e^+ Collection systems

Sami Habet

IJCLab.

JLab.

October 2022

Plan

Plan

QWT

- The QWT consist of two magnetic field region.
- The first solenoid has a strong magnetic field B_1 over a length L_1
- The second solenoid has a weaker magnetic field B_2 over a length L_2
- The phase space acceptance for a QWT is calculated from the global transfer matrix:

$$\begin{pmatrix} X \\ P_X \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} X_0 \\ P_{X_0} \end{pmatrix}$$

QWT

- From the previous matrix we can get the phase space transport over a QWT:

$$
\begin{aligned}
XX^* + \left(\frac{2}{eB_2}\right)^2 P_X P_X^* = & \left[\cos^2\chi_1 + \left(\frac{B_1}{B_2}\right)^2 \sin^2\chi_1\right] x_0 x_0^* \\
& + \left[\left(\frac{2}{eB_1}\right)^2 \sin^2\chi_1 + \left(\frac{2}{eB_2}\right)^2 \cos^2\chi_1\right] p_{x_0} p_{x_0}^* \\
& + \frac{2}{eB_1} \sin\chi_1 \cos\chi_1 \left[1 - \left(\frac{B_1}{B_2}\right)^2\right] \left(x_0^* p_{x_0} + x_0 p_{x_0}^*\right)
\end{aligned} \tag{1}
$$

- Where $X^* \equiv x + iy$ and $P_X^* \equiv p_x + ip_y$ and.

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Jefferson Lab

QWT

- Using $\chi_1 = \pi/2$, the previous formula is reduced to :

$$\left(\frac{eB_2}{2}\right)^2 XX^* + P_X P_X^* = \left(\frac{eB_2}{2}\right)^2 (x^2 + y^2) + (p_x^2 + p_y^2) = \mathrm{Cte}$$

- Which can be expressed in a cylindrical coordinate system as:

$$\left[\frac{B_1}{B_2}\right]^2 r_0^2 + \left[\frac{2}{eB_1}\right]^2 \left[P_{r_0}^2 + \frac{P_{\phi_0}}{r^2}\right] = \mathrm{Cte}$$

- The positron emitted at the converter with phase space coordinates (X_0, P_{X_0}) are transmitted only if :

$$XX^* \leq a^2$$

- Thus we can write:

$$\mathrm{Cte} - \left(\frac{2}{eB_2}\right)^2 P_X P_X^* \leq a^2$$

QWT

- At $P_{r_0}=P_{\phi_0}=0$ we can define the QWT radial acceptance:

$$r_0^{max} = \frac{B_2}{B_1} a$$

- Similarly, the radial momentum acceptance is defined by:

$$P_{r_0}^{max} = \frac{eB_1 a}{2}$$

- At a given momentum, The QWT volume acceptance is maximized:

$$\frac{dV(\chi_1)}{d\chi_1} = 0$$

- Which leads to:

$$p_m = \frac{eB_1 L_1}{n\pi}$$

QWT

- For $B_1 = 2.5\ T$ and $B_2 = 0.5\ T$ and $p_m = 60$ MeV/c, the optimal length is set at $L_1 = 25$ cm.
- The radial acceptance for an aperture radius $a = 3$ cm: :

$$r_0^{max} = \frac{B_2}{B_1}a \ = 1.5cm.$$

- The radial momentum acceptance is set at :

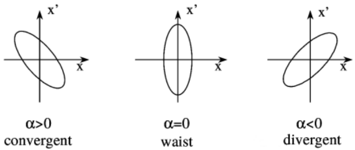
$$\theta_{r_0}^{max} = \frac{eB_1 a}{2p_m} = 0.18\ [rad] \tag{2}$$

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Jefferson Lab

QWT

- What about L_2 ?
- Our goal is to decrease the initial transverse momentum meaning we want to rotate the (x-x') phase space using $\pi/2$ rotation.



QWT

- The solenoid transfer matrix is defined as :

$$\begin{pmatrix} \cos^2(k\,l) & \frac{\sin(k\,l)\cos(k\,l)}{k} & \sin(k\,l)\cos(k\,l) & \frac{\sin^2(k\,l)}{k} \\ -k\sin(k\,l)\cos(k\,l) & \cos^2(k\,l) & -k\sin^2(k\,l) & \sin(k\,l)\cos(k\,l) \\ \sin(k\,l)\,(-\cos(k\,l)) & -\frac{\sin^2(k\,l)}{k} & \cos^2(k\,l) & \frac{\sin(k\,l)\cos(k\,l)}{k} \\ k\sin^2(k\,l) & \sin(k\,l)\,(-\cos(k\,l)) & -k\sin(k\,l)\cos(k\,l) & \cos^2(k\,l) \end{pmatrix}$$

- Where $k = \frac{B}{2B\rho}$
- Using the twiss matrix transformation : $\begin{bmatrix} \beta_{exit} \\ \alpha_{exit} \\ \gamma_{exit} \end{bmatrix} = M_{Twiss} \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix}$

QWT

- From the Solenoid matrix, the Twiss transport matrix is calculated:

$$\begin{bmatrix} \beta_{exit} \\ \alpha_{exit} \\ \gamma_{exit} \end{bmatrix} = \begin{pmatrix} -\frac{4\,\text{a0 BRo} \sin\left(\frac{B\,l}{2\,\text{BRo}}\right)\cos^3\left(\frac{B\,l}{2\,\text{BRo}}\right)}{B} + \frac{4\,\text{BRo}^2\,\text{G0} \sin^2\left(\frac{B\,l}{2\,\text{BRo}}\right)\cos^2\left(\frac{B\,l}{2\,\text{BRo}}\right)}{B^2} + \text{b0} \cos^4\left(\frac{B\,l}{2\,\text{BRo}}\right) \\ \text{a0}\left(\cos^4\left(\frac{B\,l}{2\,\text{BRo}}\right) - \sin^2\left(\frac{B\,l}{2\,\text{BRo}}\right)\cos^2\left(\frac{B\,l}{2\,\text{BRo}}\right)\right) + \frac{B\,\text{b0} \sin\left(\frac{B\,l}{2\,\text{BRo}}\right)\cos^3\left(\frac{B\,l}{2\,\text{BRo}}\right)}{2\,\text{BRo}} - \frac{2\,\text{BRo G0} \sin\left(\frac{B\,l}{2\,\text{BRo}}\right)\cos^3\left(\frac{B\,l}{2\,\text{BRo}}\right)}{B} \\ \frac{\text{a0}\,B \sin\left(\frac{B\,l}{2\,\text{BRo}}\right)\cos^3\left(\frac{B\,l}{2\,\text{BRo}}\right)}{\text{BRo}} + \frac{B^2\,\text{b0} \sin^2\left(\frac{B\,l}{2\,\text{BRo}}\right)\cos^2\left(\frac{B\,l}{2\,\text{BRo}}\right)}{4\,\text{BRo}^2} + \text{G0} \cos^4\left(\frac{B\,l}{2\,\text{BRo}}\right) \end{pmatrix}$$

- Where $L_2 = l$ is the length of the second solenoid.
- a_0, b_0 and $G0$ are thi initial Twiss parameters calculated at the exit of the 1^{st} solenoid.
- To get $\pi/2$ rotation, we want to get $\alpha_{exit} = 0$ and β_{exit} =Max
- From the β_{exit} formula, we get the maximum value at $l = 8.7$ m

Plan

Simulation : QWT initial parameters

- For the Polarized mode we have set the central momentum at 60MeV/c.
- The QWT parameters are
  - $B_1 = 2.5$ T
  - $L_1 = 0.25$ m
  - $B_2 = 0.5$ T
  - $l = L_2 = 8.7$ m
  - $r_0^{max} = 0.0015$ m
  - $\theta_0^{max} = 0.18$
- Only e^+ contained inside the accepted ellipse will be transmitted at the exit of the QWT.

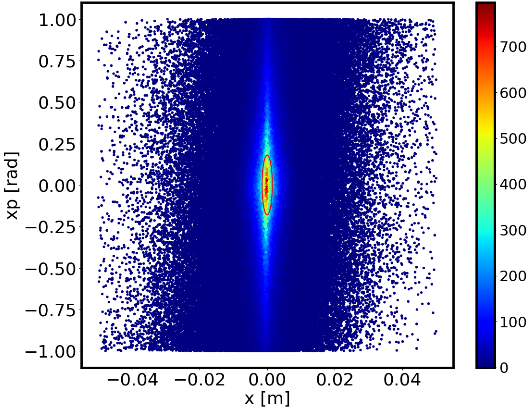
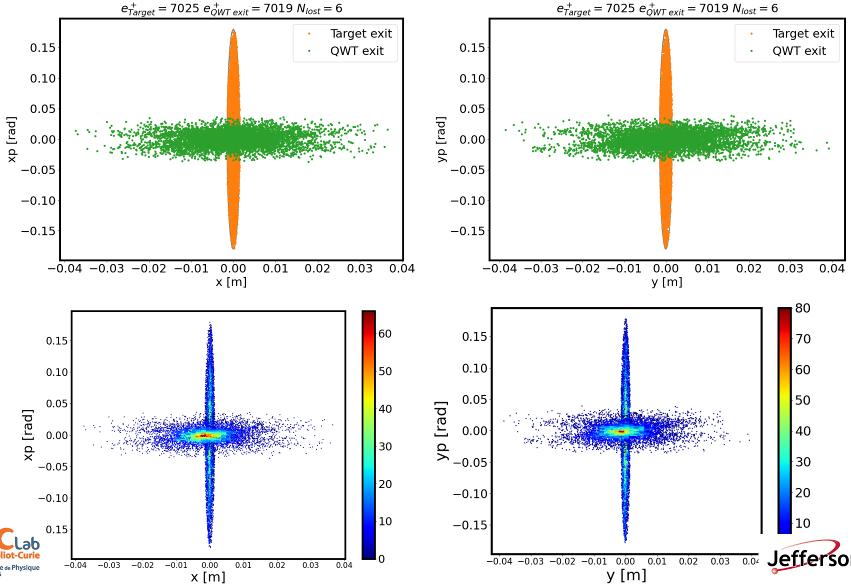


Figure: e^+ At the Target

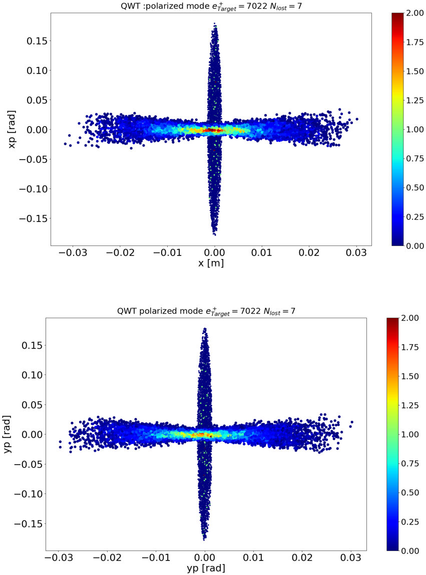
IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Jefferson Lab

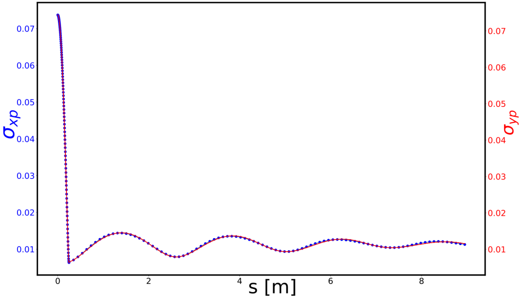
QWT : Transverse phase space rotation

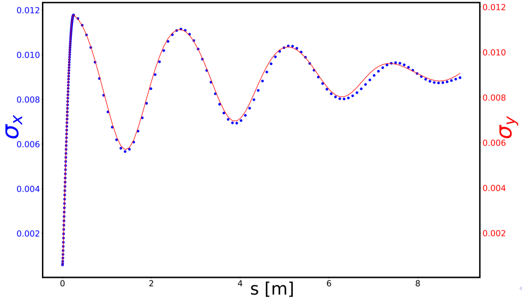


QWT: Transverse phase space rotation $L_2 = 2.45\ m$

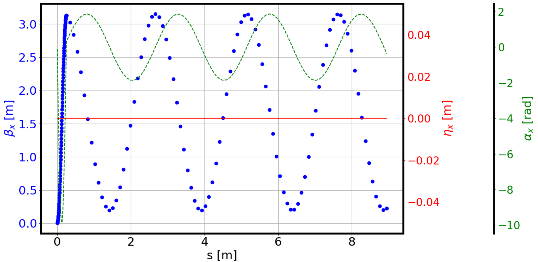


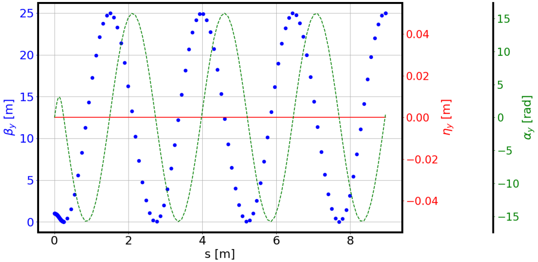
QWT : Sigmas evolution



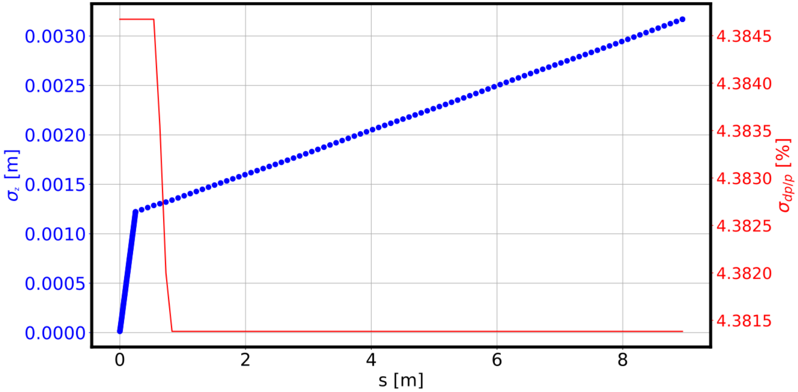


QWT : Twiss functions



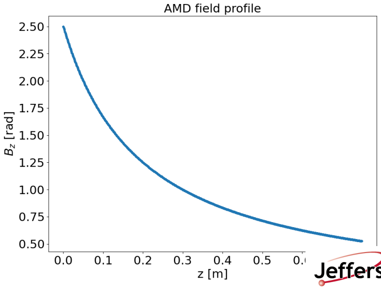


QWT : Longitudinal plane



AMD : initial parameters

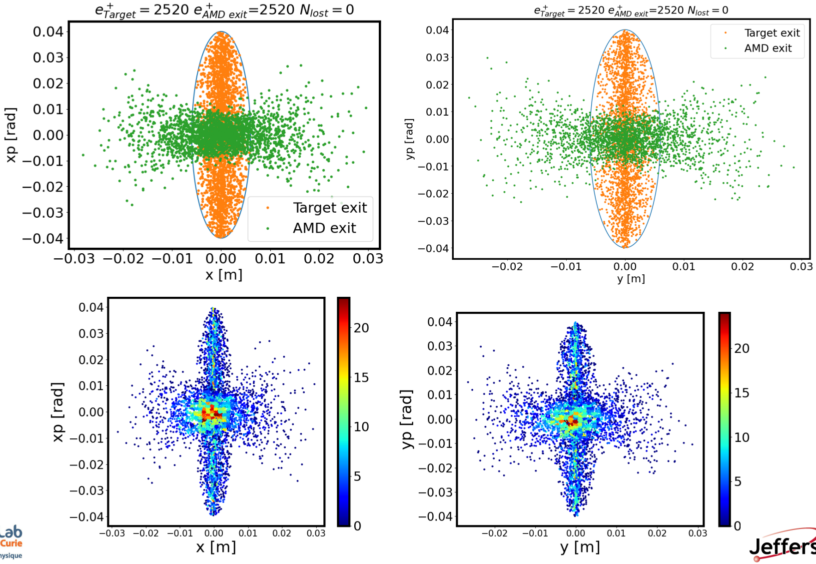
- Using the same formulas, we can define the acceptance space phase parameters for the AMD:
  - The radial acceptance : $r_0^{max} = \sqrt{\frac{B_2}{B_1}} a \; = \; 0.003 \; m.$
  - To get the transverse momentum acceptance, we let x_0 and $p_{y0} = 0$, then we get: $p_{x0}^{max} = \frac{1}{2} ea\sqrt{B_2 B_1} \; = \; 0.04 \; [rad]$
- To get a fair comparison, I will be using the same momentum cut used previously for the QWT case.
- AMD length $L_{AMD} = 0.9 \; m$
- $B_0 = 2.5 \; T$
- $B_2 = 0.5 \; T$



IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Jefferson Lab

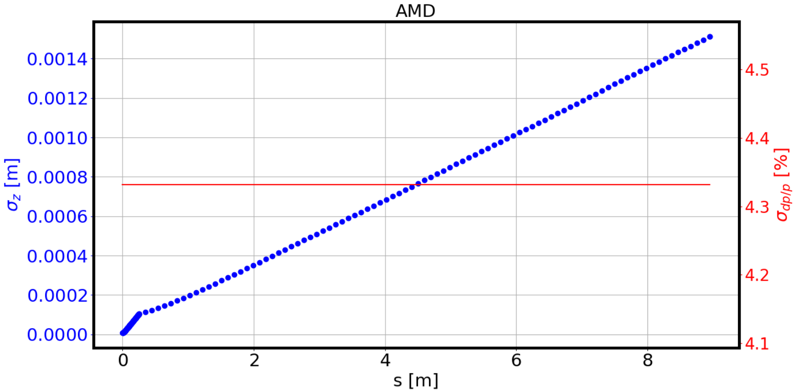
AMD : Transverse phase space rotation



IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Jefferson Lab

AMD : Longitudinal plane

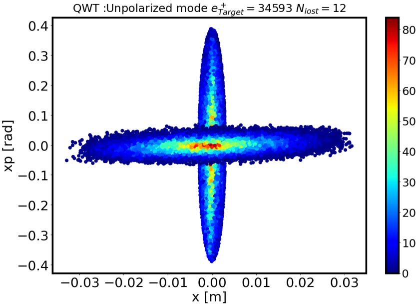


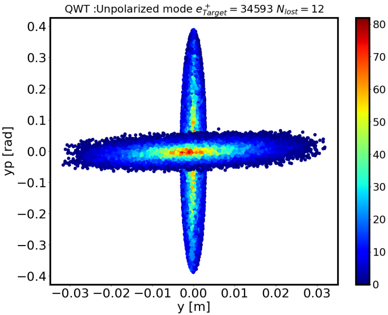
- The bunch length increase significatively in both cases : QWT and AMD

QWT : Central momentu at 20 MeV/c

- We shift the central momentum to 20 MeV/c, therefore the acceptances parametres will change too:
  - $$p_m = \frac{eB_1 L_1}{n\pi}$$
  - leading to $L_1 = 11\ cm$, $B_1 = 1.8\ T$, $B_2 = 0.2\ T$
  - The radial acceptance for the unpolarized mode : $r_0^{max} = 0.003\ m$
  - The angular acceptance : $\theta_0^{max} = 0.39\ [rad]$

QWT : Tranverse phase space rotation





Plan

QWT VS AMD

Parameters	QWT	AMD
Radial accetance r_0^{max} [m]	0.0015	0.003
Angular acceptance [rad]	0.18	0.04
p_{e^+} MeV/c	[57-63]	[57-63]
$1^{st} solenoidLength[m]$	0.25	0.75
$2^{nd} solenoidlength[m]$	8.7	8.4
Number of e^+	7025	7025
Yield e^+/e^-	0.001405	0.001405
Transmission %	99.91	100
Longitudinal σ_t [s]	$1.06\ 10^{-11}$	$4.66\ 10^{-12}$

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Jefferson Lab

Plan

1. QWT optimization
2. Simulation
   - polarized mode
   - Unpolarized mode
3. QWT VS AMD
4. Conclusion

Conclusion

- The radial acceptance is larger for the adiabatic device
- The angular acceptance is larger for the QWT
- The momentum acceptance is much larger for the adiabatic device. As a consequence, the accepted yield is higher
- The AMD is widely used with respect to high positron yield.
- The QWT may be interesting, if the yield is sufficient, to get a very clean beam, and to restrict the central momentum for easier transport.