Moeller BPM Resolution

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BPM Requirement Summary

Beam Position Monitor Requirements:

- <3 micron resolution at 960Hz (achieved in existing SEE M15)
- Known latency, high bandpass limit about 1 MHz (at least > 100 kHz, for integrate gate integrity)
- ~1% linear response over ~500 micron (i.e. 500μm ± 5μm)
- Position vs. charge, differential < 1 nm/ppb (has been achieved with careful calibration in SEE system)
- Position vs. charge, integral: 30-60uA , error in position δx <100 μm
- Low current operation: at least two x/y BPMs, ~10m apart, with ~50μm-Hz resolution at 1-10 nA

- wish: Position vs. charge, differential < 0.05 nm/ppb (possible with linear digital receiver)

Algorithms (Approximations)

Difference-over-sum

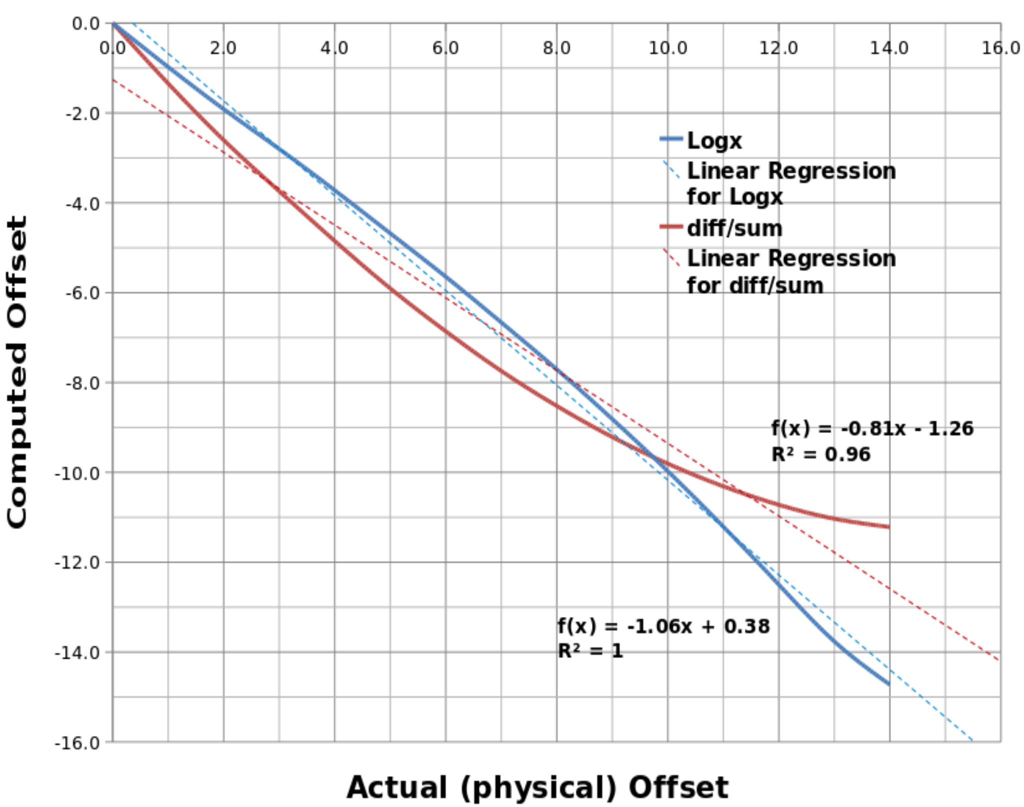
$$X = k_x \frac{V_+ - V_-}{V_+ + V_-}$$

$$Y = k_y \frac{V_+ - V_-}{V_+ + V_-}$$

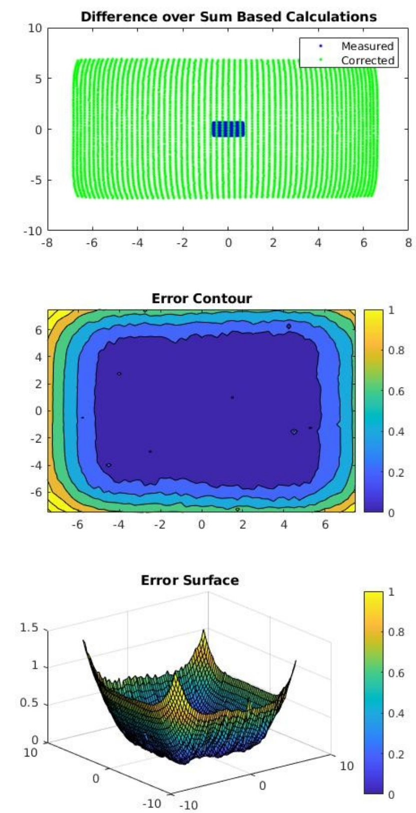
Difference of Logs

$$X = A_x \cdot [\log(V_+) - \log(V_-)]$$

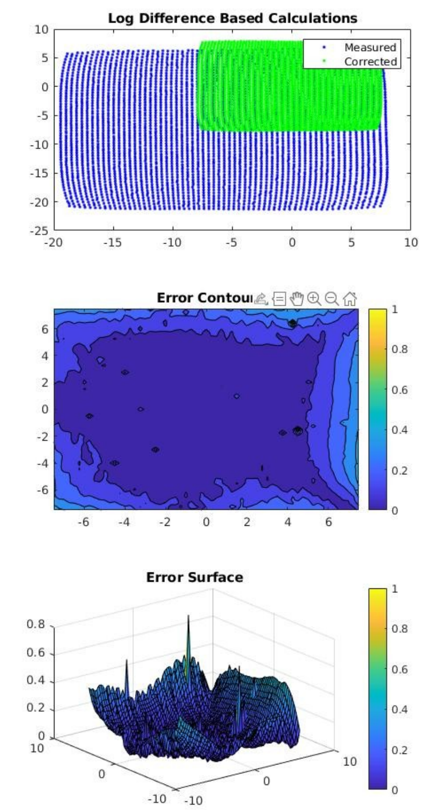
$$Y = A_y \cdot [\log(V_+) - \log(V_-)]$$



Linear Fit



Log Fit



Resolution (Naive)

Propagation of Errors

(A review!)

	Functional Form	**Uncertainty**
Rule 1:	$z = x + y$	$\delta z = \sqrt{\delta x^2 + \delta y^2}$
Rule 2:	$z = x \cdot y$	$\frac{\delta z}{z} = \sqrt{(\frac{\delta x}{x})^2 + (\frac{\delta y}{y})^2}$
Rule 3:	$q = f(x_1, x_2, \ldots, x_n)$	$\delta q = \sqrt{(\frac{\partial q}{\partial x_1} \delta x_1)^2 + \ldots + (\frac{\partial q}{\partial x_n} \delta x_n)^2}$

"An Introduction to Error Analysis," J. R. Taylor, University Science Books, 1982.

Diff-Over-Sum Resolution Analysis

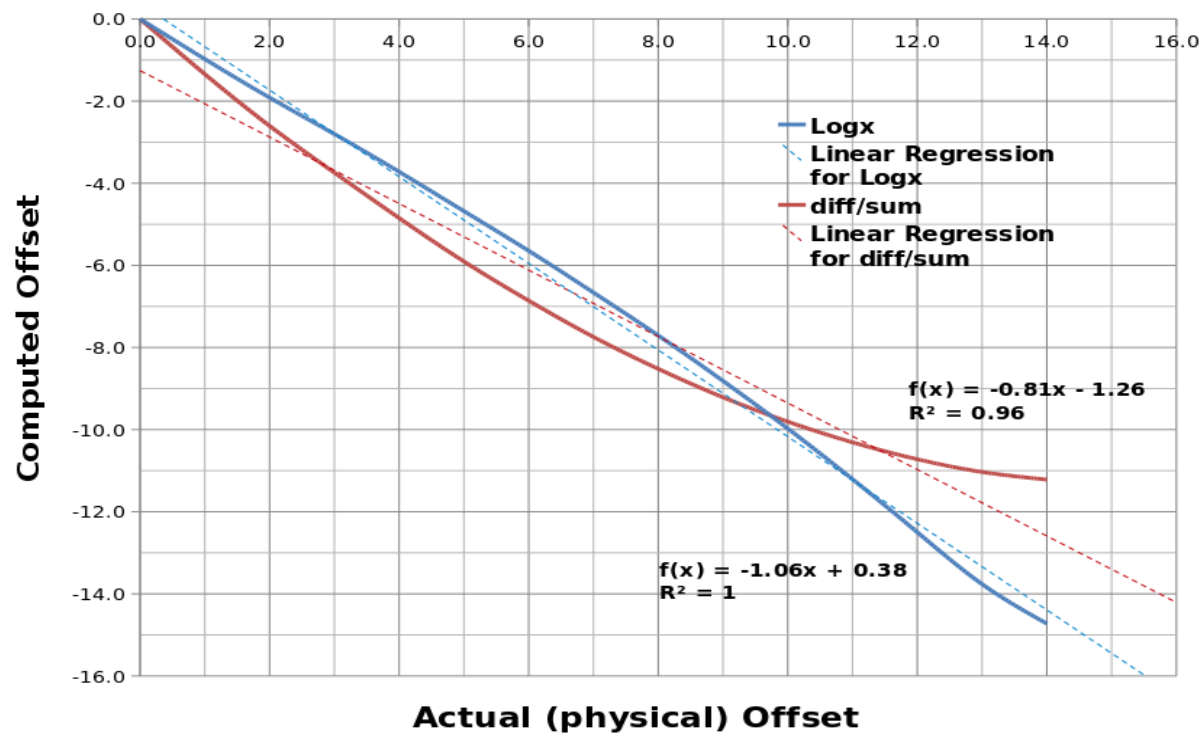
Assumption: AWGN!!

Difference-over-sum:

$$X = \frac{a}{2} \cdot \frac{V_L - V_R}{V_L + V_R}$$

$$\frac{\partial X}{\partial V_L} = \frac{a \cdot V_R}{(V_R + V_L)^2}$$

$$\frac{\partial X}{\partial V_R} = \frac{-a \cdot V_L}{(V_R + V_L)^2}$$



$$\sigma_X = \frac{a}{(V_R + V_L)^2} \cdot \sqrt{V_L^2 \delta V_R^2 + V_R^2 \delta V_L^2}$$

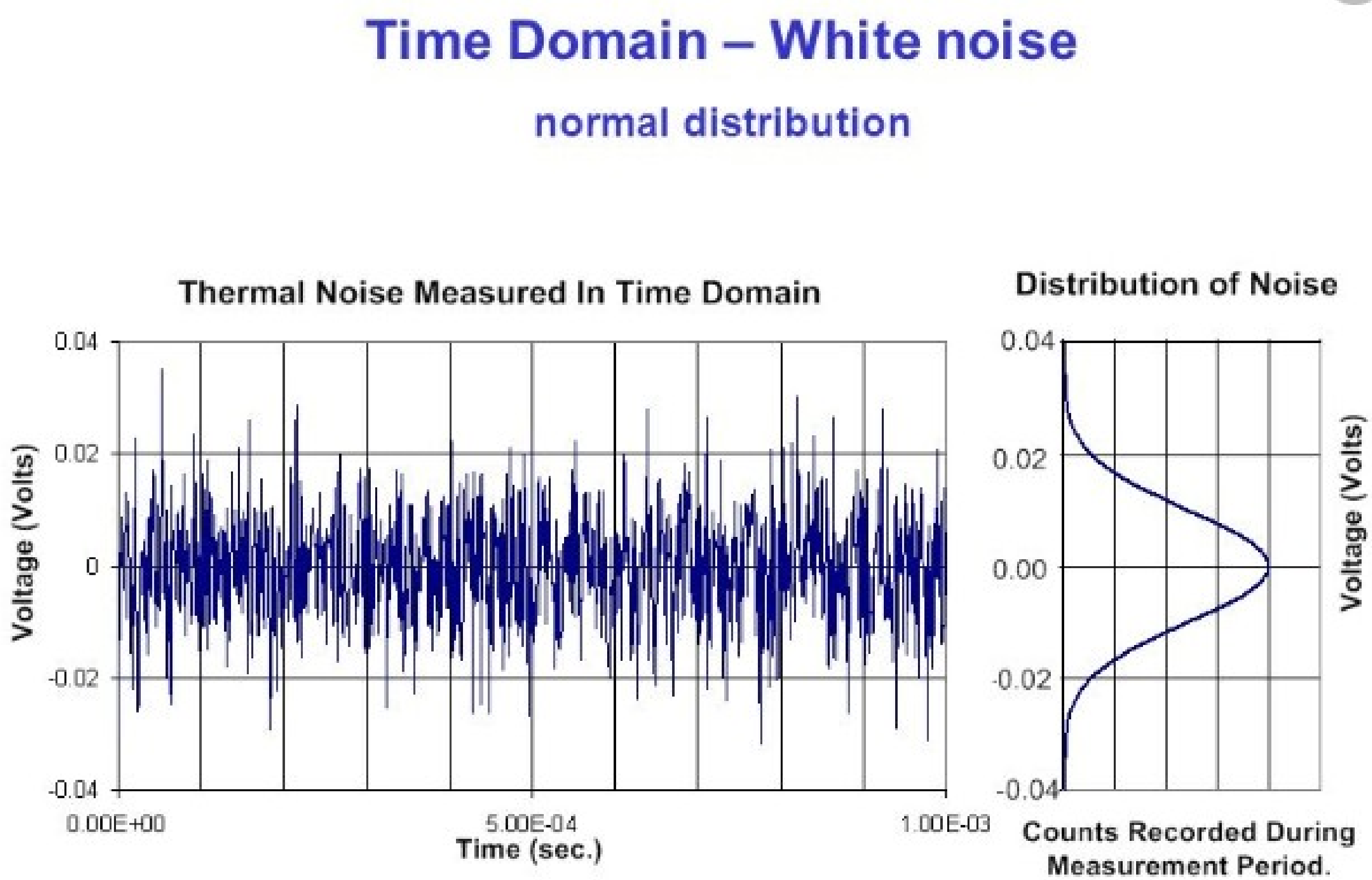
(Rule #3)

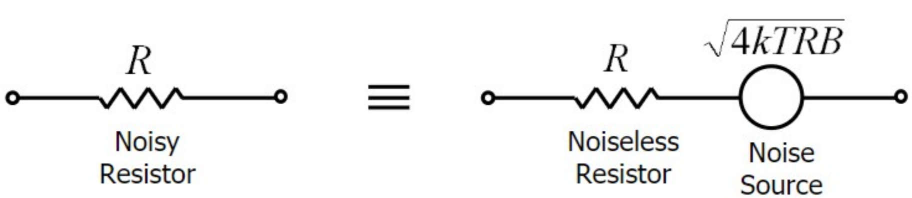
At boresight....

$$\sigma_X = \frac{a}{2} \cdot \frac{\sqrt{2}\sigma_v}{2V} = \frac{a}{2\sqrt{2}} \cdot \frac{1}{\sqrt{SNR}}$$

$$SNR = \frac{P_s}{P_n} = \frac{V_s^2}{V_n^2}$$

What is “Noise?”



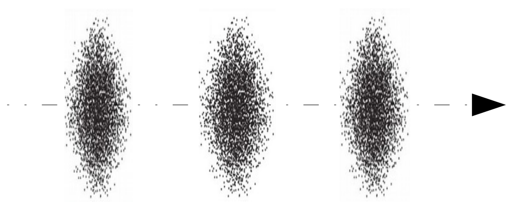


For R = 50 Ω at 300K:

$$P_n = k_B T B = -174\ dBm/Hz$$

We only have 2 knobs: T and B!…….

Gaussian Bunched Beam



$$I_b(t) = \frac{eN}{\sqrt{2\pi}\cdot\sigma}\cdot e^{-t^2/2\sigma^2}$$

σ = “bunch length”

T= period

ω_0= angular frequency

Fourier series:

$$I_b(t) = \frac{eN}{T} + \sum_{m=1}^{\infty} I_m \cos(m\omega_0 t) \quad \Rightarrow \quad I_m = \frac{2eN}{T}\cdot e^{\frac{-m^2\omega_0^2\sigma^2}{2}}$$

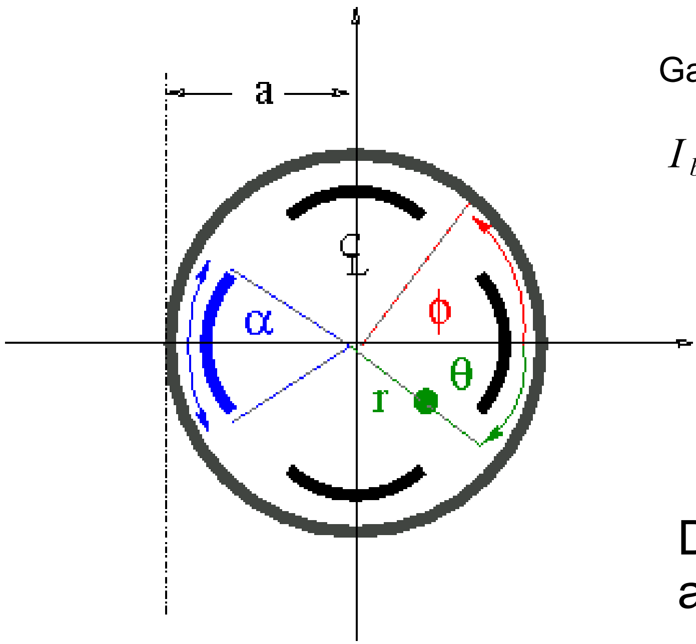
$$= \langle I_b \rangle + 2\langle I_b \rangle \sum_{m=1}^{\infty} A_m \cos(m\omega_0 t)$$

$$\langle I_b \rangle = \frac{eN}{T} = eNf_0 \qquad A_m = e^{\frac{-m^2\omega_0^2\sigma^2}{2}}$$

...we have the option to include as many terms as necessary...

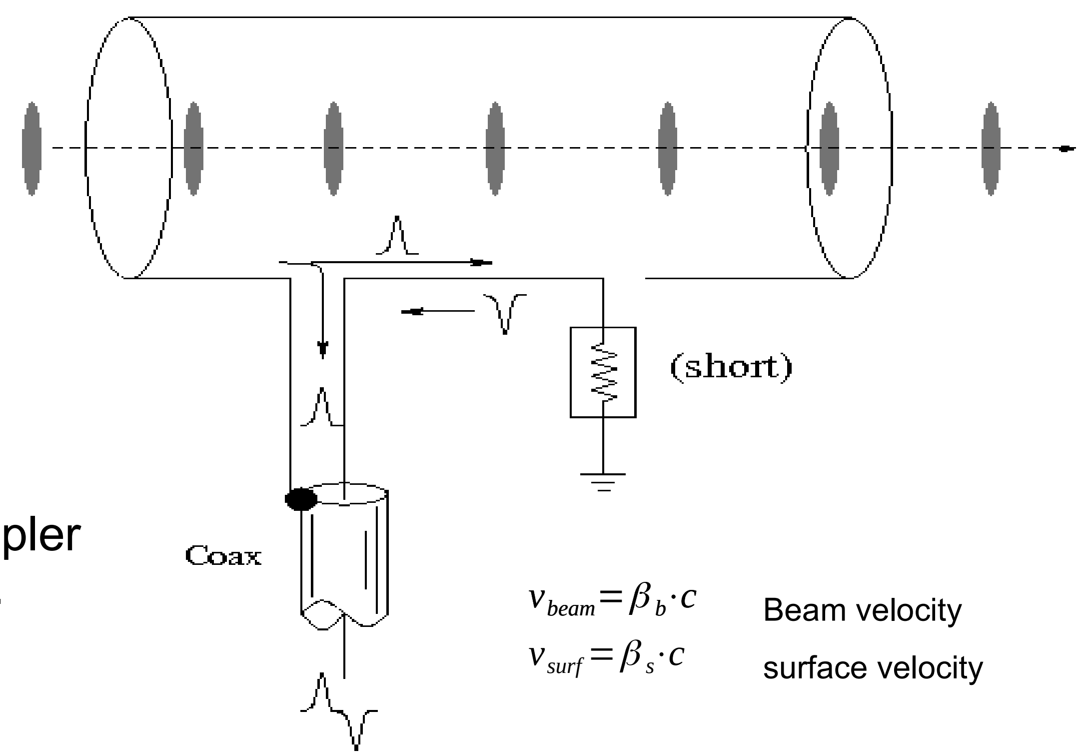
Especially wrt integration, which is easy for cos()!!

Stripline BPMs (Directional-Coupler Style)



Gaussian pulses...

$$I_{beam}(t) = I_0 \cdot e^{-t^2/2\sigma_t^2}$$



Directional Coupler architecture......

$v_{beam} = \beta_b \cdot c$ Beam velocity

$v_{surf} = \beta_s \cdot c$ surface velocity

Current

$$i_{\Im}(\phi) = \frac{I_{beam}}{2\pi a} \cdot \left(\frac{a^2 - r^2}{a^2 + r^2 - 2ar \cdot \cos(\phi - \theta)} \right)$$

$$I_{\Im} = \int_{-\alpha/2}^{+\alpha/2} a \cdot j_{\Im}(\phi)\, d\phi$$

Voltage

$$U_1(t) = \frac{1}{2} \frac{\alpha}{2\pi} \cdot R_1 \left(I_{beam}(t) - I_{beam}\left(t - \frac{2l}{c}\right) \right)$$

$$U_1(t) = \frac{Z_{strip}}{2} \frac{\alpha}{2\pi} \cdot \left(e^{-(t+\tau)^2/2\sigma_t^2} - e^{-(t-\tau)^2/2\sigma_t^2} \right) \cdot I_0$$

$$Z_t(\omega) = \frac{Z_{strip} \cdot \alpha}{4\pi} \cdot e^{-\omega^2 \sigma_t^2/2} \cdot \sin(\omega l/c) \cdot e^{i(\pi/2 - \omega l/c)}$$

Transfer Impedance

$$I_0 = \frac{eN}{\sqrt{2\pi}\,\sigma} \qquad \tau = \frac{l}{2c} \cdot \left[\frac{1}{\beta_b} + \frac{1}{\beta_s} \right]$$

α = angular extent (θ_s, prior)

BPM Output Power

In the frequency domain, RF voltage is:

$$V(\omega)=\frac{\theta_s Z}{\sqrt{2\pi}}\langle I_b\rangle A(\omega)\cdot\sin\left[\frac{\omega l}{2c}\cdot\left(\frac{1}{\beta_s}+\frac{1}{\beta_b}\right)\right]$$

… which is maximized when “sin()” argument = *π/2.*
For electron beams, $\beta_b = \beta_s = 1$. Also, $A(\omega) \sim 2$.

Output power from our DC stripline is (per electrode, for boresight beam):

$$P_s=2\left(\frac{\theta_s}{2\pi}\right)^2\cdot Z\cdot\langle I_b\rangle^2 A^2(\omega)\cdot\sin^2\left(\frac{\omega l}{c}\right)$$

Which, when optimized by 1/4-wavelength stripline electrode:

$$P_s=8\cdot\left(\frac{\theta_s}{2\pi}\right)^2\cdot Z\cdot\langle I_b\rangle^2$$

For our JLAB stripline BPM, we expect to see (and actually do!) -82 dBm for I_{beam} = 1uA. Z = 50 Ω.

Beam Position Monitoring, R. E. Shafer, Accelerator instrumentation. AIP Conference Proceedings, Volume 212, pp. 26-58 (**1990**).

I = 50 uA
B = 100kHz

RECEIVER MODEL

Cal Cell/ Mux | Downconverter

Input Field

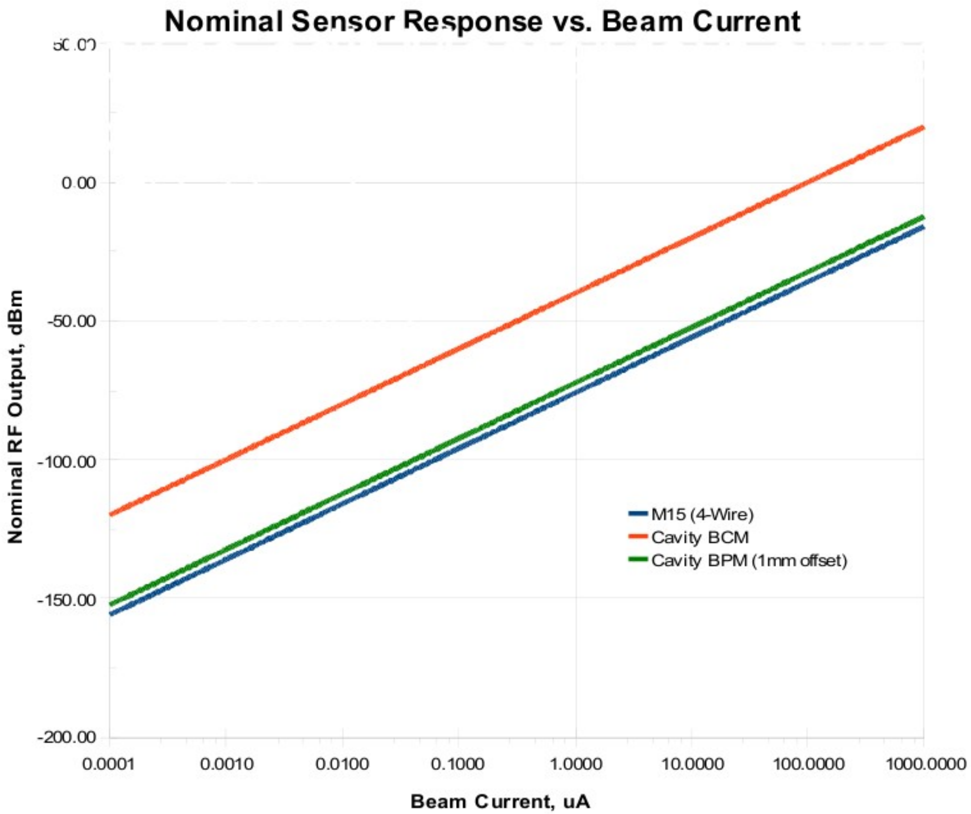
	Coax	LNA	Filter	Amp	Coax	LNA	Filter	Amp	Filter	Mixer	IF Filter	Amp	ADC	
Noise Figure	4.00	1.30	3.00	5.40	12.00	1.30	3.00	5.40	1.00	8.00	6.00	2.70	25.00	dB
Gain: Passband	-4.00	13.00	-3.00	18.00	-12.00	13.00	-3.00	18.00	-1.00	-8.00	-6.00	31.00	0.00	dB
Gain: Reject-band	-4.00	13.00	-20.00	18.00	-12.00	13.00	-20.00	18.00	-20.00	-8.00	-30.00	31.00	0.00	dB
IIP3	200.00	28.00	200.00	26.00	200.00	28.00	200.00	26.00	200.00	34.00	200.00	7.00	200.00	dBm
P1dB	200.00	23.00	200.00	20.00	200.00	23.00	200.00	20.00	200.00	22.00	200.00	20.00	200.00	dBm
Return Loss	8.00	20.00	6.00	20.00	24.00	20.00	6.00	20.00	2.00	16.00	12.00	25.00	25.00	dB

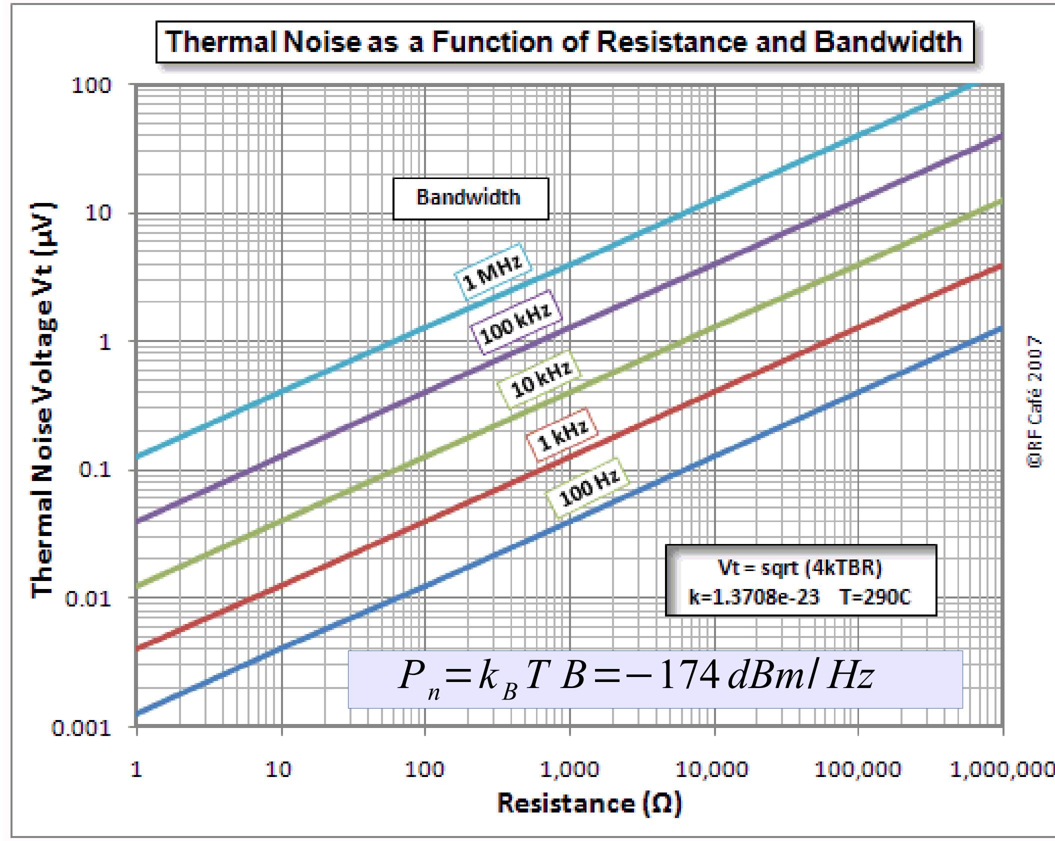
Pin Interference	0	-46.00	dBm	
Pin Passband	###	-46.00	dBm	
Input Noise BW	50	50.00	dB-Hz	(System IF BW)
Input Noise Temperature	290	290.00	K	(IEEE definition = 290K for Physical Temperature)
Input Noise Level		-124.0	dBm	
Required C/N	60	38.00	dB	(Modulator / BER -dependent…see BER sheet)
Required Sensitivity	164	-80.00	dBm	(From "Specifications" or "Standards")

Calculation Field

System Noise Figure	4.00	5.30	5.46	6.16	6.22	6.25	6.25	6.27	6.27	6.27	6.27	6.28	6.30	dB
System Noise Temp	26.42	28.41	28.63	29.58	29.67	29.70	29.70	29.72	29.72	29.72	29.72	29.74	29.76	dBK
System Gain: Passband	-4.00	9.00	6.00	24.00	12.00	25.00	22.00	40.00	39.00	31.00	25.00	43.00	43.00	dB
System Gain: Reject-band	-4.00	9.00	-11.00	7.00	-5.00	8.00	-12.00	6.00	-14.00	-22.00	-52.00	26.00	26.00	dB
IIP3: Passband	200.00	32.00	32.00	19.73	19.73	14.47	14.47	3.63	3.63	-5.56	-5.56	-5.01	-5.01	dBm
IIP3: Reject-band	200.00	32.00	32.00	30.81	30.81	28.76	28.76	28.27	28.27	28.22	28.22	11.94	11.94	dBm
Input Spurious-Free Dynamic Range	213.32	100.45	100.35	91.70	91.66	88.13	88.13	80.89	80.89	74.77	74.77	75.12	75.11	dB
Pout: Passband	-50.00	-37.00	-40.00	-22.00	-34.00	-21.00	-24.00	-6.00	-7.00	-15.00	-21.00	-3.00	-3.00	dBm
Pout: Reject-band	-50.00	-37.00	-57.00	-39.00	-51.00	-38.00	-58.00	-40.00	-60.00	-68.00	-98.00	-20.00	-20.00	dBm
Output Noise Power	-123.98	-109.68	-112.52	-93.81	-105.75	-92.73	-95.73	-77.71	-78.71	-86.71	-92.71	-74.70	-74.68	dBm
C/N Ratio	73.98	72.68	72.52	71.81	71.75	71.73	71.73	71.71	71.71	71.71	71.71	71.70	71.68	dB
Saturation?	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	
IIM3	-538.00	-202.00	-202.00	-199.61	-199.61	-195.51	-195.51	-194.54	-194.54	-194.44	-194.44	-161.89	-161.89	dBm
C/I Ratio	492.00	156.00	156.00	153.61	153.61	149.51	149.51	148.54	148.54	148.44	148.44	115.89	115.89	dB
Total Return Loss	8.00	28.00	34.00	54.00	78.00	98.00	104.00	124.00	126.00	142.00	154.00	103.00	128.00	dB

Calculated Receiver Sensitivity:	-79.68	dBm
Required Receiver Sensitivity:	-80.00	dBm
Margin:	-0.32	dB





Typical M15 output power = -46 dBm @ 50uA

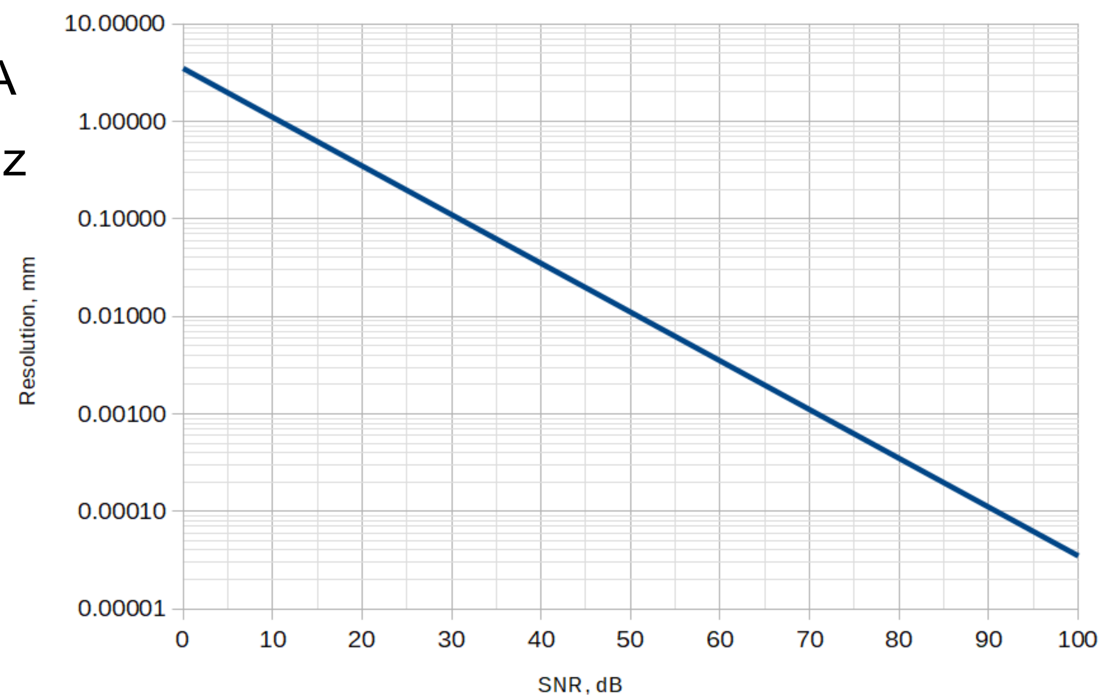
Expected noise power = -124 dBm for 100kHz

Calc. Rx SNR = 71 dB (B = 100 kHz)

Σ = 1.9 um, I = 50 uA, B = 100 kHz

So, for M15 BPM:

$$\sigma \approx \frac{0.3\,um \cdot \sqrt{\mathrm{Hz}}}{uA}$$



Note: Resolution is NOT accuracy!!

I = 10 nA
B = 1 Hz

RECEIVER MODEL

Input Field

	Coax	LNA	Filter	Amp	Coax	LNA	Filter	Amp	Filter	Mixer	IF Filter	Amp	ADC	
Noise Figure	4.00	1.30	3.00	5.40	12.00	1.30	3.00	5.40	1.00	8.00	6.00	2.70	25.00	dB
Gain: Passband	-4.00	13.00	-3.00	18.00	-12.00	13.00	-3.00	18.00	-1.00	-8.00	-6.00	60.00	0.00	dB
Gain: Reject-band	-4.00	13.00	-20.00	18.00	-12.00	13.00	-20.00	18.00	-20.00	-8.00	-30.00	60.00	0.00	dB
IIP3	200.00	28.00	200.00	26.00	200.00	28.00	200.00	26.00	200.00	34.00	200.00	-22.00	200.00	dBm
P1dB	200.00	23.00	200.00	20.00	200.00	23.00	200.00	20.00	200.00	22.00	200.00	20.00	200.00	dBm
Return Loss	8.00	20.00	6.00	20.00	24.00	20.00	6.00	20.00	2.00	16.00	12.00	25.00	25.00	dB

Pin Interference		-120.00	dBm	
Pin Passband	##	-120.00	dBm	
Input Noise BW		0.00	dB-Hz	(System IF BW)
Input Noise Temperature	290	290.00	K	(IEEE definition = 290K for Physical Temperature)
Input Noise Level		-174.0	dBm	
Required C/N	60	38.00	dB	(Modulator / BER -dependent…see BER sheet)
Required Sensitivity	164	-80.00	dBm	(From "Specifications" or "Standards")

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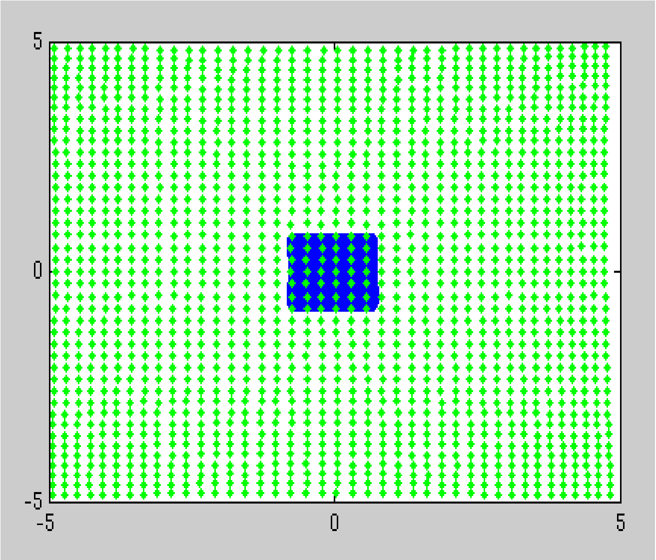
System Noise Temp	26.42	28.41	28.63	29.58	29.67	29.70	29.70	29.72	29.72	29.72	29.72	29.74	29.74	dBK
System Gain: Passband	-4.00	9.00	6.00	24.00	12.00	25.00	22.00	40.00	39.00	31.00	25.00	72.00	72.00	dB
System Gain: Reject-band	-4.00	9.00	-11.00	7.00	-5.00	8.00	-12.00	6.00	-14.00	-22.00	-52.00	55.00	55.00	dB
IIP3: Passband	200.00	32.00	32.00	19.73	19.73	14.47	14.47	3.63	3.63	-5.56	-5.56	-34.00	-34.00	dBm
IIP3: Reject-band	200.00	32.00	32.00	30.81	30.81	28.76	28.76	28.27	28.27	28.22	28.22	-17.00	-17.00	dBm
Input Spurious-Free Dynamic Range	246.65	133.78	133.68	125.03	124.99	121.46	121.46	114.22	114.22	108.10	108.10	89.13	89.13	dB
Pout: Passband	-124.00	-111.00	-114.00	-96.00	-108.00	-95.00	-98.00	-80.00	-81.00	-89.00	-95.00	-48.00	-48.00	dBm
Pout: Reject-band	-124.00	-111.00	-131.00	-113.00	-125.00	-112.00	-132.00	-114.00	-134.00	-142.00	-172.00	-65.00	-65.00	dBm
Output Noise Power	-173.98	-159.68	-162.52	-143.81	-155.75	-142.73	-145.73	-127.71	-128.71	-136.71	-142.71	-95.70	-95.70	dBm
C/N Ratio	49.98	48.68	48.52	47.81	47.75	47.73	47.73	47.71	47.71	47.71	47.71	47.70	47.70	dB
Saturation?	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	
IIM3	-760.00	-424.00	-424.00	-421.61	-421.61	-417.51	-417.51	-416.54	-416.54	-416.44	-416.44	-326.00	-326.00	dBm
C/I Ratio	640.00	304.00	304.00	301.61	301.61	297.51	297.51	296.54	296.54	296.44	296.44	206.00	206.00	dB
Total Return Loss	8.00	28.00	34.00	54.00	78.00	98.00	104.00	124.00	126.00	142.00	154.00	103.00	128.00	dB

Calculated Receiver Sensitivity:	-129.70	dBm
Required Receiver Sensitivity:	-80.00	dBm
Margin:	49.70	dB

41 dB SNR >> 60um resolution

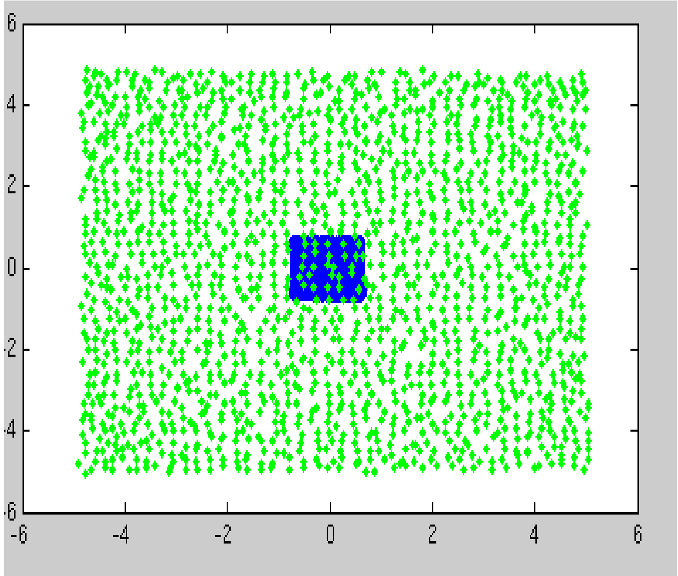
Measured Resolution Examples

(Goubau Line, per J. Musson)

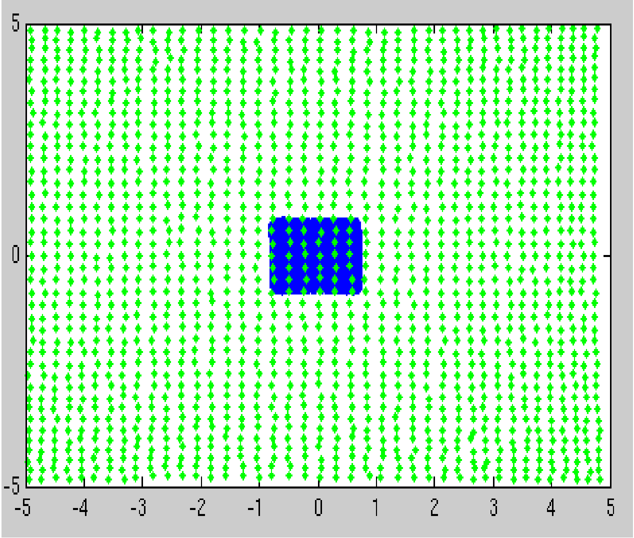


I ~ 800nA; B = 10 Hz

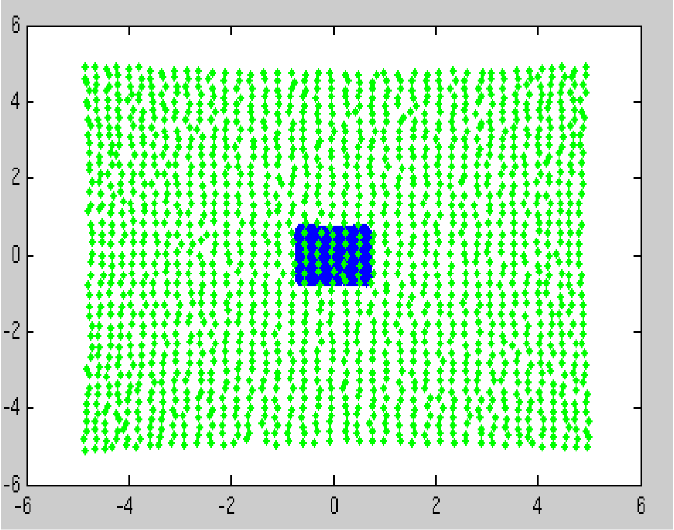
Step = 250 um



I ~ 100nA; B = 100 Hz



I ~ 100nA; B = 10 Hz



I ~ 70nA; B = 10 Hz

Statistical Communications: Formal Approach "Bandpass White Gaussian Process"

Bayes' Theorem:

Signals and noise are bivariate!!

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)} \quad (1)$$

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B \mid A) \cdot P(A) + P(B \mid \neg A) \cdot P(\neg A)} \quad (2)$$

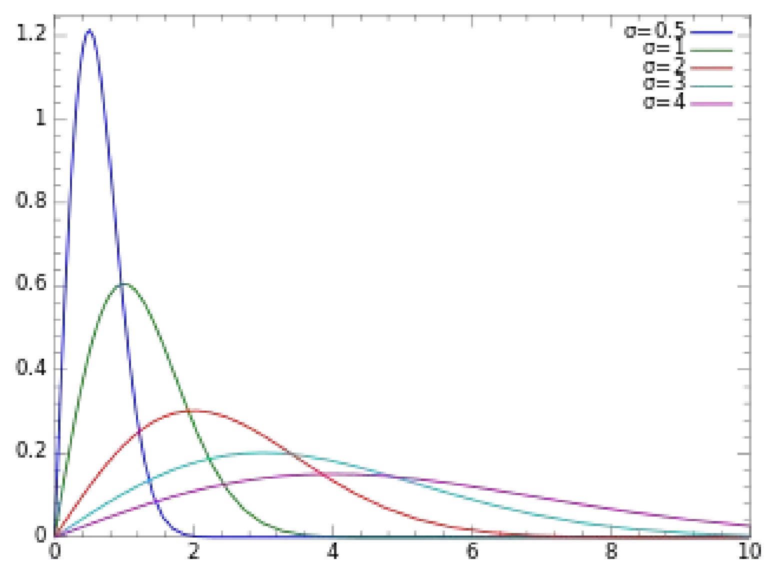
Probability functions for transmitted and received information…...

Likelyhood function of SNR (amplitude and/or phase)

Rayleigh PDF for our received signal, corrupted by AWGN:

$$p_r(r_1, r_2) = \frac{1}{2\pi\sigma_r^2} e^{-\left[\frac{(r_1 - \sqrt{\epsilon_s})^2 + r_2^2}{2\sigma_r^2}\right]}$$

(Rayleigh >> Ricean >> Gaussian)



W. W. Harman, *Principles of Statistical Theory of Communication*, McGraw-Hill, NY., NY., 1963

Joint Probability Density Function for Voltage and Phase:

$$p_{V_r,\theta_r}(V_r,\theta_r)=\frac{V_r}{2\pi\sigma_r^2}e^{-\left[\frac{V_r^2+\epsilon_s-2\sqrt{\epsilon_s}V_r\cos(\theta_r)}{2\sigma_r^2}\right]}$$

Integrate over all angles to get a PDF for Voltage:

$$p_V(V)=\int_{-\pi}^{\pi}p(V_r,\theta_r)\,d\theta=\frac{V_r}{\sigma_r^2}\cdot\frac{1}{2\pi}\cdot e^{-(V^2+\epsilon_s)/2\sigma^2}\cdot\int_{-\pi}^{\pi}e^{\left[\frac{V_r\sqrt{\epsilon_s}}{\sigma_r^2}\cos\theta\right]}d\theta$$

Integrate over all Voltages to get a PDF for Phase:

$$p_{\theta_r}(\theta_r)=\int_{0}^{\infty}p(V_r,\theta_r)\,dV=\frac{1}{2\pi\sigma_r^2}\int_{0}^{\infty}V_r\,e^{-\left[\frac{V_r^2+\epsilon_s-2\sqrt{\epsilon_s}V_r\cos(\theta_r)}{2\sigma_r^2}\right]}dV$$

(This is useful for investigating interferometric methods, LLRF resolution, etc.)

Now, sigma can be extracted, from which Confidence Intervals may be established…….
eg. 95% = Est +/- 1.96 sigma   T-scores, etc…...

Proakis, *Digital Communications* 3rd Ed., McGraw-Hill, NY., NY., 1995

Position Accuracy (cont.)

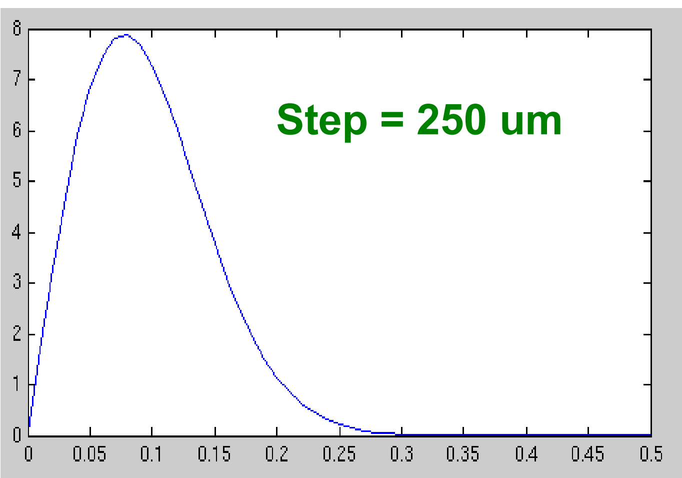
Good news...we can measure with G-Line!

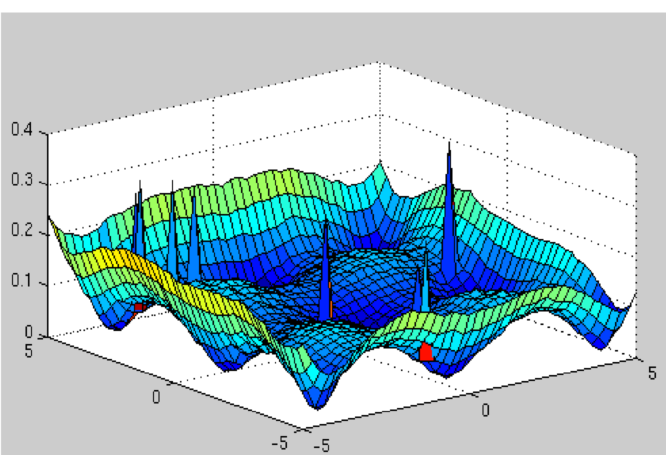
Ibeam ~ 500nA

B = 10 Hz

$\sigma = 77\, um$

R = 120 um



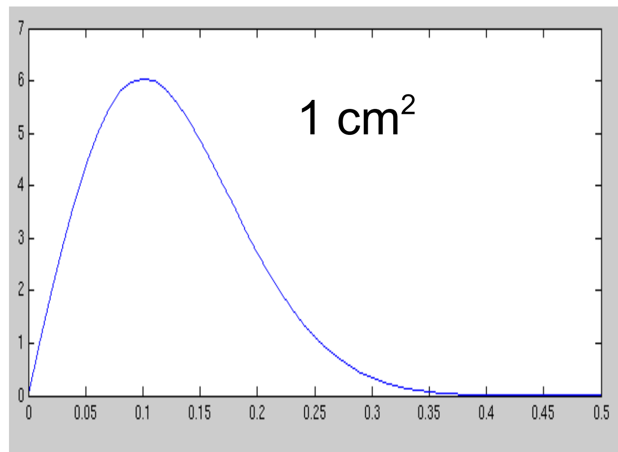


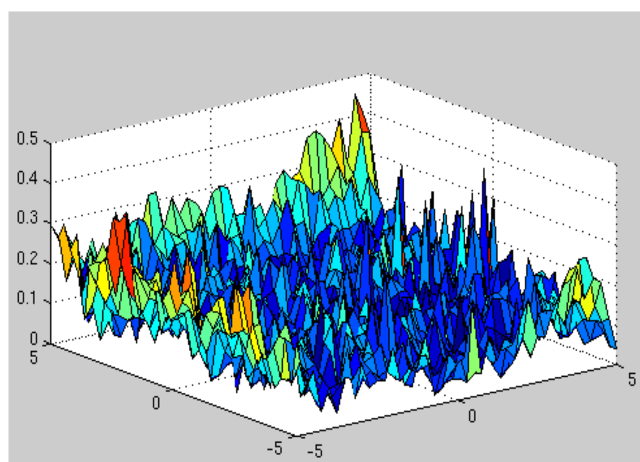
Ibeam ~ 100nA

B = 100 Hz

$\sigma = 100\, um$

R = 158 um



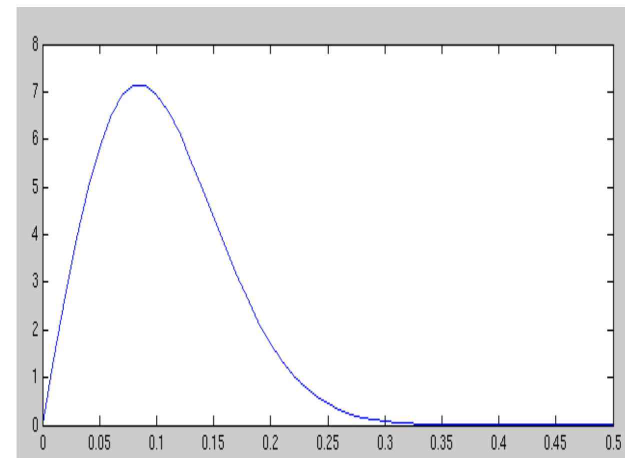


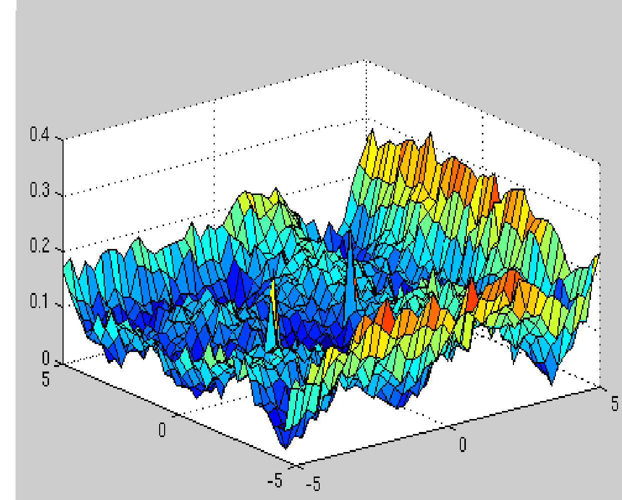
Ibeam ~ 100nA

B = 10 Hz

$\sigma = 85\, um$

R = 133 um





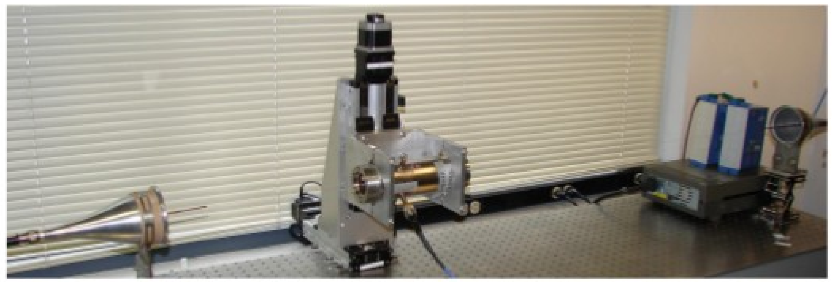
Application Of Goubau Surface Wave Transmission Line For Improves Bench Testing Of Diagnostic Beamline Elements*

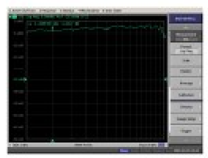
J. Musson, K. Cole, Thomas Jefferson National Accelerator Facility, Newport News, VA
S. Rubin, Rubytron, Port Chester, NY

Abstract

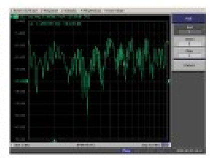
In-air test fixtures for beamline elements typically utilize an X-Y positioning stage, and a wire antenna excited by an RF source. In most cases, the antenna contains a standing wave, and is useful only for coarse alignment measurements in CW mode. A surface-wave (SW) based transmission line permits RF energy to be launched on the wire, travel through the beamline component, and then be absorbed in a load. Since SW transmission lines employ travelling waves, the RF energy can be made to resemble the electron beam, limited only by ohmic losses and dispersion. Although lossy coaxial systems are also a consideration, the diameter of the coax introduces large uncertainties in centroid location. A SW wire is easily constructed out of 200 micron magnet wire, which more accurately approximates the physical profile of the electron beam. Benefits of this test fixture include accurate field mapping, absolute calibration for given beam currents, Z-axis independence, and temporal response measurements of sub-nanosecond pulse structures. Descriptions of the surface wave launching technique, transmission line, and instrumentation are presented, along with measurement data.

Goubau Line/BPM Test Fixture

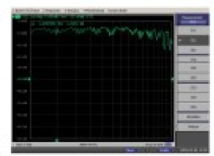




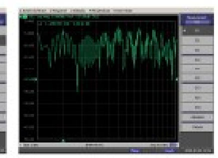
Insertion Loss (S21) plot of 1.6 mm diameter RadWire



Return Loss (S11) plot of 1.6 mm diameter RadWire



Insertion Loss (S21) plot of 160 um diameter RadWire



Return Loss (S11) plot of 160 um diameter RadWire

Goubau

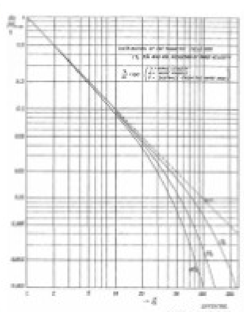
Georg Goubau (A'49) was born in Munich, Germany, on November 29, 1906. He received the Dipl. Phys. degree in 1930, and the Dr. Ing. degree in 1931, both from the Munich Technical University. From 1931 to 1939 he was employed in research and teaching in the physics department of the same university, under Professor Zenneck. During this time he was principally concerned with ionospheric investigations. He established the first German Ionospheric Research Station (Herzogstand/Kochel), and was in charge of the research work carried on at this station.

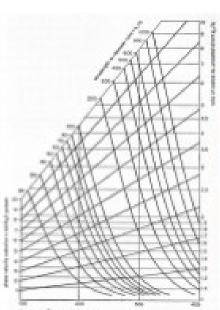
Georg Goubau

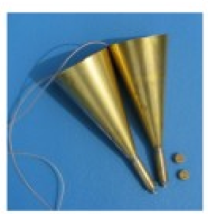
In 1939 Dr. Goubau was appointed professor and director of the department of applied physics at the Friedrich-Schiller University, in Jena, Germany. Before he arrived in this country, he was the senior author of the volumes on electronics of the FIAT Review of German Science, published by the Military Government for Germany. Dr. Goubau is now a consultant at the Signal Corps Engineering Laboratories, in Fort Monmouth, N. J.

RUBYTRON

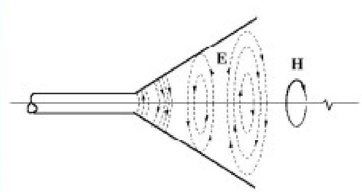
Development Of Surface Wave Lancher







.30 Caliber Brass Prototype



Surface wave evolution inside the launcher

Commercial Rubytron Inc. RadWire Lancher

Conclusions

Traditional bench testing of beamline components will be inadequate to characterize and assess performance of the 12 GeV upgrade at Jefferson lab. The use of the G-line facilitates measurements which more accurately mimic electron beam conditions. This system is particularly well-suited for our bench system, due to ease of fabrication, low-cost, and choice of operating frequency range. In addition, due to the flat 8 GHz frequency response, pulsed beam structures can be replicated, providing a platform for receiver development. Further reduction of VSWR is planned, in order to minimize dispersion of pulses resulting from reflections. Finally, the use of ~1 um X-Y stages presents a system which can be automated, improving repeatability and simplifying test procedures.

*Authored by Jefferson Science Associates, LLC under U.S. DOE Contract No. DE-AC05-06OR23177. The U.S. Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce this manuscript for U.S. Government purposes.

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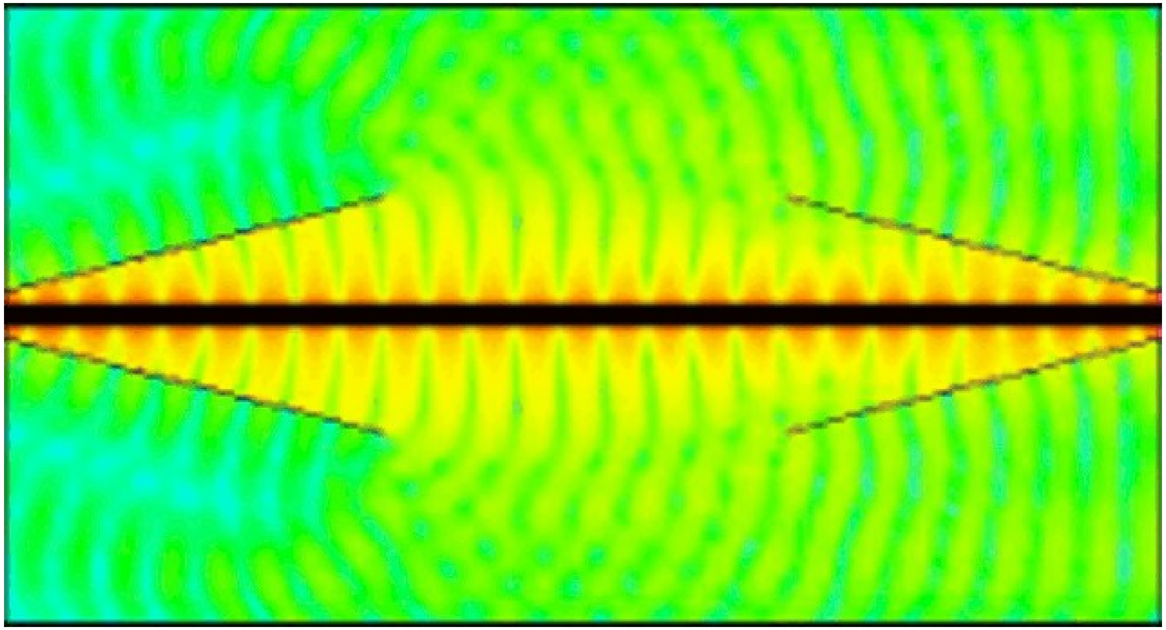
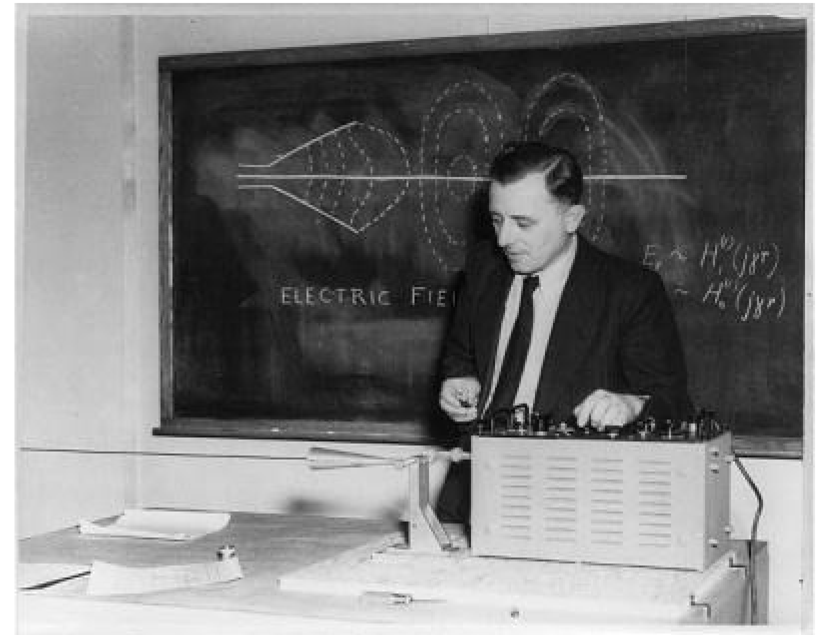
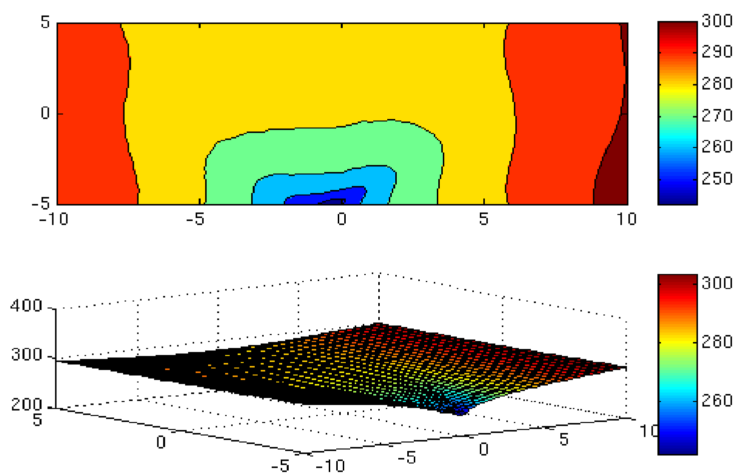


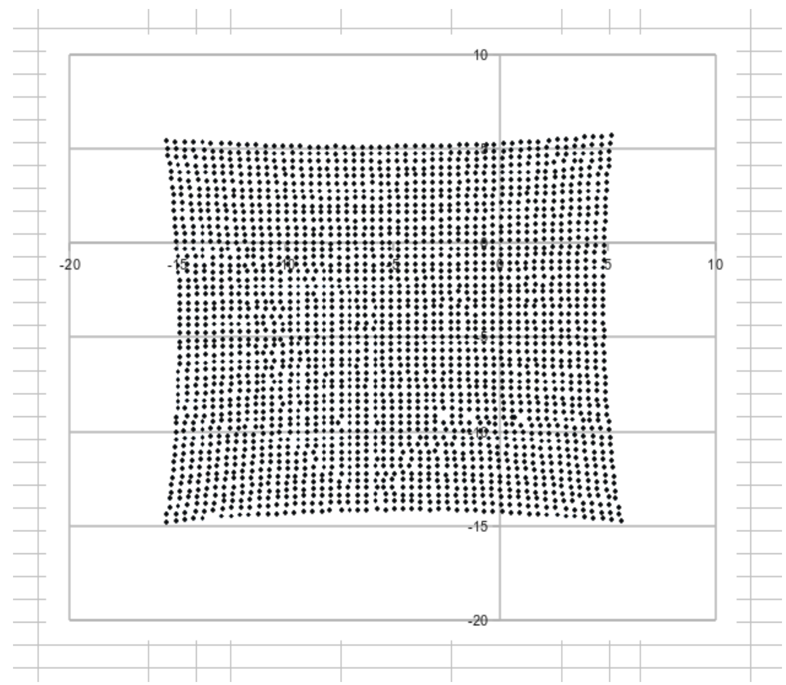
Goubau Line Animation



Georg!!



Single Button-electrode Scan



Prototype Sensor Scan

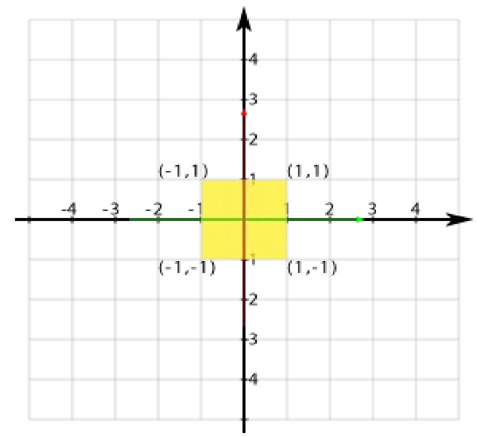
LMS 2-D Field Map Transformations

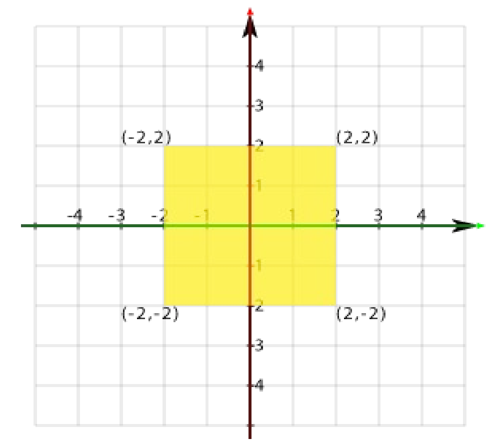
- Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Scaling

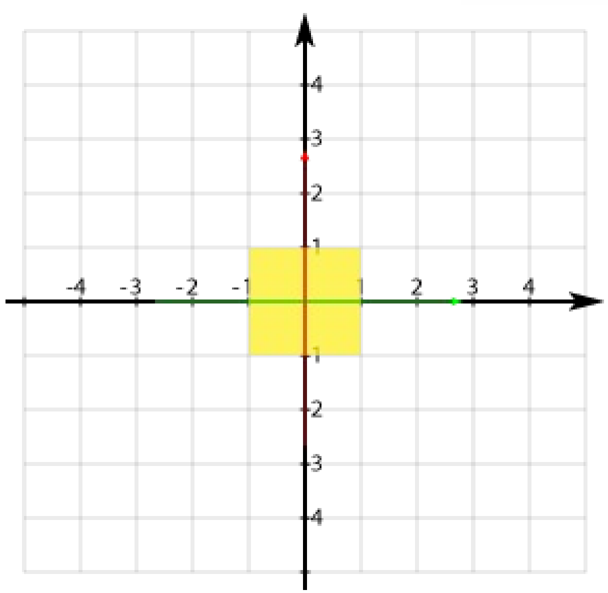
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

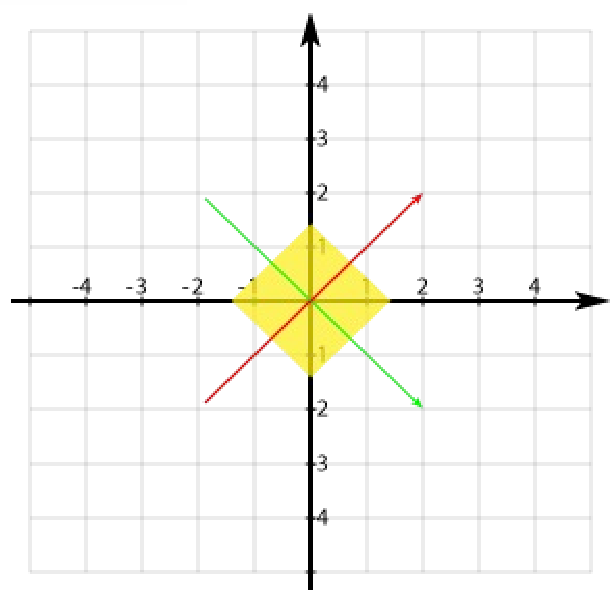


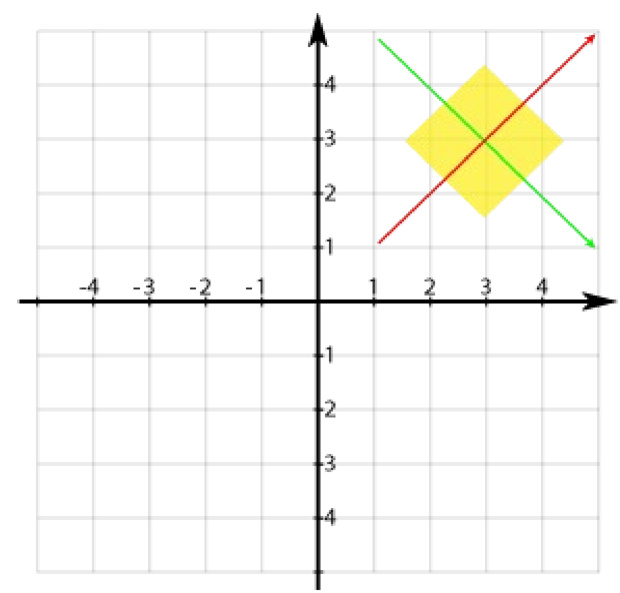


- Rotation

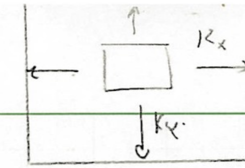
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$





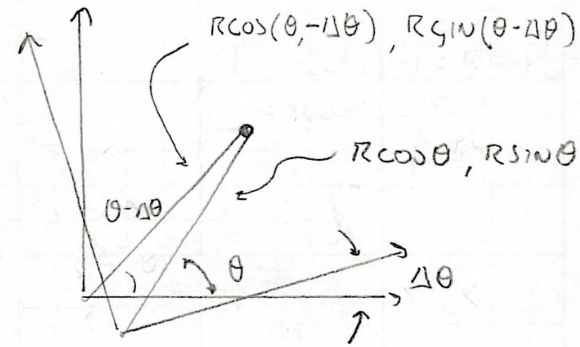


LMS Process



$$X_{MEAS} = \frac{X_+ - X_-}{X_+ + X_-} = R\cos\theta$$

$$Y_{MEAS} = \frac{Y_+ - Y_-}{Y_+ + Y_-} = R\sin\theta$$



$$X_{PROPER} = K_x \cdot X_{MEAS} = K_x \cdot R\cos(\theta - \Delta\theta)$$

Combine rotation and scaling

$$Y_{PROPER} = K_y Y_{MEAS} = K_y R\sin(\theta - \Delta\theta)$$

(MARION, 1970)

EXTRACT $\Delta\theta$:

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$X_{PROPER} = K_x R\cos\theta\cos\Delta\theta + K_x R\sin\theta\sin\Delta\theta$$

$$= K_x\cos\Delta\theta \cdot X_{MEAS} + K_x\sin\Delta\theta \cdot X_{MEAS} = \alpha_x X_{MEAS} + \beta_x Y_{MEAS}$$

$$Y_{PROPER} = K_y R \sin\theta \cos\Delta\theta - K_y R \cos\theta \sin\Delta\theta$$

$$= K_y \cos\Delta\theta \; Y_{MEAS} - K_y \sin\Delta\theta \; X_{MEAS} = \alpha_y X_{MEAS} + \beta_y Y_{MEAS}$$

$$\alpha_x = K_x \cos\Delta\theta$$

$$\beta_x = K_x \sin\Delta\theta$$

$$\alpha_y = -K_y \sin\Delta\theta$$

$$\beta_y = K_y \cos\Delta\theta$$

NOW, ADD TRANSLATION:

$$X_{PROPER} = \alpha_x X_{MEAS} + \beta_x Y_{MEAS} + \Delta X$$

$$Y_{PROPER} = \alpha_y X_{MEAS} + \beta_y Y_{MEAS} + \Delta Y$$

LET $X = X_{PROPER}$

$Y = Y_{PROPER}$

$$X_1 = \alpha_x X_{mens_1} + \beta_x Y_{mens_1} + \Delta X \qquad \alpha_x = K_x \cos \Delta\theta$$
$$X_2 = \alpha_x X_{mens_2} + \beta_x Y_{mens_2} + \Delta X \quad , \quad \beta_x = K_x \sin \Delta\theta$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$X_n = \alpha_x X_{mens_n} + \beta_x Y_{mens_n} + \Delta X$$

$$Y_1 = \alpha_y X_{mens_1} + \beta_y Y_{mens_1} + \Delta Y \qquad \alpha_y = -K_y \sin \Delta\theta$$
$$Y_2 = \alpha_y X_{mens_2} + \beta_y Y_{mens_2} + \Delta Y \quad , \quad \alpha_y = K_y \cos \Delta\theta$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$Y_n = \alpha_y X_{mens_n} + \beta_y Y_{mens_n} + \Delta Y$$

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} = \lambda \cdot \begin{bmatrix} \alpha_x \\ \beta_x \\ \Delta X \end{bmatrix} \qquad \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \lambda \cdot \begin{bmatrix} \alpha_y \\ \beta_y \\ \Delta Y \end{bmatrix}$$

,

$$\lambda = \begin{bmatrix} X_{mens_1} & Y_{mens_1} & 1 \\ X_{mens_2} & Y_{mens_2} & 1 \\ \vdots & \vdots & \vdots \\ X_{mens_n} & Y_{mens_n} & 1 \end{bmatrix}$$

Finally,

$$\begin{bmatrix} \alpha_x \\ \beta_x \\ \Delta X \end{bmatrix} = \lambda^{-1} \cdot \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$$

$$\begin{bmatrix} \alpha_y \\ \beta_y \\ \Delta Y \end{bmatrix} = \lambda^{-1} \cdot \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}$$

Use "Pseudo-Inverse:" (Moore-Penrose, Rao, Mitra, 1971)

$$\lambda^{-1} = (\lambda^T \lambda)^{-1} \cdot \lambda^T$$

So,

$$\begin{bmatrix} \alpha_x \\ \beta_x \\ \Delta X \end{bmatrix} = (\lambda^T \lambda)^{-1} \lambda^T \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$$, Least MSE (Bishop, 2006)

$$\begin{bmatrix} \alpha_y \\ \beta_y \\ \Delta X \end{bmatrix} = (\lambda^T \lambda)^{-1} \lambda^T \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}$$, Least MSE (Bishop, 2006)

Physical Significance of LMS Residuals

$$X_{scale\ factor} = \sqrt{\alpha_x^2 + \beta_x^2}$$

$$Y_{scale\ factor} = \sqrt{\alpha_y^2 + \beta_y^2}$$

Scale factors for X and Y directions

$$\theta_x = \tan^{-1}\left(\frac{\beta_x}{\alpha_x}\right)$$

$$\theta_y = \tan^{-1}\left(\frac{\beta_y}{\alpha_y}\right)$$

X and Y "effectively" rotated individually

$$\Delta\theta = \theta_y - \theta_x$$

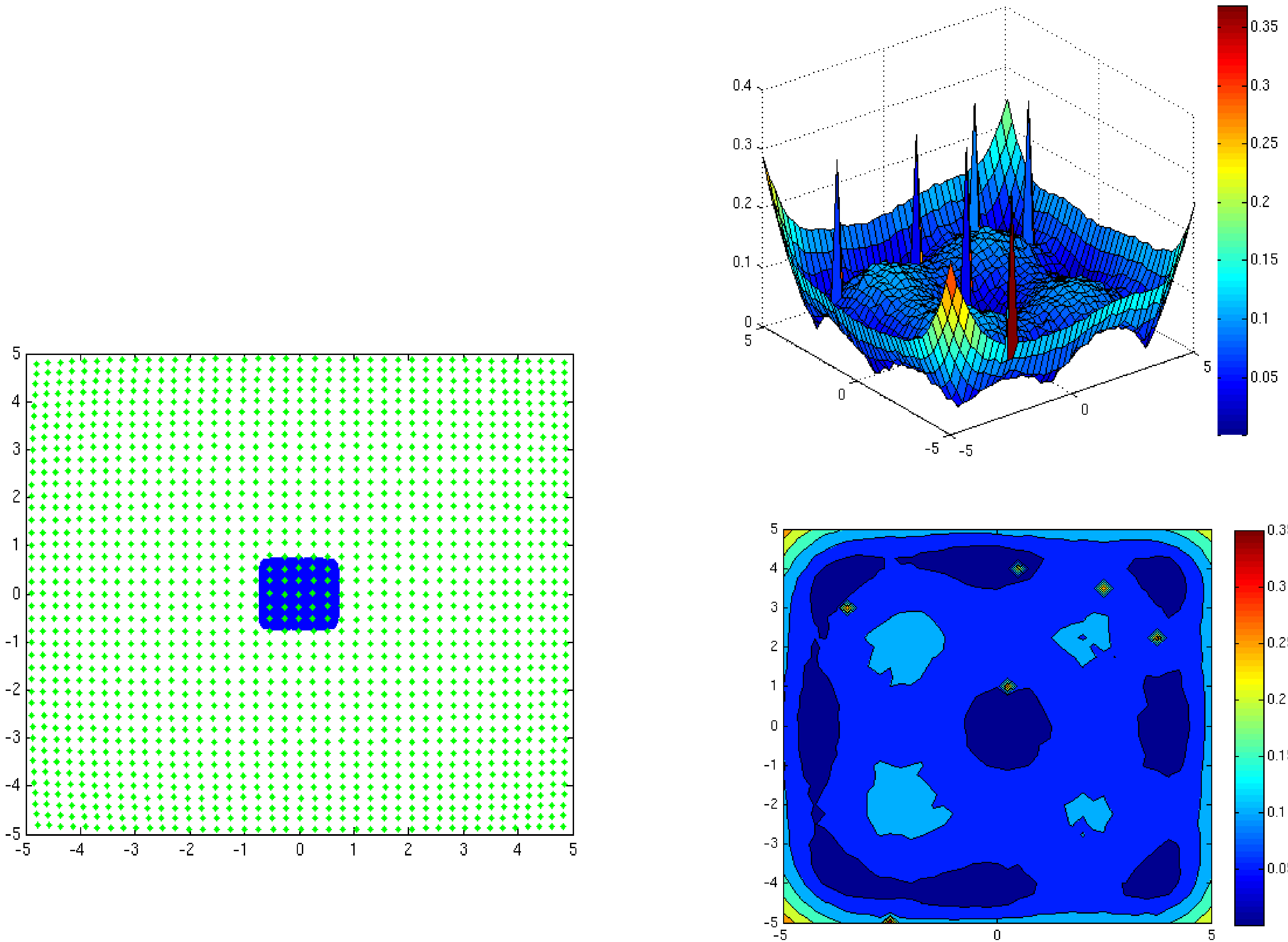
Differences in thetas represents X-Y coupling

$$\Delta_x, \Delta_y$$

Arbitrary field offset;

Merely tells us where we "should" have started the scan

Not related to physical vs. electrical centers (obtained later)



5 4 3 2 1 0 -1 -2 -3 -4 -5
-5 -4 -3 -2 -1 0 1 2 3 4 5
0.4 0.3 0.2 0.1 0
5 0 -5
-5 0 5
0.35 0.3 0.25 0.2 0.15 0.1 0.05
5 4 3 2 1 0 -1 -2 -3 -4 -5
-5 0 5
0.35 0.3 0.25 0.2 0.15 0.1 0.05

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