# Mott Experiment Run I/II Beam Energies 

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## 1 Introduction

Measurements of the electron beam momentum to determine the electron beam energies in Run I/II of the Mott Experiment are summarized in this note. Because the Run II approach was improved we begin there to provide a more global description of the methods. Results of Run I/II are summarized at the end of this note.

## 2 Run II Conditions

In Run II the beam energy was set to 5 values by changing the gradient of the second cryo-unit cavity (R028) in approximately 200 keV increments. For each setting the un-deflected beam orbit was measured when the spectrometer dipole magnet (MDL0L02) was set to send the beam straight down the 0L beam line and again when the dipole was set to deflect the beam $+25^{\circ}$ into the 5D diagnostic beam line. A pair of beam position monitors (IPM0L02, IPM0L03) measured the un-deflected orbit and another pair (IPM5D00, IPM5D01) the deflected orbit.


Fig. 1. Picture shows the layout and names of the relevant beam line elements.

Three horizontal steering coils (MHB0L02A, MHB0L02B, MBH0L03) within the spectrometer region bounded by the BPM's remained on and constant throughout all 5 measurements to compensate the $\sim 0.5$ G stray Earth field. Without the steering coils the beam would progressively deflect until reaching the beam pipe. Two steering coils (MBH0L01, MHB0L02) upstream of the spectrometer region were used to correct both the RF dipole mode and stray field between the cryo-unit and spectrometer region and null the horizontal beam position at the un-deflected monitors (IPM0L02, IPM0L03). Because the RF dipole strength and beam momentum changed these two steering coils took on modestly different values for the five gradient settings. Two steering coils (MBH5D00H, MBH5D00AH) in the deflected beam line remained $0.0 \mathrm{G}-\mathrm{cm}$ for all measurements because this beam line is better shield from stray magnetic field and is shorter. Only the spectrometer dipole magnet changed between the un-deflected and deflected measurements. The recorded cavity gradient, steering coil setpoints, spectrometer dipole, and beam positions for all 5 energies are summarized in Table 1.

Table 1. The cavity gradient (MV/m), steering coil currents (mA), spectrometer dipole field integral ( $G-\mathrm{cm}$ ) and beam position monitor readings ( mm ) for the experiment are listed. Beam positions are reported in a left-handed system ( $x=r i g h t, y=u p, z=$ forward).

| Conditions for individual measurements |  |  |  |  |  | Undeflected |  |  | Deflected |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underset{\sim}{\infty}$ |  |  |   <br> 0  <br> 0  <br>   <br>   |  | $\begin{aligned} & \text { n } \\ & \text { O } \\ & \text { o } \\ & \text { ¿ } \end{aligned}$ | 2 0 0 0 2 |  | $\begin{aligned} & \tilde{0} \\ & \dot{0} \\ & \underset{x}{0} \\ & 0 . \\ & \\ & \end{aligned}$ | 2 0 0 0 $\Sigma$ | $n$ 0 0 0 0 0 $\sum_{n}^{n}$ $n$ | $\begin{aligned} & \text { ù } \\ & \text { o } \\ & \text { ì } \\ & \text { in } \\ & \text { à } \end{aligned}$ |
| MV/m | $m A$ | $m A$ | $m A$ | $m A$ | $m A$ | G-cm | mm | mm | G-cm | mm | mm |
| 3.35 | -325.00 | -292.00 | -214.54 | -0.03 | -342.83 | 0.00 | 0.03 | 0.22 | 7109.57 | 0.00 | 3.50 |
| 3.74 | -327.00 | -293.00 | -214.54 | -0.03 | -342.83 | 0.00 | 0.08 | 0.17 | 7384.34 | 0.01 | 3.67 |
| 4.12 | -329.00 | -292.00 | -214.54 | -0.03 | -342.83 | 0.00 | 0.06 | 0.15 | 7646.01 | 0.00 | 4.06 |
| 4.50 | -332.00 | -286.00 | -214.54 | -0.03 | -342.83 | 0.00 | -0.02 | 0.00 | 7927.59 | 0.00 | 3.89 |
| 4.89 | -333.00 | -287.00 | -214.54 | -0.03 | -342.83 | 0.00 | 0.05 | 0.21 | 8185.00 | 0.03 | 3.85 |

## 3 Run II TOSCA Model

A TOSCA model predicts the electron beam of momentum $P(\mathrm{MeV} / \mathrm{c})$ to deflect the electron beam $+25.0^{\circ}$ in terms of the straight on-axis magnetic field [4]. A comparison of the spectrometer dipole magnet field measurements and the TOSCA model agree within $0.1 \%$ [4]. The relation between magnetic field and momentum is calculated with a $5^{\text {th }}$ order polynomial (Fig. 2) and computed for 5 settings in Table 2.

$$
\begin{aligned}
\mathrm{BL}= & \mathrm{M}_{0}+\mathrm{M}_{1} \mathrm{P}+\mathrm{M}_{2} \mathrm{P}^{2}+\mathrm{M}_{3} \mathrm{P}^{3}+\mathrm{M}_{4} \mathrm{P}^{4}+\mathrm{M}_{5} \mathrm{P}^{5} \\
& \mathrm{M}_{0}=+4.811 \\
\mathrm{M}_{1} & =-1416.2 \\
\mathrm{M}_{2} & =+1.2399 \\
\mathrm{M}_{3} & =-0.1646 \\
\mathrm{M}_{4} & =+0.009795 \\
\mathrm{M}_{5} & =-0.00021257
\end{aligned}
$$



Fig. 2. TOSCA prediction of momenta vs. straight field integral for $+25.0^{\circ}$ deflection.
Table 2. Table of momenta for 5 energies studied according to model of MDLOL02. The uncertainty of 0.1\% given by [4] is agreement between model and measurement.

| ORIGINAL GRADIENT WITH DIPOLE MAP AND RESULT |  |  |  |  |  | NEW DIPOLE MAP |  |  | NEW/OLD | $\frac{\text { FINAL VALUES }}{\text { TOSCA }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| R028 | MDLOLO2 |  |  | tosca |  | MDLOLO2 |  |  |  |  |  |
| GSET | BL [G-cm] |  |  | $p$ | $d p$ | BL [G-cm] |  |  |  | $p$ | $d p$ |
| MV/m | Straight | Deflected | Difference | $\mathrm{MeV} / \mathrm{c}$ | MeV/c | Straight | Deflected | Difference |  | $\mathrm{MeV} / \mathrm{c}$ | $\mathrm{MeV} / \mathrm{c}$ |
| 3.350 | 0.000 | 7109.570 | 7109.570 | 5.035 | 0.005 | -27.270 | 7089.810 | 7117.080 | 1.00106 | 5.040 | 0.005 |
| 3.740 | 0.000 | 7384.340 | 7384.340 | 5.229 | 0.005 | -27.270 | 7364.460 | 7391.730 | 1.00100 | 5.234 | 0.005 |
| 4.120 | 0.000 | 7646.010 | 7646.010 | 5.415 | 0.005 | -27.270 | 7626.300 | 7653.570 | 1.00099 | 5.420 | 0.005 |
| 4.500 | 0.000 | 7927.590 | 7927.590 | 5.614 | 0.005 | -27.270 | 7907.890 | 7935.160 | 1.00095 | 5.619 | 0.005 |
| 4.890 | 0.000 | 8185.000 | 8185.000 | 5.797 | 0.005 | -27.270 | 8165.390 | 8192.660 | 1.00094 | 5.802 | 0.005 |

## 4 Run II Earth Field Correction

The TOSCA model assumes ideal $+25.0^{\circ}$ deflection, but does not include any correction for the Earth's field and steering coil which changes the actual bend angle. The correction due to stray magnetic field and steering coils is recursively calculated by particle tracking with all fields considered,

```
kick=asin(2.9980E-4*bl[k]/(p))
xp[k]=xp[k-1]+kick
x[k]=x[k-1]+xp[k]*1
```


## Model of Stray Field and Steering Coils

A survey of vertical component of the stray magnetic field was made in December 2015 (see Appendix A) and is plotted in Fig. 3 over the span IPM0L02 to IPM0L03. To demonstrate the significance of the stray field trajectories of beam momenta ranging from $4.5-6.5 \mathrm{MeV} / \mathrm{c}$ are overlaid on Fig. 3; the point is that a beam centered at IPM0L02 will exit the beam pipe ( $x=1.75 \mathrm{~cm}$ ) before reaching IPM0L03.


Fig. 3. Stray magnetic field measurements (red diamonds) and spline fit are used to generate $B_{y}(s)$ splanning IPM0L02 to IPM0L03. The field is forced to zero for regions where mu-metal shielding is added to the beam pipe. The horizontal trajectory is plotted vs. beam position over this length for momenta (4.5, 5.0, 5.5, 6.0, $6.5 \mathrm{MeV} / \mathrm{c}$ ) with initial conditions IPMOLO2( $x=0 \mathrm{~cm}, x^{\prime}=0 \mathrm{mrad}$ ).

By including the steering coils used in the experiment (see Table 1), and after converting to magnetic field integral (see Appendix B), a model of the beam trajectories for the same range in momenta is more realistic (see Fig. 4); the beam is transported from IPM0L02 to IPM0L03 because the point-wise steering corrections compensate for the distributed stray field. Because deflection by both the steering coils and stray magnetic field scale linearly with beam momentum the sensitivity to be orbit is greatly minimized.


Fig. 4. Four fixed steering coils (MBH0L02, MHB0L02A, MHB0L02B, MBH0L03) are added to the model. The horizontal trajectories plotted vs. beam position over a range of momenta (4.5, 5.0, 5.5, 6.0, $6.5 \mathrm{MeV} / \mathrm{c}$ ) with initial conditions IPM0L02( $x=0 \mathrm{~cm}, x^{\prime}=0$ mrad) is shown.

## BPM Analysis

Prior to the experiment each beam position monitor center was calibrated (Appendix A) to the magnetic center of the nearest upstream quadrupole magnet. Beam monitor positions (IPM.XPOS) recorded during the energy measurements and reported in Table 3 are therefore relative to the quadrupoles. Corrected absolute beam positions (IPM.XCOR) are determined by first accounting for surveyed quadrupole positions (QUAD.XOFF) according to the relationship,

$$
I P M . X C O R=I P M . X P O S-Q U A D . X O F F
$$

## Application of model to determine size of correction

The orbit for each of the 5 cases may be studied separately however during Run II systematically similar conditions allow averaging the 5 measurements to compute a single correction to the bend angle. Adding uncertainties for the quadrupole survey ( 0.25 mm ) and quad centering ( 0.50 mm ) we find,

$$
\begin{aligned}
& <0 L 02 \text { _COR }>_{\text {meas }}=0.07 \pm 0.02 \pm 0.25 \pm 0.50 \mathrm{~mm}=0.07 \pm 0.56 \mathrm{~mm} \\
& <0 L 03 \_C O R>_{\text {meas }}=0.42 \pm 0.04 \pm 0.25 \pm 0.50 \mathrm{~mm}=0.42 \pm 0.56 \mathrm{~mm}
\end{aligned}
$$

Table 3. Note that the quadrupole offsets reported in [1,2] are converted from their right-handed system to the left-handed beam position monitor system.

| Constant | Undeflected |  |  |  |  |  | Deflected |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underset{\sim}{\infty}$ |  | $\begin{aligned} & \text { u } \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & 0 \\ & 0 \\ & 0 \\ & \vdots \end{aligned}$ | 0 <br> 0 <br> $x$ <br>  <br>  <br> 0 <br> 0 <br> 0 | $\begin{aligned} & n \\ & 0 \\ & 0 \\ & \underset{x}{n} \\ & 0 \\ & 0 \\ & \vdots \\ & i \end{aligned}$ |  | 0 0 0 0 0 0 0 2 | $\begin{aligned} & n \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \sum \\ & i \end{aligned}$ | 4 0 0 0 0 0 0 0 2 | 0 0 0 $\times$ 0 0 0 $i$ $i$ | IPM5D01.XPOS | 4 0 0 $\vdots$ 0 0 0 0 0 $\vdots$ |  |
| MV/m | mm | mm | mm | mm | mm | mm | mm | mm | mm | mm | mm | mm |
| 3.35 | 0.03 | -0.01 | 0.04 | 0.22 | -0.24 | 0.46 | 0.00 | -0.27 | 0.27 | 3.50 | -0.22 | 3.72 |
| 3.74 | 0.08 | -0.01 | 0.09 | 0.17 | -0.24 | 0.41 | 0.01 | -0.27 | 0.28 | 3.67 | -0.22 | 3.89 |
| 4.12 | 0.06 | -0.01 | 0.07 | 0.15 | -0.24 | 0.39 | 0.00 | -0.27 | 0.27 | 4.06 | -0.22 | 4.28 |
| 4.50 | -0.02 | -0.01 | -0.01 | 0.00 | -0.24 | 0.24 | 0.00 | -0.27 | 0.27 | 3.89 | -0.22 | 4.11 |
| 4.89 | 0.05 | -0.01 | 0.06 | 0.21 | -0.24 | 0.45 | 0.03 | -0.27 | 0.30 | 3.85 | -0.22 | 4.07 |

Using the tracking model, beam positions with total uncertainties and range of momenta $4.5-6.7 \mathrm{MeV} / \mathrm{c}$ the beam position $<0 L 03$ _COR $>_{\text {meas }}$ is satisfied within total uncertainty when conditions at IPM0L02 satisfy,

$$
\begin{aligned}
& X=+0.07 \pm 0.56 \mathrm{~mm} \text { (from measurement) } \\
& X^{\prime}=+0.04 \pm 0.16 \mathrm{mrad} \text { (from model) }
\end{aligned}
$$

A plot of the trajectories over range of momenta $4.5-6.7 \mathrm{MeV} / \mathrm{c}$ for these initial conditions is shown in Fig. 5. Including the uncertainty of the beam position and angle at IPMOL02 the distribution of position and angle at MDLOLO2 is calculated using the model (where first uncertainty derives from conditions at IPM0L02 and IPM0L03, and second uncertainty derives from using range of momenta),

$$
\begin{aligned}
& X=-1.20 \pm 0.51 \pm 0.19 \mathrm{~mm}=-1.20 \pm 0.54 \mathrm{~mm} \\
& X^{\prime}=-0.92 \pm 0.16 \pm 0.14 \mathrm{mrad}=-0.92 \pm 0.21 \mathrm{mrad}
\end{aligned}
$$

Because the point-wise steering coil corrections cancel the stray magnetic field and scale with momentum the average beam positions and angles have a small spread at the dipole.

A survey of stray magnetic field for the 5D diagnostics beam line was also measured (see Appendix D) and is plotted in Fig. 5. Overlaid on this plot are trajectories (beginning at center of dipole) for the range of momenta using the model-calculated values at MDLOL02 ( $x=-1.20 \mathrm{~mm}, x^{\prime}=-0.92 \mathrm{mrad}$ ).

Using the values calculated at MDL0L02 we continue tracking the beam through the 5D beam line to determine the additional deflection (adding or subtracting) required to produce the measured beam positions at IPM5D00 or IPM5D01,

$$
\begin{aligned}
& <5 D 00 \_C O R>_{\text {meas }}=0.27 \pm 0.01 \pm 0.25 \pm 0.50 \mathrm{~mm}=0.27 \pm 0.56 \mathrm{~mm} \\
& <5 D 01 \_C O R>_{\text {meas }}=3.96 \pm 0.29 \pm 0.25 \pm 0.50 \mathrm{~mm}=3.96 \pm 0.63 \mathrm{~mm}
\end{aligned}
$$



Fig. 5. The trajectories for 5 different momenta (4.5, 5.0, 5.5, 6.0, $6.5 \mathrm{MeV} / \mathrm{c}$ ) with initial conditions IPMOL02 ( $x=+0.07 \mathrm{~mm}, x^{\prime}=+0.04 \mathrm{mrad}$ ) is shown.

The difference in deflection is attributed to the dipole bending the beam by an amount different than $+25.0^{\circ}$, that is, it is a correction to the ideal angular deflection. The results obtained by including the model value of MDLOL2 the following additional deflection for the two beam position monitors is determined,

$$
\begin{aligned}
& I P M 5 D 00 \Rightarrow \theta=1.43 \pm 0.90 \mathrm{mrad}=0.0819^{\circ} \pm 0.0516^{\circ} \\
& I P M 5 D 01 \Rightarrow \theta=1.30 \pm 0.28 \mathrm{mrad}=0.0745^{\circ} \pm 0.0160^{\circ}
\end{aligned}
$$

The uncertainty associated with IPM5D01 is smaller because this beam position monitor is further from the dipole magnet. The weighted average of the two measurements is finally,

$$
<\theta>=1.311 \pm 0.267 \mathrm{mrad}=0.0751^{\circ} \pm 0.015^{\circ} .
$$

This additional deflection computed suggests the dipole deflected the beam (independent of the orbit due to the initial conditions at IPMOL02, the stray field and the steering coils) by an amount $\langle\theta\rangle=25.0751^{\circ} \pm 0.015^{\circ}$.


Fig. 5. The trajectories for 5 different momenta (4.5, 5.0, $5.5,6.0,6.5 \mathrm{MeV} / \mathrm{c}$ ) with initial conditions MDLOLO2 ( $x=-1.20 \mathrm{~mm}, x^{\prime}=-0.92 \mathrm{mrad}$ ) is shown.

## 5 Run II Systematic Uncertainties

There are five contributions to the systematic uncertainty, summarized in Table 4.
Table 4. Error budget for beam momentum method is listed.

| Contribution | Value (\%) |
| :--- | ---: |
| TOSCA Model (Ref [4]) | 0.10 |
| Power Supply Calibration (5 mA) | 0.18 |
| Power Supply Regulation (1.5 mA) | 0.06 |
| Tracking Model Correction | 0.06 |
| Absolute map uncertainty | 0.10 |
| Total |  |

- TOSCA model - Ref [4] describes the model which provides the straight-line BL required to bend a beam of momentum P by $25.0^{\circ}$ and agrees with the magnet measure to $0.1 \%$. The magnet measurement is used as the basis for setting the magnet BL in the EPICS control system.
- Power Supply Calibration - The dipole is powered with a standard 10 Amp trim power supply with reported [5] absolute calibration better than 5 mA . For Run II the dipole was operated with currents ranging approximately from $2800-3240 \mathrm{~mA}$. If one considers only the trim power supply the contributing uncertainty is $0.18 \%$.
- Power Supply Drift - The power supply drift is specified at 1.5 mA over a 24 hour period which is of order the duration between the extrapolation measurement and the energy measurement.
- Tracking Model Correction - The model correction taking into account the stray magnetic field and steering coils suggests the average bend angle for the five measurements is $25.0751^{\circ} \pm 0.015^{\circ}$; the correction incurs an uncertainty of $0.06 \%$.
- Absolute magnetic field - A precision magnetic probe with accuracy $10^{-4}$ is mounted in the magnet gap to provide absolute calibration between the magnet mapping and the magnetic field as installed. A new field map using the spare DL magnet was generated taking into account the Earth's magnetic field. The resulting correction is applied in Table 2. During the course of this work an absolute magnetic field error of 7 G -cm was determined Ref [6]. The contribution of this error contributes most to the weakest bending field applied and thus contribute less than $7 \mathrm{G}-\mathrm{cm} / 7109 \mathrm{G}-\mathrm{cm}$, or $0.1 \%$ to the error budget.


## 6 Run II Momentum and Beam Energy Summary

The corrected beam momenta ( $P_{c}$ ), kinetic energy ( T ) and uncertainties are computed according to the following relationships and reported in Table 5,

$$
\begin{aligned}
& P_{C}=P_{T}(25.0 /<\theta>) \\
& \delta \mathrm{T} / \mathrm{T}=(1-1 / \gamma) \delta P_{C} / P_{C}
\end{aligned}
$$

Table 5. Run II Beam Momentum and Kinetic Energy

| Condition |  | Momentum |  |  | Kinetic Energy |  |
| :---: | :---: | ---: | ---: | ---: | ---: | :---: |
| $R O 28$ | MDLOLO2 | TOSCA | Corrected |  | Final |  |
| $G S E T$ | $B L$ | $P_{T}$ | $P_{C}$ | $\delta P_{C}$ | $T$ | $\delta T$ |
| $M V / m$ | $G-c m$ | $M e V / c$ | $M e V / c$ | $M e V / c$ | $M e V$ | MeV |
| 3.350 | 7109.570 | 5.040 | 5.025 | 0.012 | 4.540 | 0.012 |
| 3.740 | 7384.340 | 5.234 | 5.219 | 0.013 | 4.733 | 0.012 |
| 4.120 | 7646.010 | 5.420 | 5.404 | 0.013 | 4.917 | 0.013 |
| 4.500 | 7927.590 | 5.619 | 5.603 | 0.013 | 5.115 | 0.013 |
| 4.890 | 8185.000 | 5.802 | 5.785 | 0.014 | 5.297 | 0.014 |

## 7 Run I Conditions

The beam energy for Run I was setup using the 2D beam line view screen. Later the beam was setup into the Mott polarimeter using the target view screen. Finally, the beam was setup into the 5D spectrometer using pulsed beam. We did not have a refined procedure in Run I that carefully took into account the beam position monitor survey offsets, the Earth's magnetic field or the systematic setting of the steering coils. We ultimately determine the beam momentum from four instances when dipole field integral is well known in different beam lines (2D, 3D, 5D), but with large uncertainty in deflection angle.

## 8 Run I TOSCA Model and Systematic Uncertainties

The four Run I measurements are summarized in Table 6, where the momentum is determined from TOSCA similar to Run II. A worst-case uncertainty is prescribed to the angular deflection assuming the beam is within $15 \mathrm{~mm}(+/-7.5 \mathrm{~mm})$ over 1 meter of travel spanning the dipole.

Table 6. Instances of Run I beam momentum setup or measurement.

| $\begin{aligned} & \text { 드 } \\ & \text { 흥 } \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { O} \\ & \hline \boldsymbol{U} \end{aligned}$ |  | $\begin{aligned} & \text { \% } \\ & \frac{0}{0} \\ & 0 \end{aligned}$ | 1 0 0 0 0 0 0 $\Sigma$ | $\begin{aligned} & \Sigma \\ & \mathbf{N} \\ & 0 \\ & 0 \\ & 0 \\ & \Sigma \end{aligned}$ | $\overparen{T}$ 0 0 0 0 | $\underset{\infty}{\infty}$ | $\begin{aligned} & \frac{0}{\lambda} \\ & \sum_{0}^{0} \end{aligned}$ | $\underset{\perp}{\underset{\Sigma}{\Sigma}}$ | $\underset{\substack{\text { S }}}{\substack{\text { ¢ }}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2D <br> Setup | 3317364 | $\begin{array}{r} \text { 01/16/16 } \\ \text { 06:00 PM } \end{array}$ | -30.0 | -9151.00 | -3.83087 | 5.470 | 0.027 | 0.146 | 4.983 | 0.163 |
| $\begin{aligned} & \text { 3D } \\ & \text { Mott } \end{aligned}$ | 3317368 | $\begin{array}{r} \text { 01/16/16 } \\ \text { 06:00 PM } \end{array}$ | -12.5 | -3890.92 | -1.65405 | 5.380 | 0.064 | 0.343 | 4.893 | 0.382 |
| $5 \mathrm{D}$ <br> Measure | 3318152 | $\begin{array}{r} \text { 01/19/15 } \\ 04: 30 \text { PM } \end{array}$ | 25.0 | 7266.00 | 2.96539 | 5.146 | 0.032 | 0.164 | 4.660 | 0.180 |
| 5D <br> Measure | 3318237 | $\begin{array}{r} \hline 01 / 20 / 15 \\ \text { 12:02 AM } \end{array}$ | 25.0 | 7343.00 | 2.99683 | 5.201 | 0.032 | 0.166 | 4.715 | 0.183 |

This worst-case uncertainty of $15 \mathrm{mrad}\left(0.86^{\circ}\right)$ is applied to both (un) deflected rays and dominates the remaining systematic uncertainty of $0.28 \%$ (see Table 2). Finally, the kinetic energy calculated for each case is shown in Table 6.

Table 7. Run I error budget for beam momentum.

| Contribution | Value (\%) |
| :--- | ---: |
| TOSCA Model (Ref [4]) | 0.10 |
| Power Supply Calibration (5 mA) | 0.18 |
| Power Supply Regulation (1.5 mA) | 0.06 |
| Absolute map uncertainty (worst) | 0.18 |
| Total |  |

## 9 Run I Energy Summary

The 4 cases are combined with weighted averaging (see Fig. 6) to determine the Run I beam energy of $4.806+/-0.097 \mathrm{MeV}$, with a relative uncertainty of $2.03 \%$.


## 10 Summary

The Run I/II beam energies and their corresponding uncertainties are summarized in Table 8.

Table 8. Summary of Run I/II beam energies.

| Condition | T <br> (MeV) | סT <br> (MeV) |
| :--- | :--- | :--- |
| Run I (R028=3.35) | 4.540 | 0.012 |
| Run I (R028=3.74) | 4.733 | 0.012 |
| Run I (R028=4.12) | 4.917 | 0.013 |
| Run I (R028=4.50) | 5.115 | 0.013 |
| Run I (R028=4.89) | 5.297 | 0.014 |
| Run II | 4.806 | 0.097 |

## 11 References

[1] Jefferson Lab Alignment Group, Data Transmittal \#L1456, May 252012 (note the information for magnets MQJ0L03 and MQJ0L03A appear reversed).
[2] Jefferson Lab Alignment Group, Data Transmittal \#L1462 Rev. 2, Jan 222013.
[3] Magnet Measurement Facility, /u/group/MagTest/DataBase/DL
[4] J. Benesch, "A detailed examination of the MDL field map and the TOSCA model of this " 5 MeV " dipole", JLab-TN-15-017.
[5] Private communication, Simon Wood testing 10A trim in IN02B26-9 (MDL0L02), July 2013.
[6] Private communication, R. Suleiman testing MDL0L02, March 2016.

## Appendix A

The sequence of steps for each measurement is described here:

1. Setup beam to Faraday Cup \#2
2. Quad center and update .SOF for IPM0L02, IPM0L03, IPM5D00, IPM5D01
3. Set MDL0L02 to zero and cycle
4. Set magnets on 2D and 3D lines to zero and cycle
5. Set skew and normal quads from IPM0L02 to IPM0L03 to zero and cycle
6. (Optional) Zero beam positions at IPM0L02 and IPM0L03
7. Record undeflected orbit conditions:
a. MDLOL02
b. beam positions IPM0L02/IPM0L03
c. all corrector settings in 0L line
8. Set steering coils (stray fields) between MDL0L02 and IPM0L03 to zero
9. Set MDL0L02 to deflect beam to IPM5D00 and IPM5D01 so both within 5 mm .
10. Cycle MDL0L02 and iterate with step \#9 until satisfied
11. Record deflected orbit conditions:
a. MDLOL02
b. beam positions IPM5D00/IPM5D01
c. all corrector settings in 5D line

## Appendix B - Stray Magnetic Field 0L Region

The stray magnetic field between ITV0L02 and IFY0L03 was surveyed between Dec. 22,2015 and Jan. 12, 2016. With magnets unpowered and degaussed the vertical component of stray magnetic field was measured every 5 cm both above and below the beam pipe and averaged. Additionally the upstream (US) and downstream (DS) location of beam line elements and regions of beam line that are covered in magnetic shielding were measured with an uncertainty of 0.5 cm in a relative coordinate system.

The component and shielding positions are summarized first followed by magnetic field measurement results. The magnetic field measurement results are shown for Run II and following additional magnetic shielding added on January 12, 2016. The differences are noted in red.

| Element | US (cm) | DS (cm) |
| :--- | ---: | ---: |
| ITVOLO2/VIPOLO2 | 1 | 1 |
| MQJOLO2 | 22 | 33 |
| IPMOLO2 | 47 | 67 |
| MQSOLO2 | 54 | 67 |
| MHBOLO2 | 73 | 80 |
| MQJOLO2A | 83 | 94 |
| IBCOLO2 | 130 | 145 |
| MDLOLO2 | 199 | 209 |
| MHBOLO2A | 244 | 251 |
| MQSOLO2B | 398 | 410 |
| MHBOLO2B | 413 | 419 |
| MHEOLO3V | 513 | 525 |
| MHEOLO3H | 527 | 540 |
| MBHOLO3 | 547 | 557 |
| ITVOLO3/VIPOLO3 | 584 | 584 |
| MQUOLO3A | 602 | 613 |
| IPMOLO3 | 629 | 649 |
| MQSOLO3 | 636 | 649 |
| MQJOLO3 | 665 | 676 |
| IHAOLO3 | 695 | 695 |
| IFYOLO3 | 732 | 732 |


| Mu-Metal | US (cm) | DS (cm) |
| :---: | ---: | ---: |
| 1 | 107 | 112 |
| 2 | 116 | 224 |
| 3 | 148 | 188 |
| 4 | 260 | 274 |
| 5 | 277 | 289 |
| 6 | 305 | 397 |
| 7 | 423 | 471 |
| 8 | 477 | 499 |


| Z (cm) | Run II |  |  | After Jan 12, 2016 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} B y_{-} T \\ (G) \end{gathered}$ | $\begin{gathered} B y_{-} B \\ (G) \end{gathered}$ | <By> (G) | $\begin{gathered} B y_{-} T \\ (G) \end{gathered}$ | $\begin{gathered} B y_{-} B \\ (G) \end{gathered}$ | <By> <br> (G) |
| 0 | 1.1 | 4.0 | 2.6 | 1.0 | 0.9 | 1.0 |
| 5 | 1.1 | -0.5 | 0.3 | 1.0 | 0.8 | 0.9 |
| 10 | 2.0 | 3.0 | 2.5 | 0.9 | 0.9 | 0.9 |
| 15 | 2.2 | 3.0 | 2.6 | 0.7 | 0.7 | 0.7 |
| 20 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 25 | 0.5 | 0.3 | 0.4 | 0.5 | 0.3 | 0.4 |
| 30 | 0.2 | 0.5 | 0.4 | 0.2 | 0.5 | 0.4 |
| 35 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 |
| 40 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 45 | 0.9 | 1.1 | 1.0 | 0.9 | 1.1 | 1.0 |
| 50 | 1.1 | 1.2 | 1.2 | 1.1 | 1.2 | 1.2 |
| 55 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 |
| 60 | 1.0 | 1.1 | 1.1 | 1.0 | 1.1 | 1.1 |
| 65 | 1.3 | 1.2 | 1.3 | 1.3 | 1.2 | 1.3 |
| 70 | 1.1 | 1.2 | 1.2 | 1.1 | 1.2 | 1.2 |
| 75 | 1.1 | 1.0 | 1.1 | 1.1 | 1.0 | 1.1 |
| 80 | 0.5 | 0.6 | 0.6 | 0.5 | 0.6 | 0.6 |
| 85 | 0.2 | 0.3 | 0.3 | 0.2 | 0.3 | 0.3 |
| 90 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| 95 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 100 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 |
| 105 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 110 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 |
| 115 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 120 | 0.9 | 1.1 | 1.0 | 0.9 | 1.1 | 1.0 |
| 125 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 130 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 135 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 140 | 0.9 | 0.8 | 0.9 | 0.9 | 0.8 | 0.9 |
| 145 | 0.9 | 0.8 | 0.9 | 0.9 | 0.8 | 0.9 |
| 150 | 1.0 | 0.9 | 1.0 | 1.0 | 0.9 | 1.0 |
| 155 | 1.1 | 0.9 | 1.0 | 1.1 | 0.9 | 1.0 |
| 160 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 165 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 |
| 170 | 1.0 | 0.9 | 1.0 | 1.0 | 0.9 | 1.0 |
| 175 | 1.0 | 1.1 | 1.1 | 1.0 | 1.1 | 1.1 |
| 180 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 185 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 190 | 0.9 | 1.0 | 1.0 | 0.9 | 1.0 | 1.0 |
| 195 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 200 | 0.7 | 0.9 | 0.8 | 0.7 | 0.9 | 0.8 |


| 205 | 0.8 | 0.9 | 0.9 | 0.8 | 0.9 | 0.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 210 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 |
| 215 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 220 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 225 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 230 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 235 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 240 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 245 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 250 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 255 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 260 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 265 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 270 | 0.9 | 1.0 | 1.0 | 0.9 | 1.0 | 1.0 |
| 275 | 0.9 | 1.1 | 1.0 | 0.9 | 1.1 | 1.0 |
| 280 | 1.0 | 1.1 | 1.1 | 1.0 | 1.1 | 1.1 |
| 285 | 1.0 | 1.2 | 1.1 | 1.0 | 1.2 | 1.1 |
| 290 | 1.2 | 1.1 | 1.2 | 1.2 | 1.1 | 1.2 |
| 295 | 1.7 | 1.0 | 1.4 | 1.7 | 1.0 | 1.4 |
| 300 | 1.2 | 1.1 | 1.2 | 1.2 | 1.1 | 1.2 |
| 305 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 310 | 0.9 | 1.0 | 1.0 | 0.9 | 1.0 | 1.0 |
| 315 | 0.8 | 0.9 | 0.9 | 0.8 | 0.9 | 0.9 |
| 320 | 0.9 | 0.8 | 0.9 | 0.9 | 0.8 | 0.9 |
| 325 | 0.9 | 0.8 | 0.9 | 0.9 | 0.8 | 0.9 |
| 330 | 0.9 | 0.7 | 0.8 | 0.9 | 0.7 | 0.8 |
| 335 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 340 | 0.9 | 0.8 | 0.9 | 0.9 | 0.8 | 0.9 |
| 345 | 0.8 | 0.9 | 0.9 | 0.8 | 0.9 | 0.9 |
| 350 | 0.9 | 1.0 | 1.0 | 0.9 | 1.0 | 1.0 |
| 355 | 0.8 | 0.9 | 0.9 | 0.8 | 0.9 | 0.9 |
| 360 | 0.6 | 0.8 | 0.7 | 0.6 | 0.8 | 0.7 |
| 365 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 |
| 370 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| 375 | 0.7 | 0.8 | 0.8 | 0.7 | 0.8 | 0.8 |
| 380 | 0.7 | 0.8 | 0.8 | 0.7 | 0.8 | 0.8 |
| 385 | 0.7 | 0.8 | 0.8 | 0.7 | 0.8 | 0.8 |
| 390 | 0.8 | 0.7 | 0.8 | 0.8 | 0.7 | 0.8 |
| 395 | 0.9 | 0.7 | 0.8 | 0.9 | 0.7 | 0.8 |
| 400 | 0.9 | 0.6 | 0.8 | 0.9 | 0.6 | 0.8 |
| 405 | 0.9 | 0.7 | 0.8 | 0.9 | 0.7 | 0.8 |


| 410 | 0.7 | 0.5 | 0.6 | 0.7 | 0.5 | 0.6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 415 | 0.9 | 0.7 | 0.8 | 0.9 | 0.7 | 0.8 |
| 420 | 0.8 | 0.7 | 0.8 | 0.8 | 0.7 | 0.8 |
| 425 | 0.9 | 0.7 | 0.8 | 0.9 | 0.7 | 0.8 |
| 430 | 0.8 | 0.7 | 0.8 | 0.8 | 0.7 | 0.8 |
| 435 | 0.9 | 0.7 | 0.8 | 0.9 | 0.7 | 0.8 |
| 440 | 0.9 | 0.7 | 0.8 | 0.9 | 0.7 | 0.8 |
| 445 | 0.7 | 0.5 | 0.6 | 0.7 | 0.5 | 0.6 |
| 450 | 0.7 | 0.6 | 0.7 | 0.7 | 0.6 | 0.7 |
| 455 | 0.6 | 0.5 | 0.6 | 0.6 | 0.5 | 0.6 |
| 460 | 0.7 | 0.5 | 0.6 | 0.7 | 0.5 | 0.6 |
| 465 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| 470 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| 475 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| 480 | 0.7 | 0.6 | 0.7 | 0.7 | 0.6 | 0.7 |
| 485 | 0.7 | 0.6 | 0.7 | 0.7 | 0.6 | 0.7 |
| 490 | 0.7 | 0.6 | 0.7 | 0.7 | 0.6 | 0.7 |
| 495 | 0.7 | 0.6 | 0.7 | 0.7 | 0.6 | 0.7 |
| 500 | 0.7 | 0.6 | 0.7 | 0.7 | 0.6 | 0.7 |
| 505 | 0.7 | 0.6 | 0.7 | 0.7 | 0.6 | 0.7 |
| 510 | 0.7 | 0.6 | 0.7 | 0.7 | 0.6 | 0.7 |
| 515 | 0.7 | 0.6 | 0.7 | 0.7 | 0.6 | 0.7 |
| 520 | 0.7 | 0.6 | 0.7 | 0.7 | 0.6 | 0.7 |
| 525 | 0.7 | 0.6 | 0.7 | 0.7 | 0.6 | 0.7 |
| 530 | 0.7 | 0.6 | 0.7 | 0.7 | 0.6 | 0.7 |
| 535 | 0.7 | 0.6 | 0.7 | 0.7 | 0.6 | 0.7 |
| 540 | 0.9 | 0.7 | 0.8 | 0.9 | 0.7 | 0.8 |
| 545 | 0.8 | 0.6 | 0.7 | 0.8 | 0.6 | 0.7 |
| 550 | 0.9 | 0.7 | 0.8 | 0.9 | 0.7 | 0.8 |
| 555 | 1.0 | 0.9 | 1.0 | 0.8 | 0.6 | 0.7 |
| 560 | 1.3 | 1.2 | 1.3 | 0.8 | 0.6 | 0.7 |
| 565 | 1.7 | 3.2 | 2.5 | 0.5 | 0.5 | 0.5 |
| 570 | 1.8 | 4.2 | 3.0 | 0.5 | 0.5 | 0.5 |
| 575 | 1.5 | 4.6 | 3.1 | 0.5 | 0.7 | 0.6 |
| 580 | 1.0 | -5.3 | -2.2 | 0.5 | 0.5 | 0.5 |
| 585 | 0.6 | 3.9 | 2.3 | 0.5 | 0.6 | 0.6 |
| 590 | 0.1 | 3.6 | 1.9 | 0.6 | 0.9 | 0.8 |
| 595 | -0.3 | -2.0 | -1.2 | 0.5 | 0.6 | 0.6 |
| 600 | -0.2 | -0.6 | -0.4 | 0.4 | 0.4 | 0.4 |
| 605 | 0.0 | 0.3 | 0.2 | 0.0 | 0.3 | 0.2 |
| 610 | 0.3 | 0.2 | 0.3 | 0.3 | 0.2 | 0.3 |


| 615 | 0.3 | 0.5 | 0.4 | 0.3 | 0.5 | 0.4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 620 | 0.6 | 0.5 | 0.6 | 0.6 | 0.5 | 0.6 |
| 625 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 |
| 630 | 0.8 | 0.7 | 0.8 | 0.8 | 0.7 | 0.8 |
| 635 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 |
| 640 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 |
| 645 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 |
| 650 | 0.8 | 0.9 | 0.9 | 0.8 | 0.9 | 0.9 |
| 655 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 |
| 660 | 0.6 | 0.5 | 0.6 | 0.6 | 0.5 | 0.6 |
| 665 | 0.3 | 0.2 | 0.3 | 0.3 | 0.2 | 0.3 |
| 670 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |
| 675 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 680 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| 685 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| 690 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 695 | 0.7 | 0.5 | 0.6 | 0.7 | 0.5 | 0.6 |
| 700 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 705 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 710 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 715 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 720 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 725 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 730 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 735 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
|  |  |  |  |  |  |  |

## Appendix C - Haimson Steering Magnet Calibration

In addition to the spectrometer dipole magnet two sizes (BH and HB) of Haimsonstyle steering coils were used to make point-wise corrections to compensate for the extended stray magnetic fields. The magnetic field profiles for both styles and both planes (horizontal or vertical) were measured at the Magnet Measurement Facility. Results for the four profiles are summarized in Table 2. A probe uncertainty of 0.05 Gauss for the 100-point profile results in an uncertainty of $2 \mathrm{G}-\mathrm{m}$.

Table 2. The standard configuration at CEBAF is the inner coils produce horizontal $(\mathrm{H})$ deflection and outer coils produce vertical (V) deflection.

| Haimson Part No. | JLab Type | Integrated <br> Dipole Field <br> $(G-\mathrm{cm} / \mathrm{A})$ | Integrated Dipole <br> Field Uncertainty <br> $(G-\mathrm{cm})$ |
| :---: | :--- | :---: | :---: |
| 334 | BH-Horizontal | 179 | 2 |
| 335 | BH-Vertical | 143 | 2 |
| 825 | HB-Horizontal | 173 | 2 |
| 826 | HB-Vertical | 148 | 2 |

## Appendix D- Stray Magnetic Field 5D Region

The stray magnetic field between MDL0L02 and IDL5D01 was surveyed during December 21-23, 2015. With magnets unpowered and degaussed the vertical component of stray magnetic field was measured every 5 cm both above and below the beam pipe and averaged. Additionally the upstream (US) and downstream (DS) location of beam line elements and regions of beam line that are covered in magnetic shielding were measured with an uncertainty of 0.5 cm in a relative coordinate system.

The component and shielding positions are summarized first followed by magnetic field measurement results; note the center of dipole (MDL0L02) corresponds to $S=20 \mathrm{~cm}$. The magnetic field measurement results are shown for Run II.

| Element Name | US (cm) | DS (cm) |
| :--- | ---: | ---: |
| MDLOLO2 | 20 | 20 |
| MBH5D00 | 37 | 47 |
| ITV5D00 | 62 | 62 |
| MQD5DD00 | 77.5 | 99.5 |
| IPM5D00 | 102 | 116 |
| MBH5D00A | 104 | 115 |
| MQD5D01 | 217.5 | 239.5 |
| IPM5D01 | 242 | 256 |
| MBH5D01 | 244 | 254 |
| IFY5D01/ITV5D01 | 281 | 281 |
| MBH5D01A | 304 | 314 |
| IDL5D01 | 340 | 349 |


| Mu-Metal | US (cm) | DS (cm) |
| ---: | ---: | ---: |
| 1 | 118 | 126 |
| 2 | 140 | 174 |
| 3 | 175.5 | 206.5 |


| $S$ (cm) | Run II |  |  |
| :---: | :---: | :---: | :---: |
|  | $\begin{gathered} B y_{-} T \\ (G) \end{gathered}$ | $B y_{-} B$ <br> (G) | <By> <br> (G) |
| 0 | 0.40 | 0.40 | 0.40 |
| 5 | 0.50 | 0.40 | 0.45 |
| 10 | 0.50 | 0.50 | 0.50 |
| 15 | 0.60 | 0.50 | 0.55 |
| 20 | 0.70 | 0.60 | 0.65 |
| 25 | 0.70 | 0.60 | 0.65 |
| 30 | 0.50 | 0.50 | 0.50 |
| 35 | 0.40 | 0.40 | 0.40 |
| 40 | 0.50 | 0.50 | 0.50 |
| 45 | 0.50 | 0.50 | 0.50 |


| 50 | 0.50 | 0.50 | 0.50 |
| :---: | :---: | :---: | :---: |
| 55 | 0.50 | 0.50 | 0.50 |
| 60 | 0.10 | 0.20 | 0.15 |
| 65 | 0.10 | 0.10 | 0.10 |
| 70 | -0.10 | -0.10 | -0.10 |
| 75 | 0.00 | 0.00 | 0.00 |
| 80 | 0.30 | 0.40 | 0.35 |
| 85 | 0.50 | 0.50 | 0.50 |
| 90 | 0.60 | 0.50 | 0.55 |
| 95 | 0.50 | 0.50 | 0.50 |
| 100 | 0.50 | 0.60 | 0.55 |
| 105 | 0.50 | 0.60 | 0.55 |
| 110 | 0.50 | 0.50 | 0.50 |
| 115 | 0.50 | 0.40 | 0.45 |
| 120 | 0.40 | 0.60 | 0.50 |
| 125 | 0.50 | 0.50 | 0.50 |
| 130 | 0.50 | 0.50 | 0.50 |
| 135 | 0.40 | 0.40 | 0.40 |
| 140 | 0.50 | 0.50 | 0.50 |
| 145 | 0.50 | 0.50 | 0.50 |
| 150 | 0.50 | 0.20 | 0.35 |
| 155 | 0.50 | 0.20 | 0.35 |
| 160 | 0.50 | 0.20 | 0.35 |
| 165 | 0.50 | 0.20 | 0.35 |
| 170 | 0.50 | 0.20 | 0.35 |
| 175 | 0.50 | 0.40 | 0.45 |
| 180 | 0.50 | 0.40 | 0.45 |
| 185 | 0.40 | 0.60 | 0.50 |
| 190 | 0.50 | 0.40 | 0.45 |
| 195 | 0.40 | 0.40 | 0.40 |
| 200 | -0.20 | 0.00 | -0.10 |
| 205 | -0.10 | -0.10 | -0.10 |
| 210 | 0.00 | 0.00 | 0.00 |
| 215 | 0.00 | 0.00 | 0.00 |
| 220 | 0.40 | 0.40 | 0.40 |
| 225 | 0.80 | 0.80 | 0.80 |
| 230 | 0.80 | 0.60 | 0.70 |
| 235 | 0.90 | 0.90 | 0.90 |
| 240 | 0.90 | 1.00 | 0.95 |
| 245 | 1.00 | 1.00 | 1.00 |
| 250 | 1.00 | 1.00 | 1.00 |


| 255 | 0.70 | 0.70 | 0.70 |
| ---: | ---: | ---: | ---: |
| 260 | 0.00 | 0.50 | 0.25 |
| 265 | -0.40 | 1.70 | 0.65 |
| 270 | -0.40 | 0.20 | -0.10 |
| 275 | 0.50 | 0.50 | 0.50 |
| 280 | 0.60 | 0.60 | 0.60 |
| 285 | 0.60 | 0.50 | 0.55 |
| 290 | 0.60 | 0.50 | 0.55 |
| 295 | 0.60 | 0.50 | 0.55 |
| 300 | 0.60 | 0.50 | 0.55 |
| 305 | 0.60 | 0.50 | 0.55 |
| 310 | 0.60 | 0.60 | 0.60 |
| 315 | 0.70 | 0.60 | 0.60 |
| 320 | 0.70 | 0.70 | 0.70 |
| 325 | 0.70 | 0.70 | 0.70 |
| 330 | 0.70 | 0.70 |  |

