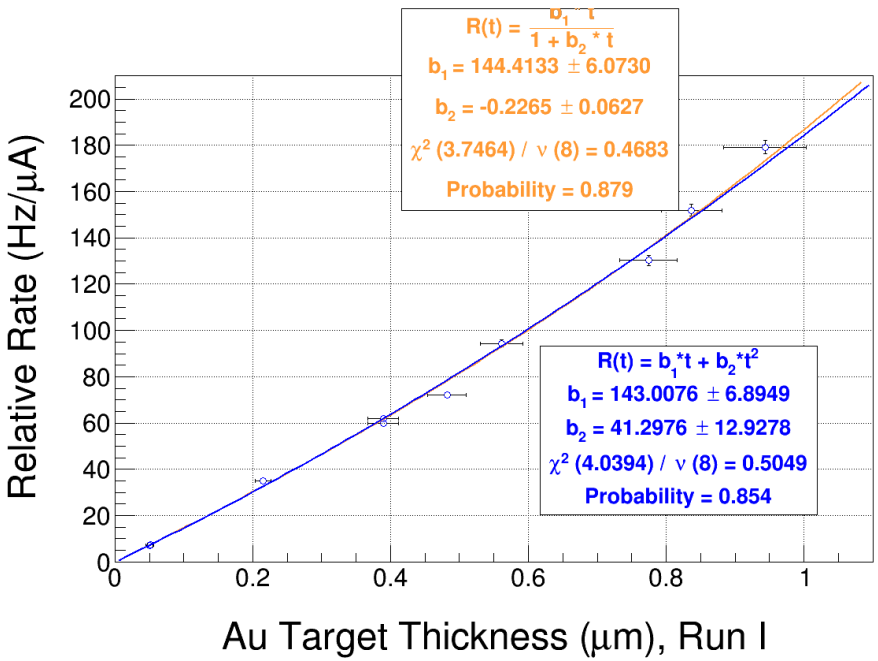



Without Pade(2,0)

Run 1 Ao(R)= 44.06(09)
Run 2 Ao(R)= 44.15(11)

# Root R(t)

| R vs. T | a1 | Δa1 | a2 | b1 | Chi sq | |
|---|---|---|---|---|---|---|
| Pade (2,0) | 143.01 | 6.9 | 41.3 | | 0.50 | Root |
| Pade(1,1) | 144.41 | 6.1 | | -0.23 | 0.47 | Root |
| Pade (2,0) | 130.65 | 6.4 | 43.36 | | 0.84 | Rt run2 |
| Pade(1,1) | 132.15 | 5.6 | | -0.26 | 0.88 | Rt run2 |

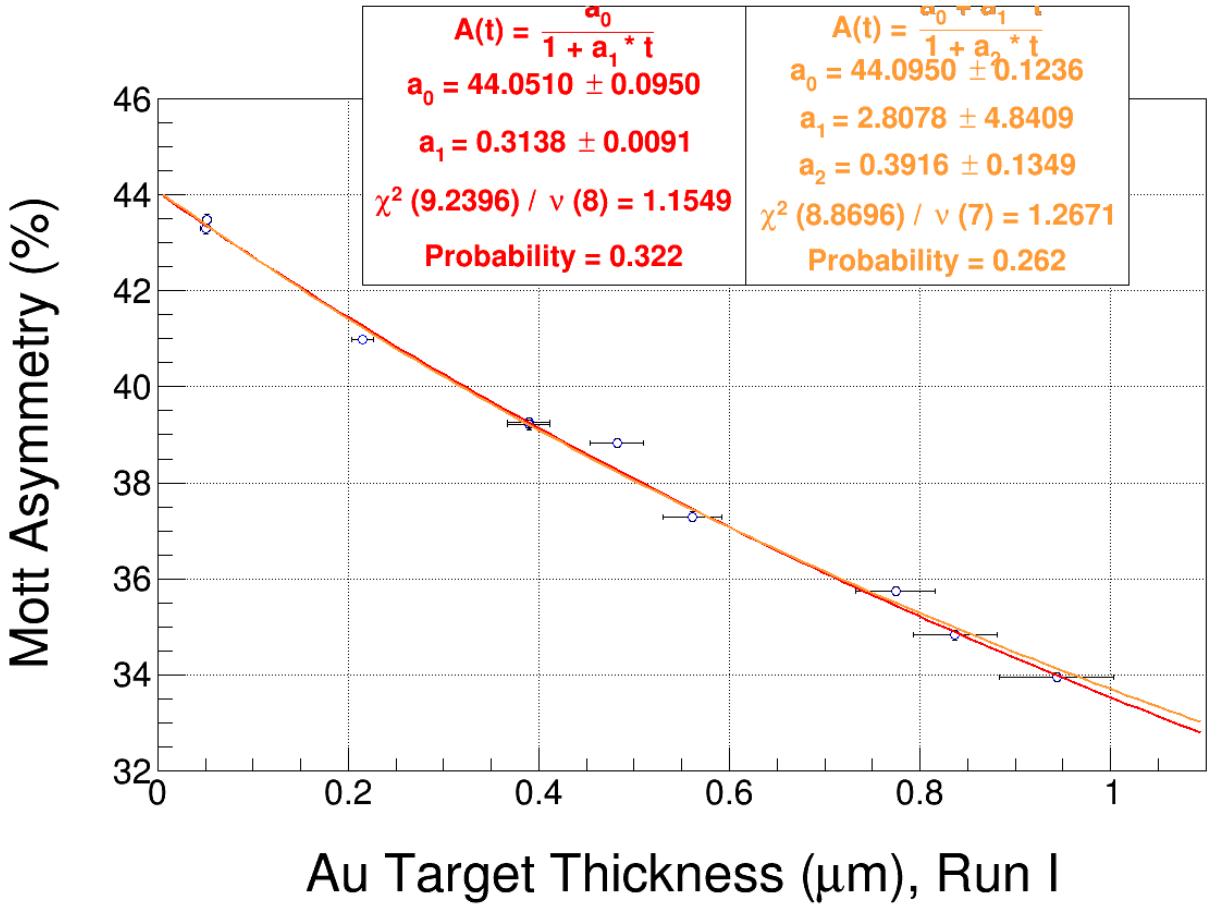



What to say about this? Compare the Pade(2,0) to the values predicted by GEANT4????

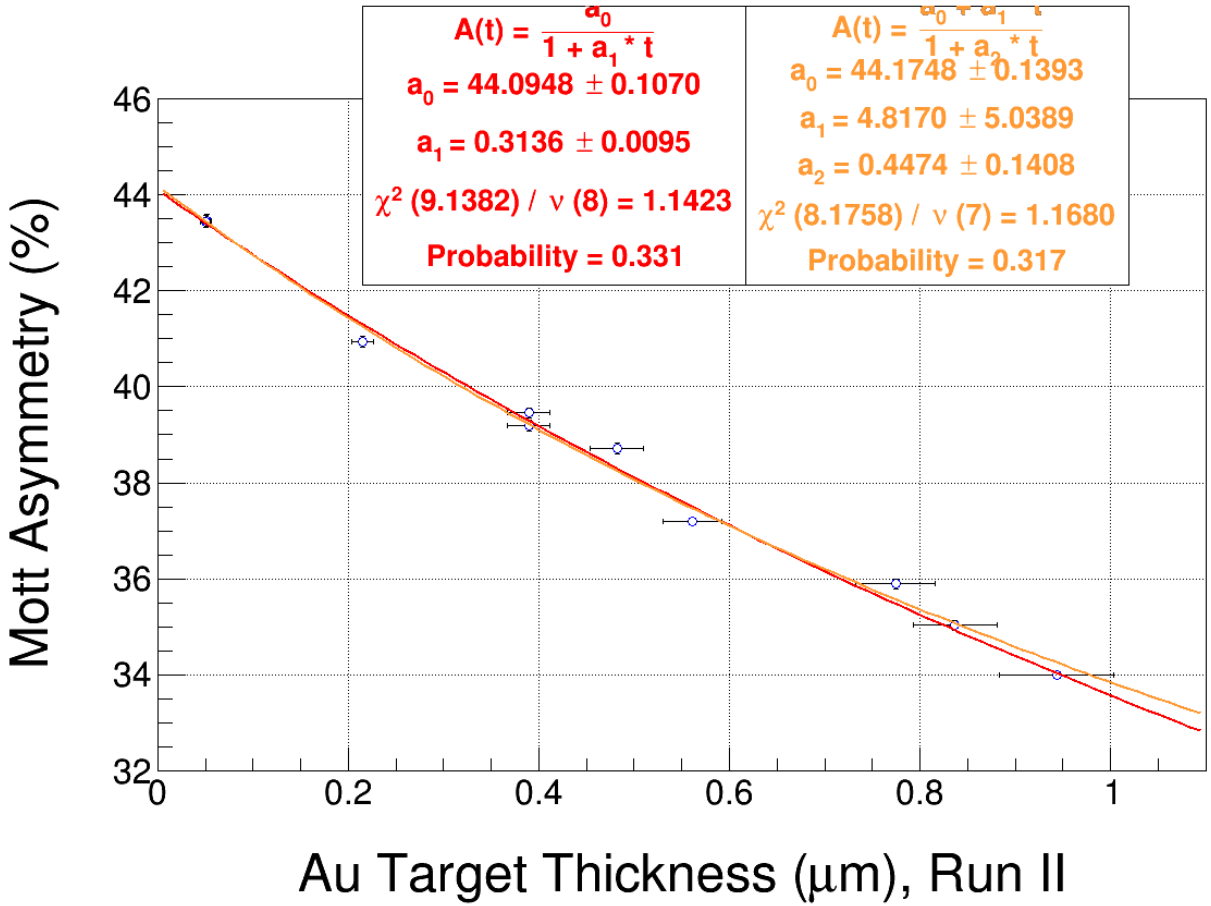
# Summary

- Analysis “final” A(t), A(R), R(t), -0.5-2σ, no bkg.
- Pade analysis to find non-rejected Pade orders
- Further elimination due to poor reduced$\chi 2$, curvature as thickness increases
- Root & mathematica fits compared
  - use Root fits, mathematica agrees well
  - Use microns as units rather than nm (done)
- Compare R(t) experimental fit results to GEANT4 predictions??

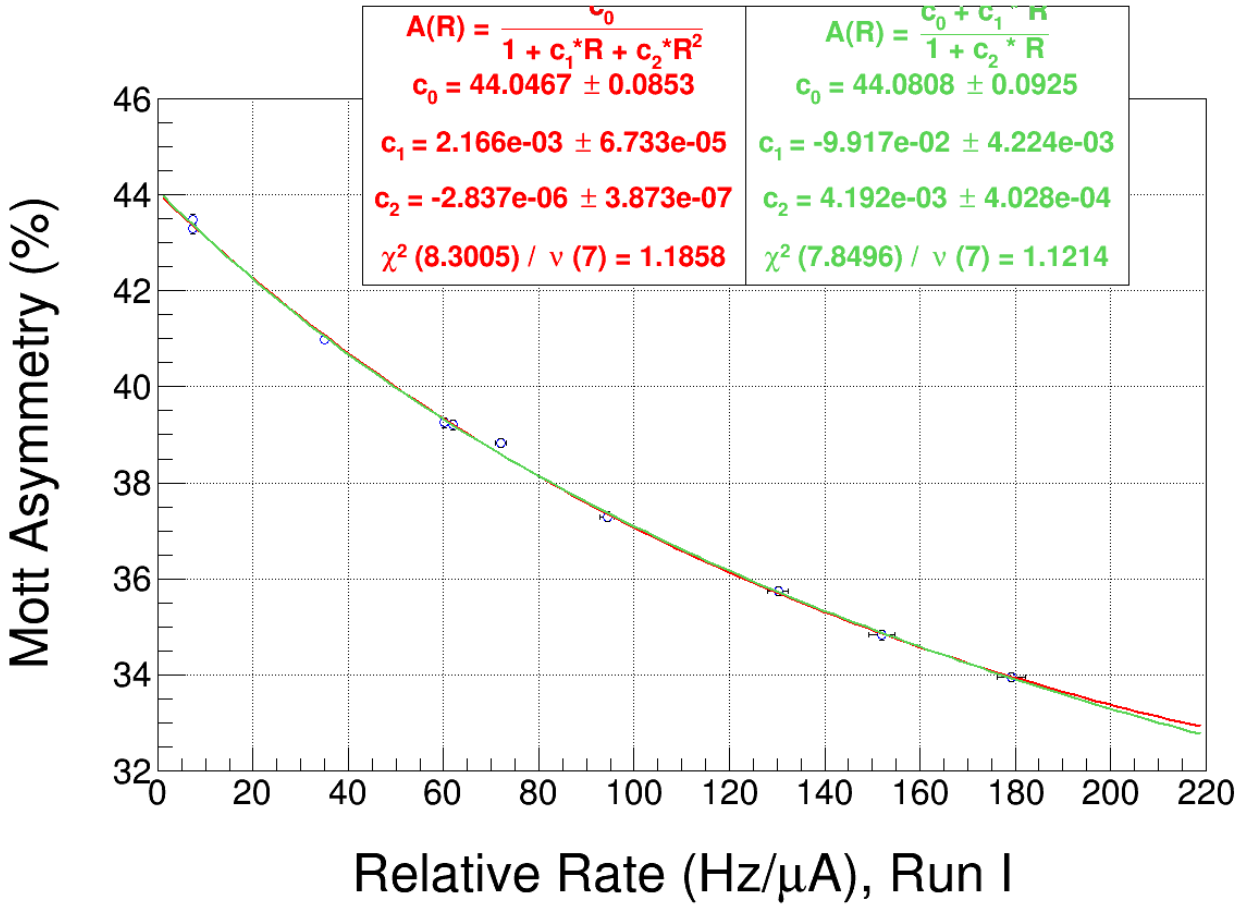
# A(t), Run 1


$A(t) = \frac{a_0}{1 + a_1 * t}$
$a_0 = 44.0510 \pm 0.0950$
$a_1 = 0.3138 \pm 0.0091$
$\chi^2$ (9.2396) / $\nu$ (8) = 1.1549
Probability = 0.322
$A(t) = \frac{a_0 + a_1 * t}{1 + a_2 * t}$
$a_0 = 44.0950 \pm 0.1236$
$a_1 = 2.8078 \pm 4.8409$
$a_2 = 0.3916 \pm 0.1349$
$\chi^2$ (8.8696) / $\nu$ (7) = 1.2671
Probability = 0.262
Mott Asymmetry (%)
46
44
42
40
38
36
34
32
0
0.2
0.4
0.6
0.8
1
Au Target Thickness (μm), Run I

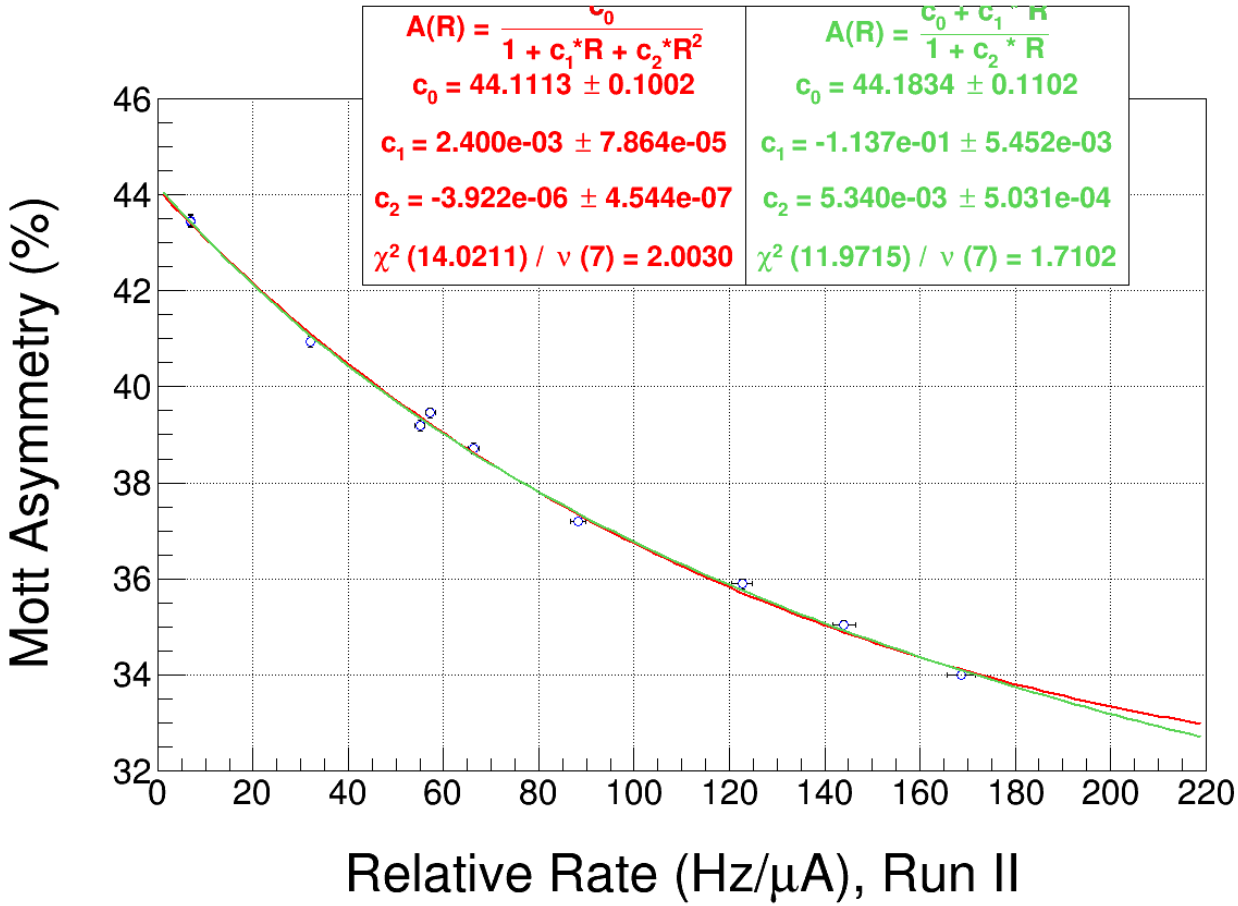
# A(t) Run 2


$A(t) = \frac{a_0}{1 + a_1 * t}$
$a_0 = 44.0948 \pm 0.1070$
$a_1 = 0.3136 \pm 0.0095$
$\chi^2$ (9.1382) / $\nu$ (8) = 1.1423
Probability = 0.331
$A(t) = \frac{a_0 + a_1 * t}{1 + a_2 * t}$
$a_0 = 44.1748 \pm 0.1393$
$a_1 = 4.8170 \pm 5.0389$
$a_2 = 0.4474 \pm 0.1408$
$\chi^2$ (8.1758) / $\nu$ (7) = 1.1680
Probability = 0.317
Mott Asymmetry (%)
46
44
42
40
38
36
34
32
0
0.2
0.4
0.6
0.8
1
Au Target Thickness (μm), Run II

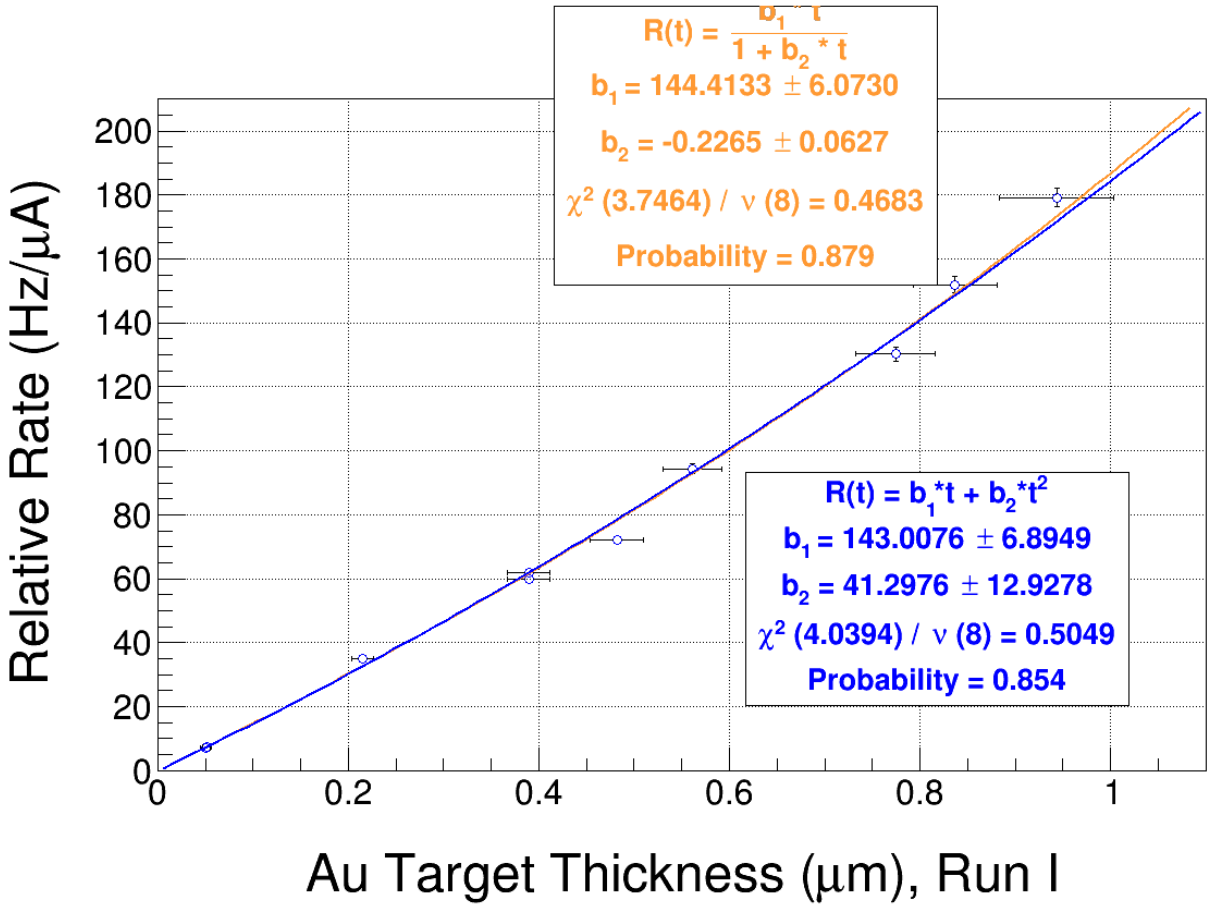
# A(R) Run 1


A(R) = c0 / (1 + c1*R + c2*R2)
c0 = 44.0467 ± 0.0853
c1 = 2.166e-03 ± 6.733e-05
c2 = -2.837e-06 ± 3.873e-07
χ2 (8.3005) / ν (7) = 1.1858
A(R) = (c0 + c1 * R) / (1 + c2 * R)
c0 = 44.0808 ± 0.0925
c1 = -9.917e-02 ± 4.224e-03
c2 = 4.192e-03 ± 4.028e-04
χ2 (7.8496) / ν (7) = 1.1214
Mott Asymmetry (%)
Relative Rate (Hz/μA), Run I
46
44
42
40
38
36
34
32
0
20
40
60
80
100
120
140
160
180
200
220

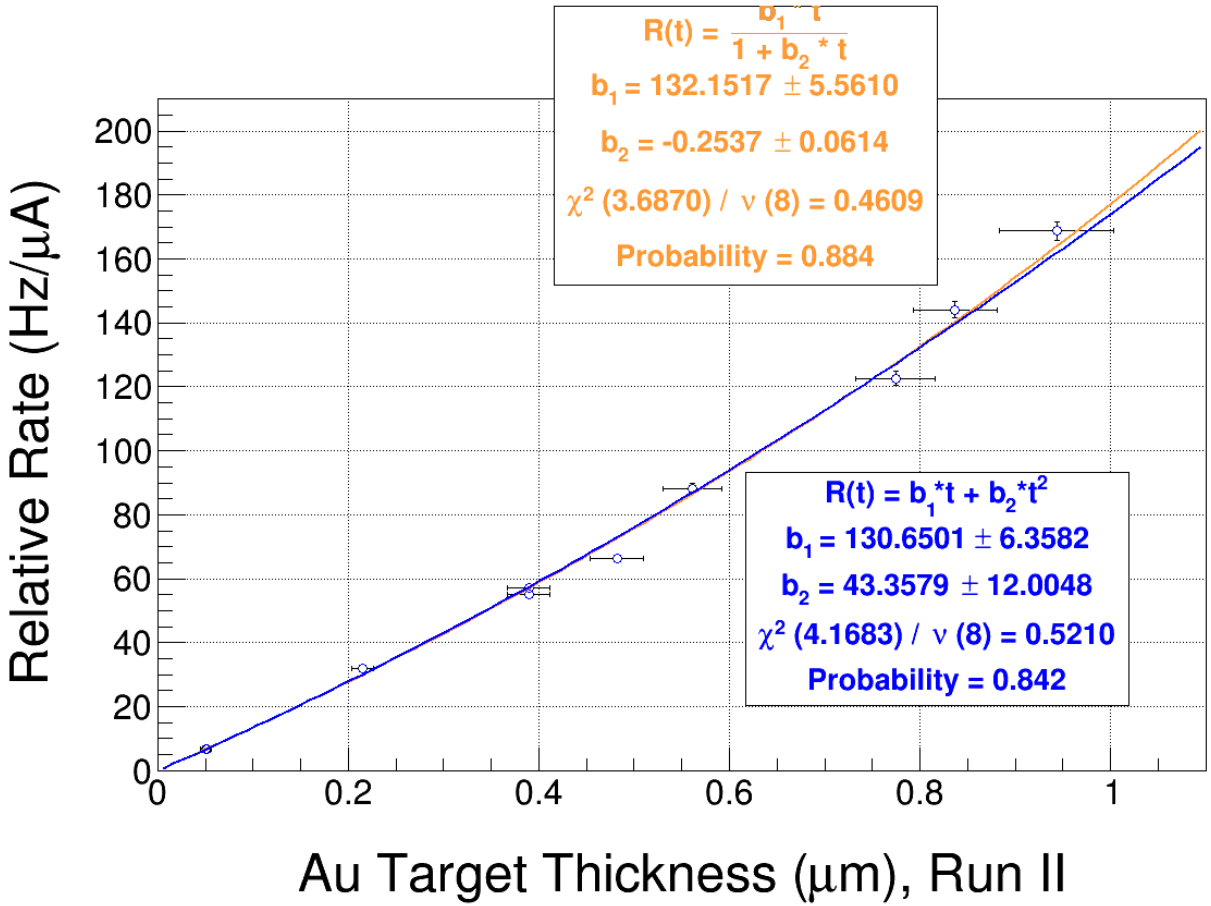
# A(R) Run 2


$A(R) = \frac{c_0}{1 + c_1 * R + c_2 * R^2}$
$c_0 = 44.1113 \pm 0.1002$
$c_1 = 2.400e-03 \pm 7.864e-05$
$c_2 = -3.922e-06 \pm 4.544e-07$
$\chi^2$ (14.0211) / $\nu$ (7) = 2.0030
$A(R) = \frac{c_0 + c_1 * R}{1 + c_2 * R}$
$c_0 = 44.1834 \pm 0.1102$
$c_1 = -1.137e-01 \pm 5.452e-03$
$c_2 = 5.340e-03 \pm 5.031e-04$
$\chi^2$ (11.9715) / $\nu$ (7) = 1.7102
Mott Asymmetry (%)
Relative Rate (Hz/μA), Run II
46
44
42
40
38
36
34
32
0
20
40
60
80
100
120
140
160
180
200
220

# R(t) Run 1


R(t) = b1 * t / (1 + b2 * t)
b1 = 144.4133 ± 6.0730
b2 = -0.2265 ± 0.0627
χ2 (3.7464) / ν (8) = 0.4683
Probability = 0.879
R(t) = b1*t + b2*t2
b1 = 143.0076 ± 6.8949
b2 = 41.2976 ± 12.9278
χ2 (4.0394) / ν (8) = 0.5049
Probability = 0.854
Relative Rate (Hz/μA)
Au Target Thickness (μm), Run I
0
20
40
60
80
100
120
140
160
180
200
0
0.2
0.4
0.6
0.8
1

# R(t) Run 2


R(t) = b1 * t / 1 + b2 * t
b1 = 132.1517 ± 5.5610
b2 = -0.2537 ± 0.0614
χ2 (3.6870) / ν (8) = 0.4609
Probability = 0.884
R(t) = b1*t + b2*t2
b1 = 130.6501 ± 6.3582
b2 = 43.3579 ± 12.0048
χ2 (4.1683) / ν (8) = 0.5210
Probability = 0.842
Relative Rate (Hz/μA)
Au Target Thickness (μm), Run II
0
20
40
60
80
100
120
140
160
180
200
0
0.2
0.4
0.6
0.8
1