

β Calculation

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➤ Focal length

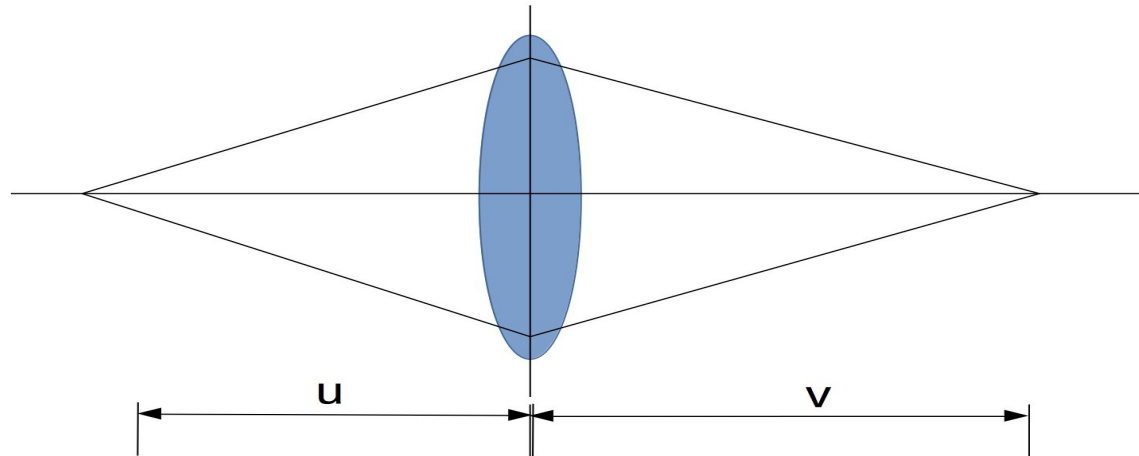
- For a solenoid

$$\frac{1}{f} = \frac{e^2 B_z^2 dz}{4\beta_z^2 \gamma^2 m^2 c^2}$$

- From optics, lens equations

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

where, u-distance to the object, v-distance to the image

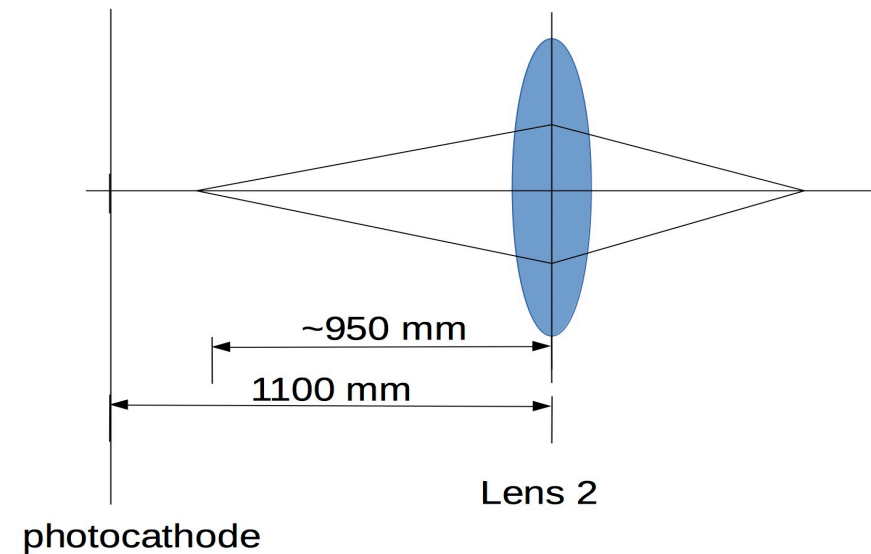
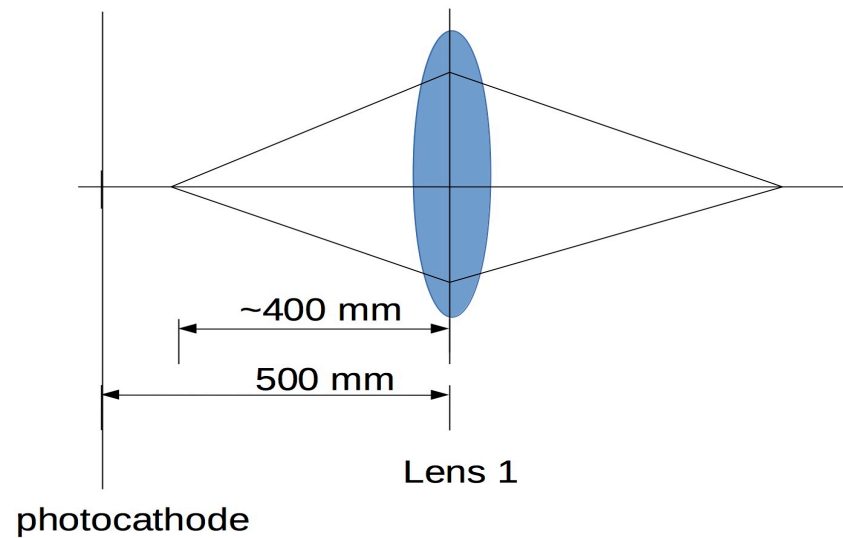


➤ Calculated focal length values from lens equation and from current

Lens	Viewer	$f = \frac{uv}{u+v}$ (mm)	f from the current (mm)
1	1	333.33	271.515
1	2	375.00	306.64
2	1	293.33	284.42
2	2	495.00	461.276

- Calculated distance to the photocathode using the focal length from the current

Lens	Viewer	Distance to object (mm)
1	1	372.7118
1	2	385.4327
2	1	984.3225
2	2	946.2632



Accelerator Physics

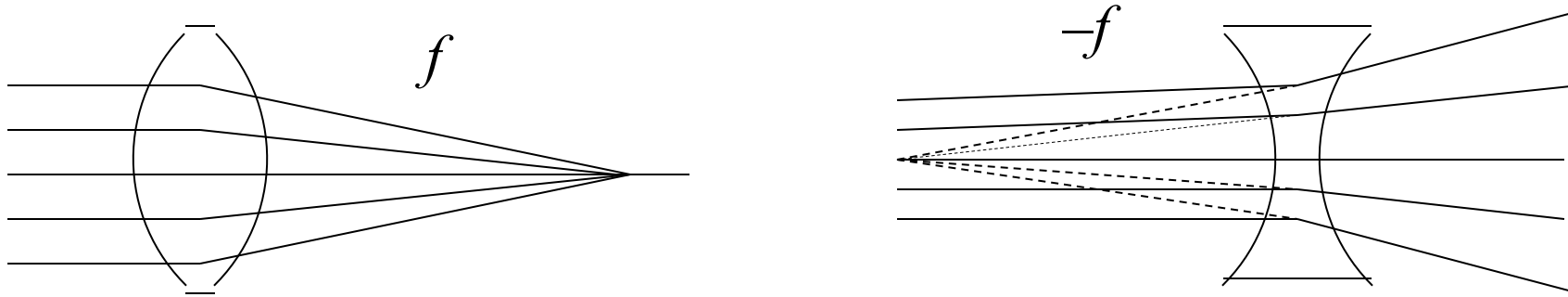
G. A. Krafft

Jefferson Lab

Old Dominion University

Lecture 5

Thin Lenses



Thin Focusing Lens (limiting case when argument goes to zero!)

$$\begin{pmatrix} x(s_{lens} + \varepsilon) \\ \frac{dx}{ds}(s_{lens} + \varepsilon) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x(s_{lens} - \varepsilon) \\ \frac{dx}{ds}(s_{lens} - \varepsilon) \end{pmatrix}$$

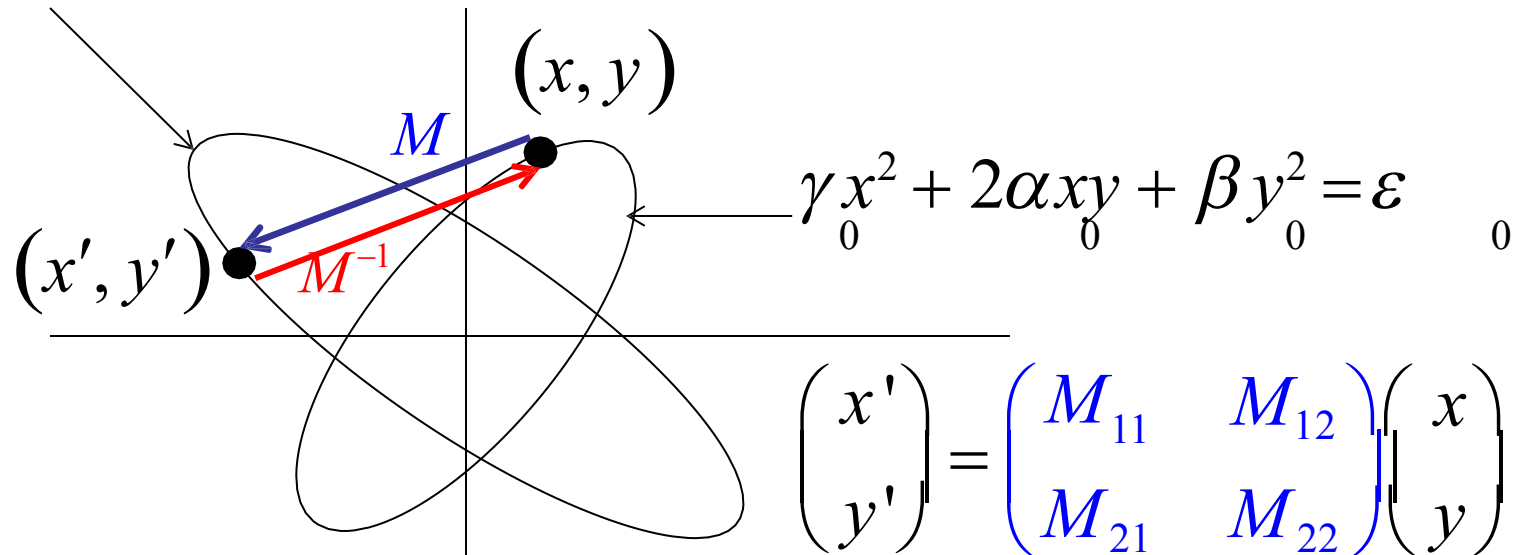
Thin Defocusing Lens: change sign of f

Effect of Transformation

Let the final ellipse be $\gamma x^2 + 2\alpha xy + \beta y^2 = \varepsilon$

The transformed coordinates must solve this equation.

$$\gamma x'^2 + 2\alpha x'y' + \beta y'^2 = \varepsilon$$



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} M_{11}^{-1} & M_{12}^{-1} \\ M_{21}^{-1} & M_{22}^{-1} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

The transformed coordinates must also solve the initial equation transformed.

Because

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (M^{-1})_{11} & (M^{-1})_{12} \\ (M^{-1})_{21} & (M^{-1})_{22} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

The transformed initial ellipse is

$$\gamma x'^2 + 2\alpha x'y' + \beta y'^2 = \varepsilon_0$$

$$\gamma = (M^{-1})_{11}^2 \gamma_0 + 2(M^{-1})_{11}(M^{-1})_{21} \alpha_0 + (M^{-1})_{21}^2 \beta_0$$

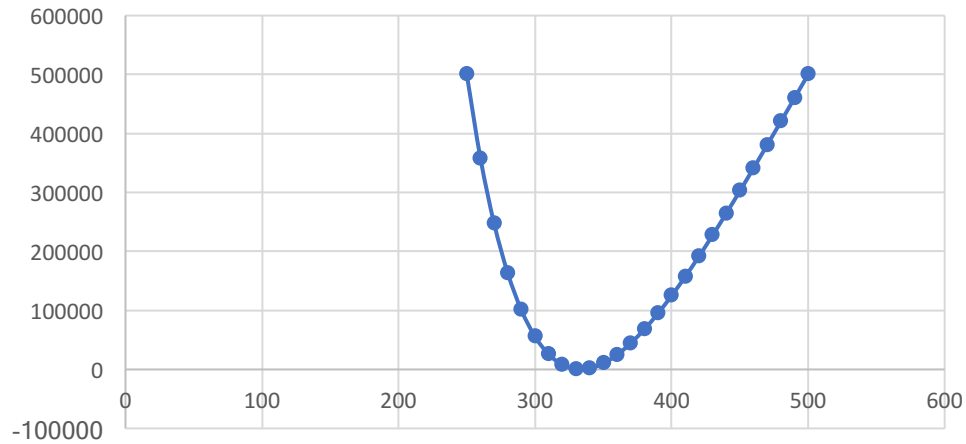
$$\alpha = (M^{-1})_{11}(M^{-1})_{12} \gamma_0 + ((M^{-1})_{11}(M^{-1})_{22} + (M^{-1})_{12}(M^{-1})_{21}) \alpha_0 + (M^{-1})_{21}(M^{-1})_{22} \beta_0$$

$$\beta = (M^{-1})_{12}^2 \gamma_0 + 2(M^{-1})_{12}(M^{-1})_{22} \alpha_0 + (M^{-1})_{22}^2 \beta_0$$

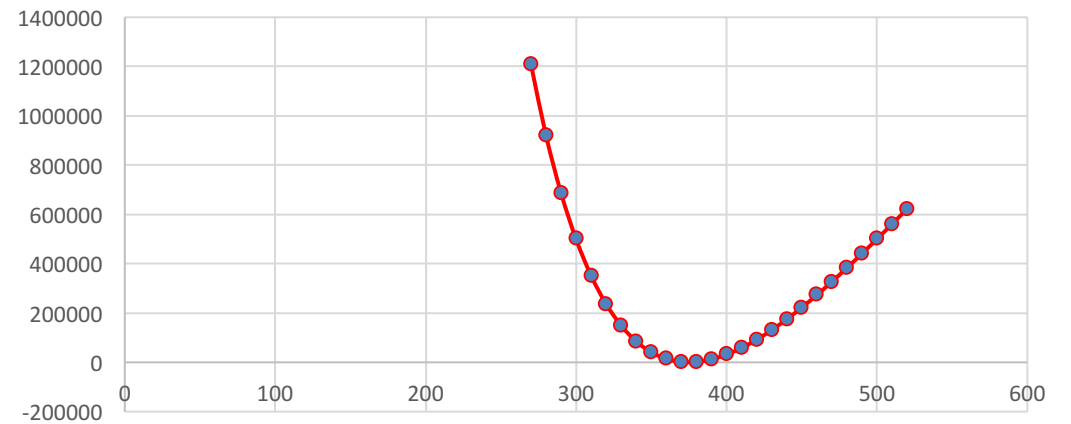
$$\beta = \frac{1}{0.5} \left(-u - v + \frac{uv}{f} \right)^2 + 0.5 \left(1 - \frac{v}{f} \right)^2$$

$$\gamma_0 = \frac{1}{0.5} \text{ mm}^{-1}, \quad \beta_0 = 0.5 \text{ mm}, \quad \alpha_0 = 0$$

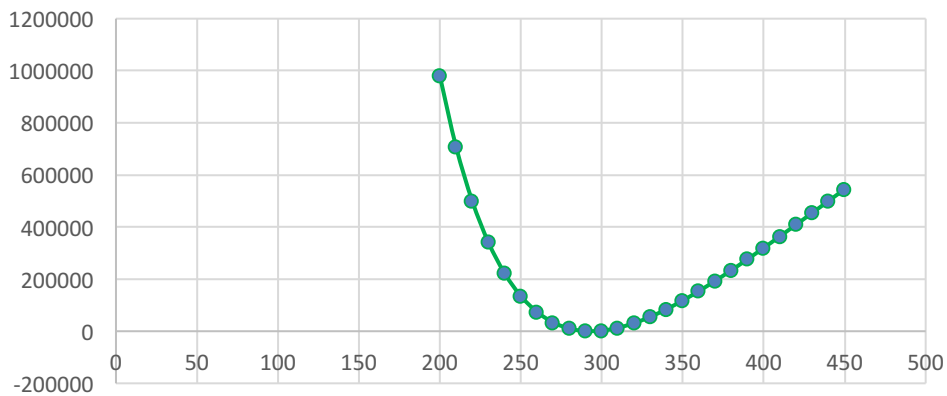
Lens 1 -viewer 1 (f=333.33 mm)



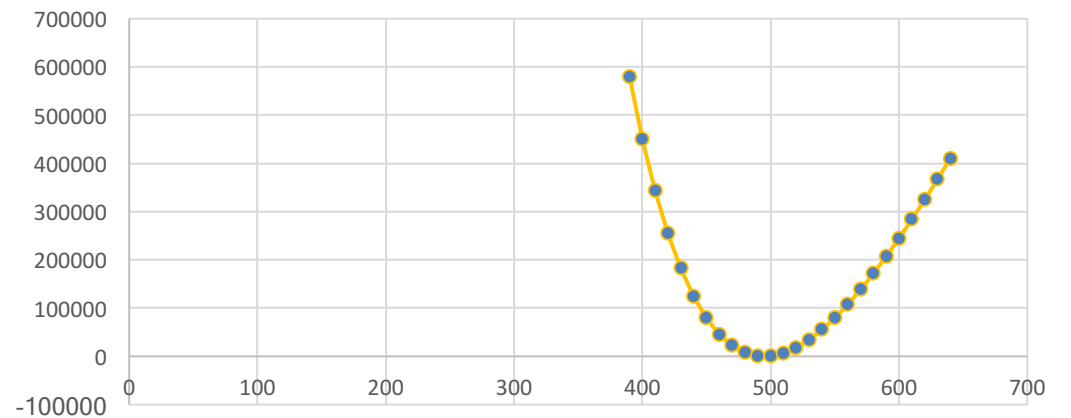
Lens 1-viewer 2 (f=375 mm)



Lens 2-viewer 1 (f=293.33 mm)



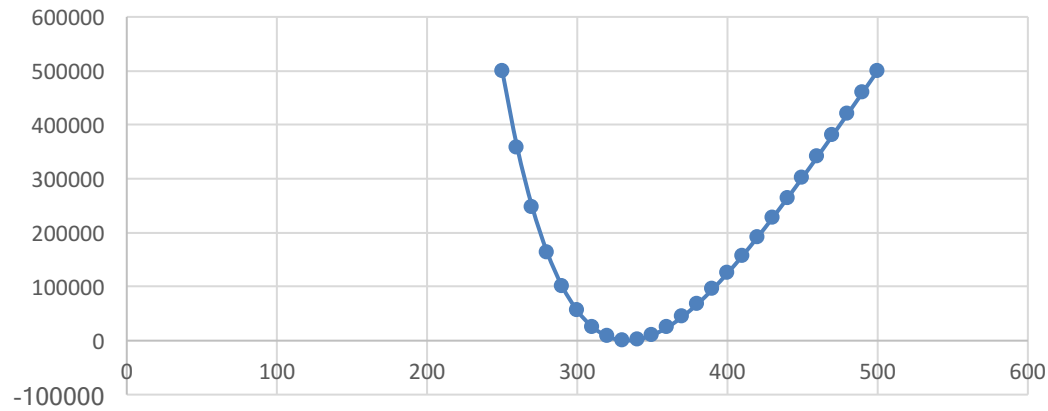
Lens 2-viewer 2 (f=495 mm)



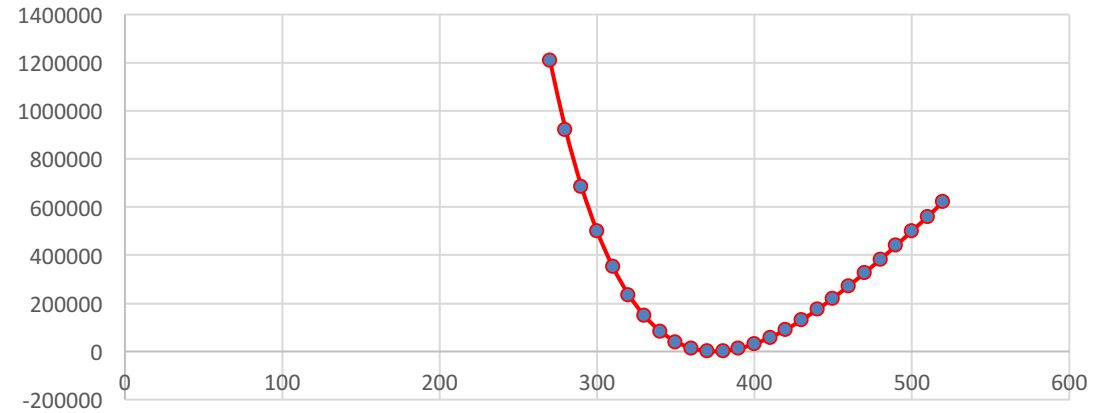
$$\beta = \frac{1}{0.5} \left(-u - v + \frac{uv}{f}\right)^2 + 2 \times 2.5 \times 10^{-3} \left(-u - v + \frac{uv}{f}\right) \left(1 - \frac{v}{f}\right) + 0.5 \left(1 - \frac{v}{f}\right)^2$$

$$\gamma_0 = \frac{1}{0.5} \text{ mm}^{-1} \quad , \quad \beta_0 = 0.5 \text{ mm} \quad , \quad \alpha_0 = \frac{\beta}{2f} = 2.5 \times 10^{-3}$$

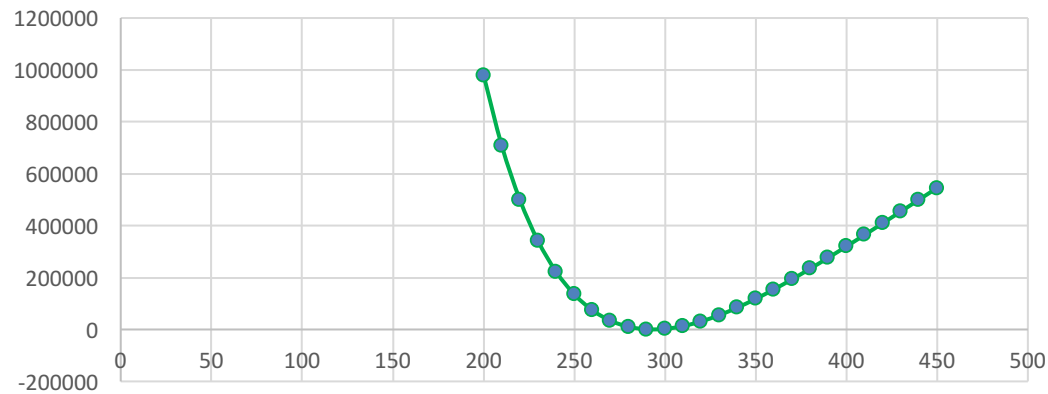
Lens 1-viewer 1 (f=333.33 mm)



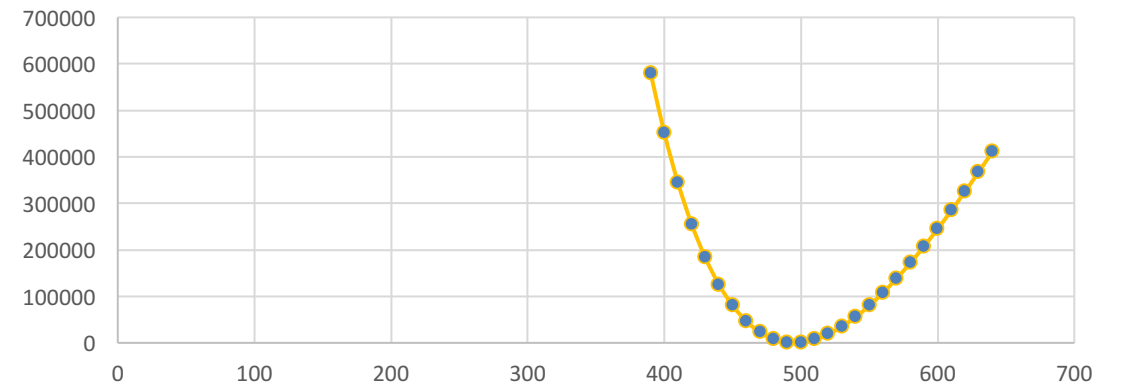
Lens 1-viewer 2 (f=375 mm)



Lens 2-viewer 1 (f=293.33 mm)



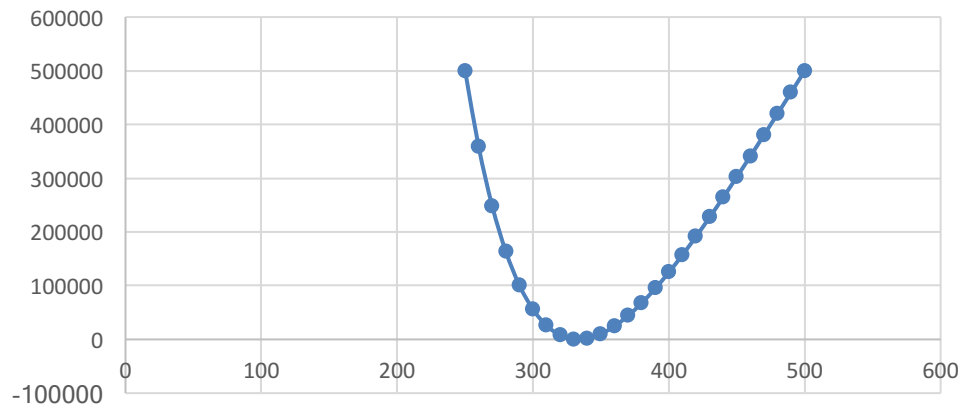
Lens 2-viewer 2 (f=495 mm)



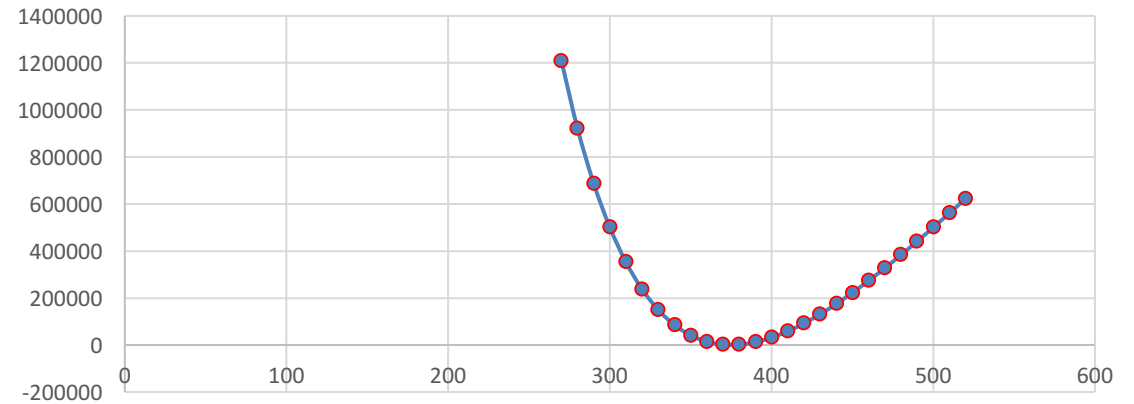
$$\beta = \frac{1}{0.5} \left(-u - v + \frac{uv}{f}\right)^2 - 2 \times 2.5 \times 10^{-3} \left(-u - v + \frac{uv}{f}\right) \left(1 - \frac{v}{f}\right) + 0.5 \left(1 - \frac{v}{f}\right)^2$$

$$\gamma_0 = \frac{1}{0.5} \text{ mm}^{-1} , \beta_0 = 0.5 \text{ mm} , \alpha_0 = -2.5 \times 10^{-3}$$

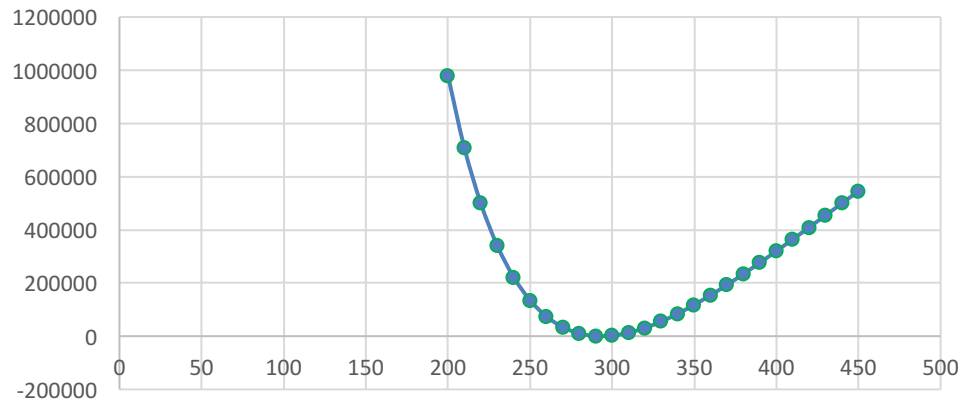
Lens 1 -viewer 1 (f=333.33 mm)



Lens 1 -viewer 2 (f=375 mm)



Lens 2 -viewer 1 (f=293.33 mm)



Lens 2 -viewer 2 (f=495 mm)

