Beamline Design for a Stern-Gerlach Deflection Experiment in CEBAF 1D Spectrometer Line

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1 Beamline Optics

This note describes a preliminary beamline design for detection and measurement of the Stern-Gerlach deflection of relativistic 0.5 MeV ($\gamma = 2$) polarized electrons in the CEBAF 1D Spectrometer beamline. Stern-Gerlach (S-G) deflection of higher energy electrons is also considered.

Beamline parameters for the proposed beamline are given in Table 1. The quadrupole layout is shown in Figure 1. By design the line is symmetric about the center at point C0, and

Table 1: Lattice parameters for the Stern-Gerlach beamline in the 1D Spectrometer beamline served by the CEBAF injector. Asterisks indicate quadrupoles whose strengths can be much weaker, and tailored to lead the beam gracefully to the beam dump. The assume kinetic energy is 500 KeV or, approximately $\gamma = 2$.

index	loc. cm	Loc. name	Quad name	Quad len. mm	Inv.foc.len. 1/m	dB_y/dx T/m	$\Delta\theta_{SG}$ 10^{-10} r
s0	0	B0					
s1	0.3	B1	qBC1	6	894.0	433.5	2.000
s2	2.8	B2	qBC2	6	51.76	25.1	0.1158
s3	10.3	B3	qBC3	6	-11.67	-5.66	-0.026
s4	17.8	C0					
s5	25.3	C1	qCD1	6	-11.67	-5.66	-0.026
s6	32.8	C2	qCD2	6	51.76	*	*
s7	35.3	C3	qCD3	6	894.0*	*	*
s8	35.6	C4					

the optics plots in the following figures respect this symmetry. In fact, the strengths of the quadrupoles beyond C0, especially the final quadrupole, qCD3, can be much weaker without affecting performance, depending on the distance to the beam dump. This is indicated by asterisks in the table. The only really strong quadrupole is qBC1. This quadrupole is taken to be identical to a permanent magnet quadrupole described in Table III of a paper by Li and Musumeci[1].

Optical properties of the beamline are shown in the following figures.

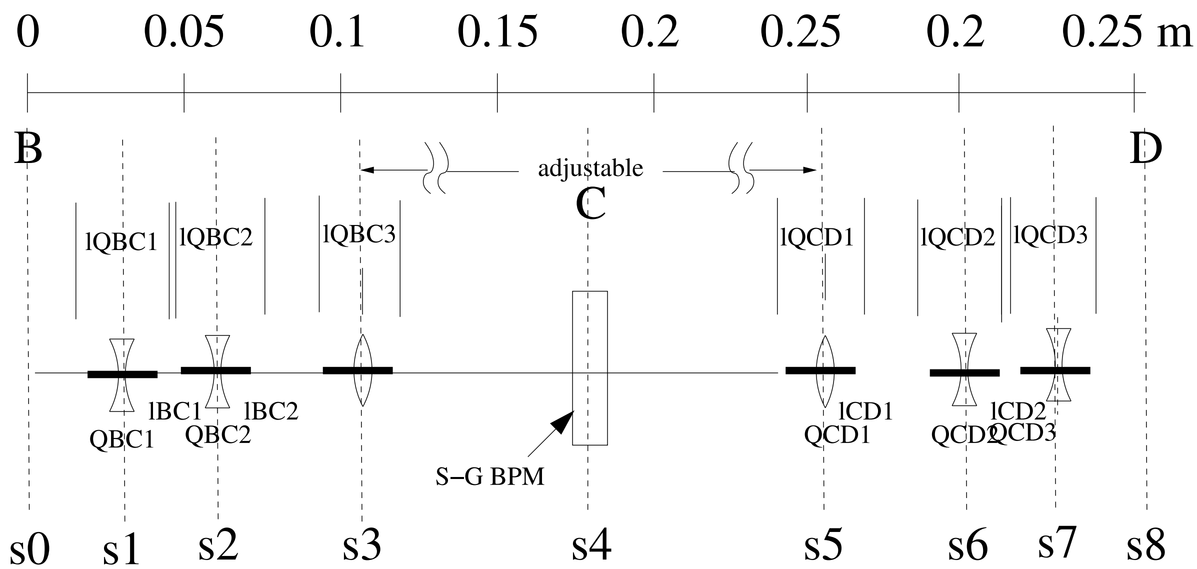

Figure 1: A six-quadrupole beam line for detection and measurement of Stern-Gerlach deflection of a polerized electron beam. The heavy black lines indicating quadrupole lengths are not-to-scale, except, of course, that they cannot overlap. The dimensions on the plot are approximately valid for a 500 KeV (kinetic energy) electron beam. But, by scaling all lengths and quadrupole focal lengths, the same design is also applicable to other energies, for example 5 MeV. In order for the S-G deflection to be purely vertical the quadrupoles have to be "skew", i.e. at 45° relative to "erect".

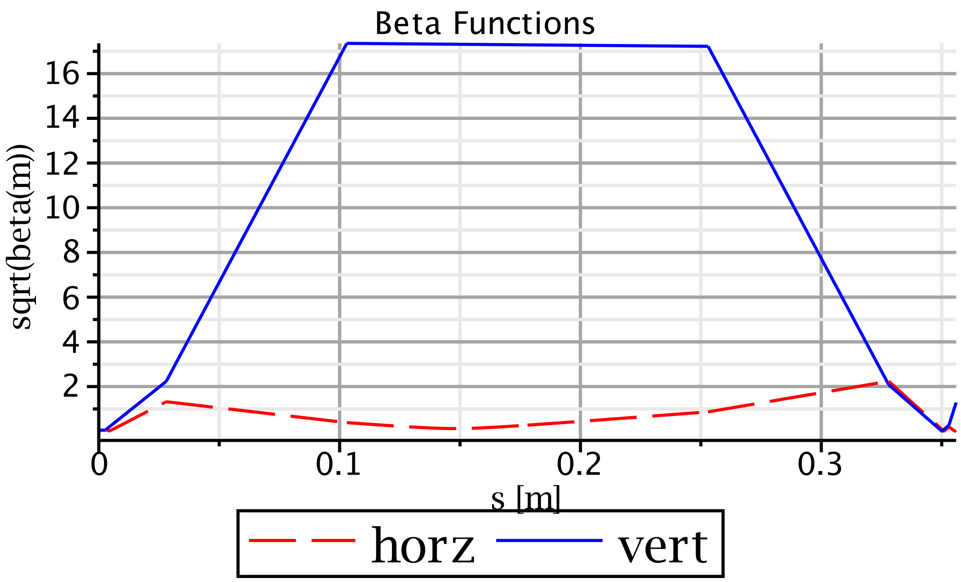

Figure 2: Vertical, $\beta_y(s)$, and horizontal, $\beta_x(s)$, beta functions for S-G detection and measurement in the CEBAF 1D Spectrometer Line.

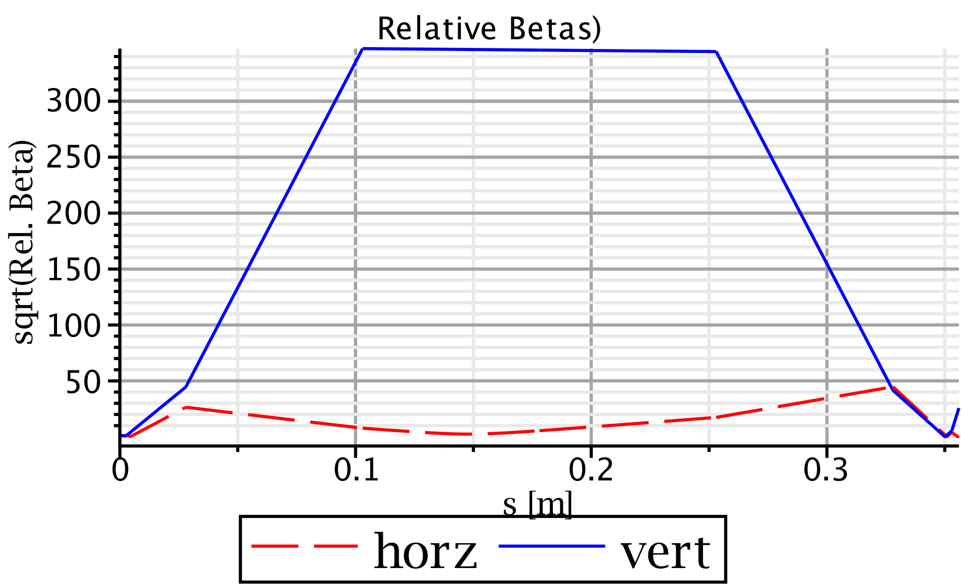

Figure 3: (Square root) beta function ratios $\sqrt{\beta_x(s)/\beta_x(s_0)}$ and $\sqrt{\beta_y(s)/\beta_y(s_0)}$. The initial beta functions, $\beta(s_0)$ are determined by the beam emittances (which vary inversely with the relativistic beam energy factor γ). The initial beta function values are therefore proportional to γ. This limits the degree to which the S-G signal can be enhanced by reducing γ, while limiting the transverse beam dimensions. As a result the maximum S-G displacement depends only weakly on γ.

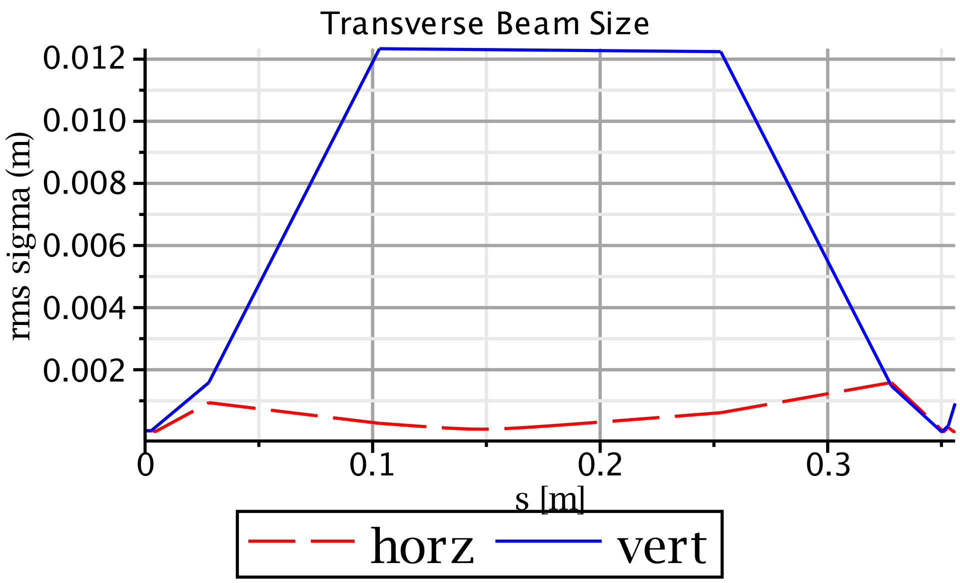

Figure 4: Transverse rms beam sizes as functions of longitudinal position s. For designs in this tech-note, the maximum rms beam size is constrained to be approximately $\sigma_y = 1\,\mathrm{cm}$.

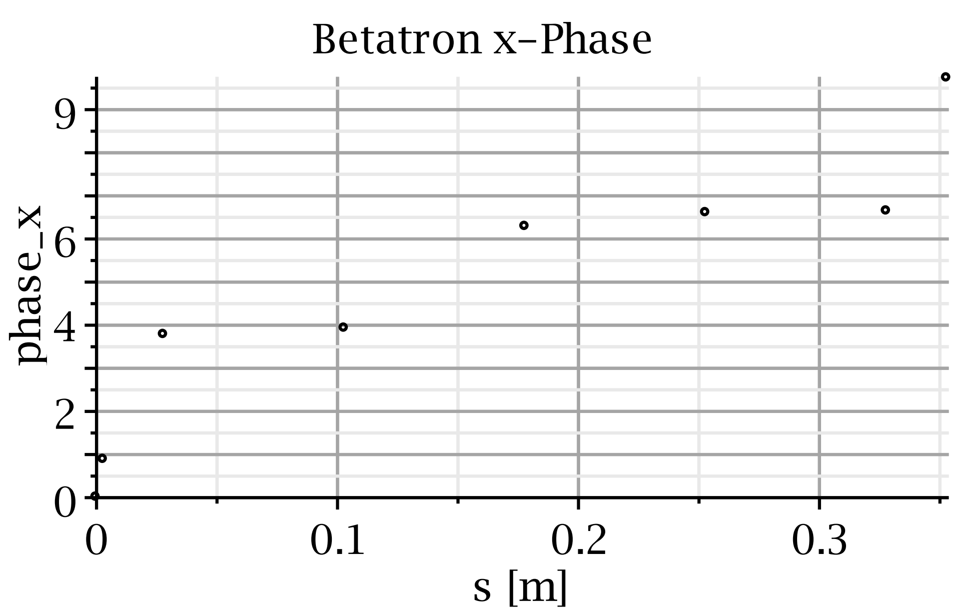

Figure 5: Horizontal betatron phase advance as function of longitudinal position s.

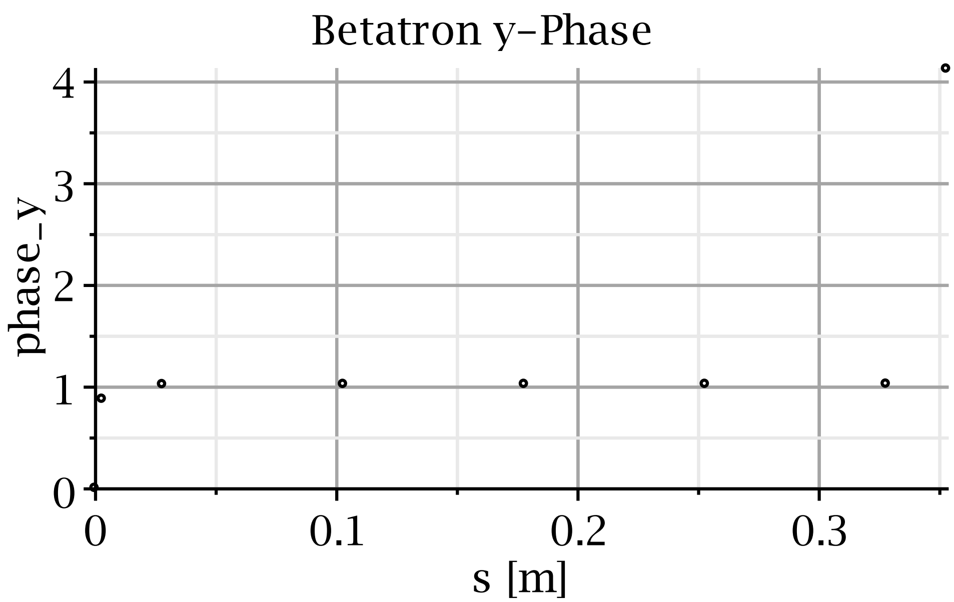

Figure 6: Vertical betatron phase advance $\psi_y(s)$, as function of longitudinal position s. Because $\psi_y(s)$ is essentially constant over the central, high-β region, the Stern-Gerlach displacement does not increase notably over this central region, in spite of the substantial angular deflection $\Delta\theta_y^{SG}$ caused by the (very strong) qB1 quadrupole. Whatever S-G displacement there is, is mostly already present at the qB3 quadrupole.

2 Calculated Stern-Gerlach Displacement

The ratio of Stern-Gerlach to electromagnetic force is determined by a ratio of coupling constants:

$$\frac{\mu_B/c}{e} = 1.930796 \times 10^{-13}\,\mathrm{m}, \tag{1}$$

where, except for anomalous magnetic moment and sign, Bohr magneton μ_B is the electron magnetic moment.

The Stern-Gerlach deflection in a quadrupole is strictly proportional to the inverse focal lengths of the quadrupole;

$$\boxed{\Delta\theta_y^{SG} = -\frac{\mu_x^*}{ec\beta}q_x, \quad \text{and} \quad \Delta\theta_y^{SG} = \frac{\mu_y^*}{ec\beta}q_y,} \tag{2}$$

These formulas are boxed to emphasize their universal applicability to all cases of polarized beams passing through quadrupoles. For all practical (electron beam) cases $\beta \approx 1$.

The S-G deflection at fixed magnet excitation is proportional to $1/\gamma$. Yet, superficially, these formulas show no *explicit* dependence on γ. This is only because the angular deflections are expressed in terms of quadrupole inverse focal lengths. For a given quadrupole at fixed quadrupole excitation, the inverse focal length scales as $1/\gamma$. Expressing the S-G deflection in terms of inverse focal lengths has the effect of "hiding" the $1/\gamma$ Stern-Gerlach deflection dependence, which comes from the beam stiffness.

μ_x^* and μ_y^* differ from the Bohr magnetron μ_B only by $\sin\theta$ and $\cos\theta$ factors respectively For a single quadrupole, the Stern-Gerlach-induced angular deflection is

$$\Delta\theta_y^{SG} = (1.93 \times 10^{-13}\,\mathrm{m})\, q_y. \tag{3}$$

The transverse displacement Δy_j at downstream location "j" caused by angular displacement $\Delta\theta_{y,i}$ at upstream location "i" is given by

$$\Delta_{y,j} = q_y\,(1.93 \times 10^{-13}\,\mathrm{m})\,\sqrt{\beta_{y,j}\beta_{y,i}}\,\sin(\psi_{y,j} - \psi_{y,i}). \tag{4}$$

where $\psi_{y,j} - \psi_{y,i}$ is the vertical betatron phase advance from "i" to "j". Currently the conditions at starting point $s0$ are not well known. I tentatively assume, for both planes,

$$\begin{aligned} \epsilon &= \frac{1.0 \times 10^{-6}\,\mathrm{m}}{\gamma}, \\ \sigma^{B0} &= \frac{50 \times 10^{-6}\,\mu\mathrm{m}}{\sqrt{\gamma}}, \\ \beta^{B0} &= \frac{\sigma^2}{\epsilon}, \quad \alpha^{B0} = 0, \quad \psi^{B0} = 0. \end{aligned} \tag{5}$$

```
epsilon_x := 1.0e-6/gamma;
epsilon_y := 1.0e-6/gamma;
sigmaB0x := 50e-6/sqrt(gamma);
sigmaB0y := 50e-6/sqrt(gamma);
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```
betB0x := sigmaB0x^2/epsilon_x;
betB0y := sigmaB0y^2/epsilon_y;
alfB0x := 0.0:
alfB0y := 0.0:
phB0x := 0.0:
phB0y := 0.0:
```

The resulting S-G deflections are shown in Table 2. As explained earlier the S-G displacement

Table 2: .Stern-Gerlach displacements, measured in Å units, at points along the beamline, for kinetic energy $K_e = 500\,\mathrm{KeV}$.

source	displ. at B2	displ. at B3	displ. at C0	displ. at C1
qBC1		0.3942	0.3938	0.3925
qBC2		0.0088	0.0098	0.1085
qBC3		0	-0.0020	-0.0039
total		0.4029	0.4012	0.3994

is essentially constant over the central region. Stretching the central region, even by a large amount, has little effect on the S-G displacement. This would, however allow the S-G detection BPM's to be long, to increase their sensitivity, or even multiple, to lower the noise floor.

3 Energy Dependence of Stern-Gerlach Deflection

To investigate the dependence on electron energy, the kinetic energy was increased by a factor of 9, yielding $\gamma = 9.8$. All longitudinal positions and quad lengths were tripled. This left the tuned-up quadrupole magnetic field gradients approximately constant, because the quadrupole strengths were approximately tripled. These changes left the beamline still fairly well tuned up. Just one plot, namely Figure 7, illustrates the outcome of these few changes.

The resulting parameters are given in Table 3. The most important changes were that the rms transverse size *increased* from 12 to 14 mm and the maximum field gradient *decreased* from 433 T/m to 275 T/m. Meanwhile the Stern-Gerlach deflection decreased by only 12 percent. Because of the reduced beam emittances with $\gamma = 10$, the quadrupole bore could be decreased, allowing the field gradient to increase, say by a factor of two. After re-optimization the S-G signal would surely be found to be approximately independent of energy (contrary to earlier discussion).

Table 3: Lattice parameters for the Stern-Gerlach beamline in the 1D Spectrometer beamline served by the CEBAF injector, assuming $\gamma = 10$, or approximately, electron energy $\mathcal{E}_e = 5\,\mathrm{MeV}$.

index	loc. cm	Loc. name	Quad name	Quad len. mm	Inv.foc.len. 1/m	dB_y/dx T/m	$\Delta\theta_{SG}$ 10^{-10} r
s0	0	B0					
s1	0.9	B1	qBC1	18	298.0	275.3	0.5784
s2	8.4	B2	qBC2	18	17.25	15.9	0.0334
s3	30.9	B3	qBC3	18	-3.89	-3.59	-0.026
s4	53.4	C0					
s5	75.9	C1	qCD1	18	-3.89	-3.59	-0.026
s6	98.4	C2	qCD2	18	17.25	*	*
s7	105.9	C3	qCD3	19	*	*	*
s8	106.8	C4					

Table 4: .Stern-Gerlach displacements, measured in Å units, at points along the beamline, for kinetic energy $K_e = 4.5\,\mathrm{MeV}$.

source	displ. at B2	displ. at B3	displ. at C0	displ. at C1
qBC1		0.3419	0.3412	0.3405
qBC2		0.0075	0.0085	0.0094
qBC3		0	-0.0017	-0.0034
total		0.3495	0.3480	0.3464

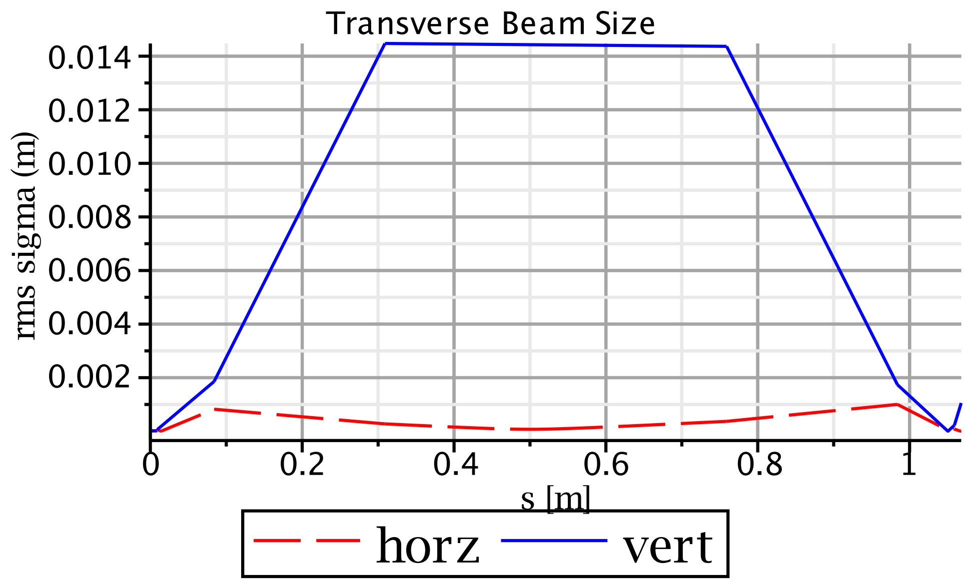

Figure 7: Transverse rms beam sizes as functions of longitudinal position s for $K_e = 4.5\,\mathrm{MeV}$, $\gamma = 9.8$.

4 Uncertainty and Conclusions

The greatest uncertainty in the calculation concerns Eqs. (5), and the corresponding lines of code listed below these equations. To magnify the Stern-Gerlach deflection one wants the vertically-deflecting quadrupole to be strong. This automatically causes β_y to increase, which increases the beam height. Accepting the limitation that the rms beam height cannot exceed approximately 10 mm, this limits the downstream S-G displacement. In detail this limitation depends on the initial beta function/emittance/beam height assumptions. The entries, on my part, to Eqs. (5) were little more than guesses. Discussion of this aspect of the calculation will be appreciated.

The Stern-Gerlach energy dependence has been much discussed in the past. The importance of the transverse beam size has not previously, as far as I know, been properly appreciated in those discussions. It is now my opinion that, as long as the transverse beam dimensions are dominated by adiabatic damping (with increased energy) that the achievable S-G-induced betatron beam deflection is more or less independent of energy.

As far as the proof-of-principle test at CEBAF, the most convenient energy appears to be at 500 KeV, but this is for reasons of economy and accessibility, not because the S-G signal is strongest at low energy.

Also somewhat surprising is that, for the assumed electron beam parameters, the beamline optics cannot be designed to enable the S-G deflection to much exceed about one Angstrom. As I have argued previously and repeatedly, especially with Reza's suggested toggling polarization beam preparation, it should not be hard to detect such a small betatron amplitude.

References

[1] R. Li and P. Musumeci, *Single-Shot MeV Transmission Electron Microscopy with Picosecond Temporal Resolution,* Physical Review Applied 2, 024003, 2014.