Chicane optimization

Sami Habet

IJCLab.

Jefferson Laboratory.

December 2021

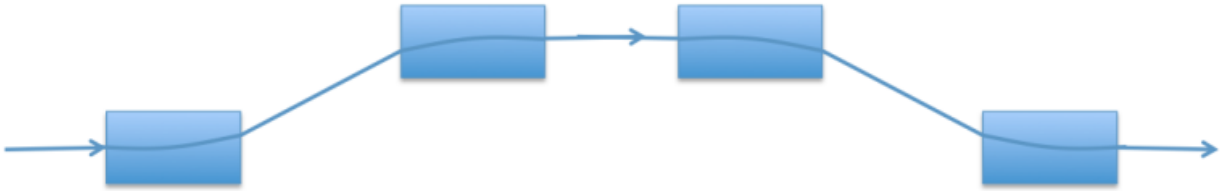
Plan

1. Chicane definitions
2. Case 1 : Without Dispersion
3. Case 2 : Optics insertions
4. Conclusion & Questions

Plan

1. Chicane definitions
2. Case 1 : Without Dispersion
3. Case 2 : Optics insertions
4. Conclusion & Questions

Chicane definitions



- The main reason to use a chicane:
  - Compress the beam.
  - Separate the electrons from the positron beam.
  - Momentum selection.

Chicane definition

- Transfer matrix:

$$\begin{bmatrix} x_{exit} \\ x'_{exit} \\ y_{exit} \\ y'_{exit} \\ z_{exit} \\ \delta_{exit} \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{44} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \\ z_0 \\ \delta_0 \end{bmatrix}$$

Dipole matrix

- The dipole matrix is defined as :

$$M_{dipole}(\rho\theta) = \begin{bmatrix} \cos\theta & \rho\sin\theta & 0 & \rho(1-\cos\theta) \\ -\frac{1}{\rho} & \cos\theta & 0 & \sin\theta \\ -\sin\theta & -\rho(1-\cos\theta) & 1 & (\frac{\rho\theta}{\gamma^2}) - \rho(\theta - \sin\theta) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

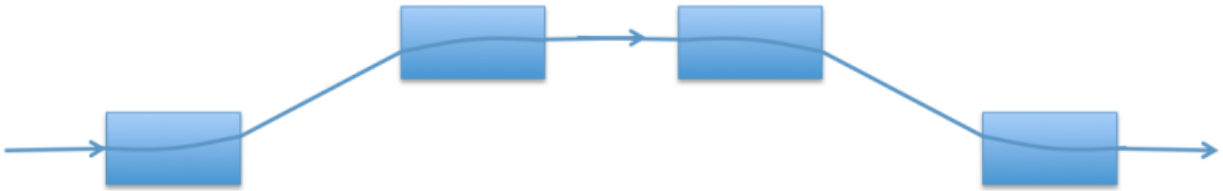
- $L_{dipole} = \rho\theta$
- ρ is the bend radius.
- θ is the bend angle.
- $[y, y']$ and [x,x',z,δ] elements are decoupled.

Drift Matrix

- The drift matrix is defined as:

$$M_{drift} = \begin{bmatrix} 1 & L_{drift} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_{drift}/\gamma^2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{chicane} = M_{dipole}(-\rho, -\theta) \times M_{drift} \times M_{dipole}(\rho, \theta) \times M_{drift} \times M_{dipole}(\rho, \theta) \times M_{drift} \times M_{dipole}(-\rho, -\theta)$$



$$z_{exit\,chicane} = R_{51}x_0 + R_{52}x'_0 + R_{53}y_0 + R_{54}y'_0 + R_{55}z_0 + R_{56}\delta_0$$

Plan

1. Chicane definitions
2. Case 1 : Without Dispersion
3. Case 2 : Optics insertions
4. Conclusion & Questions

Achromaticity condition

To simplify the mathematics we apply:

$$\text{Achromaticity criterion : D} = \begin{bmatrix} \eta_{x\,exit} \\ \eta'_{x\,exit} \\ \eta_{y\,exit} \\ \eta'_{y\,exit} \end{bmatrix} = \begin{bmatrix} \eta_{x\,entrance} \\ \eta'^{x\,entrance} \\ \eta_{y\,entrance} \\ \eta'^{y\,entrance} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$D_{exit} = \begin{bmatrix} M_x & 0 \\ 0 & M_y \end{bmatrix} \times D_{entrance} + \begin{bmatrix} R_{16} \\ R_{26} \\ R_{36} \\ R_{46} \end{bmatrix}$$

$$R_{16} = R_{26} = R_{36} = R_{46} = 0$$

Achromaticity condition

$$M_{chicane} = \begin{bmatrix} 1 & R_{12} & R_{13} & R_{14} & R_{15} & 0 \\ R_{21} & 1 & R_{23} & R_{24} & R_{25} & 0 \\ R_{31} & R_{32} & 1 & R_{24} & R_{25} & 0 \\ R_{41} & R_{42} & R_{43} & 1 & R_{25} & 0 \\ R_{51} & R_{52} & R_{53} & R_{54} & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z_{exit\,chicane} = R_{51}x_0 + R_{52}x'_0 + R_{53}y_0 + R_{54}y'_0 + R_{55}z_0 + R_{56}\delta_0$$

Symplectic matrix

- Since the transfert matrix is symplectic:

$$M^T J M = J$$

- Then, we get:

$$R_{16} = R_{52},\ R_{26} = R_{52},\ R_{36} = R_{54},\ R_{46} = R_{53}$$

Symplectic matrix

$$M_{chicane} = \begin{bmatrix} 1 & R_{12} & R_{13} & R_{14} & R_{15} & 0 \\ R_{21} & 1 & R_{23} & R_{24} & R_{25} & 0 \\ R_{31} & R_{32} & 1 & R_{24} & R_{25} & 0 \\ R_{41} & R_{42} & R_{43} & 1 & R_{25} & 0 \\ 0 & 0 & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Case 1 : Uncoupled calculation

- **Using z & $\frac{\Delta P}{P}$ space, we get:**

$$\begin{bmatrix} z_{exit} \\ \delta_{exit} \end{bmatrix} = \begin{bmatrix} 1 & R_{56} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_0 \\ \delta_0 \end{bmatrix}$$

- We get:

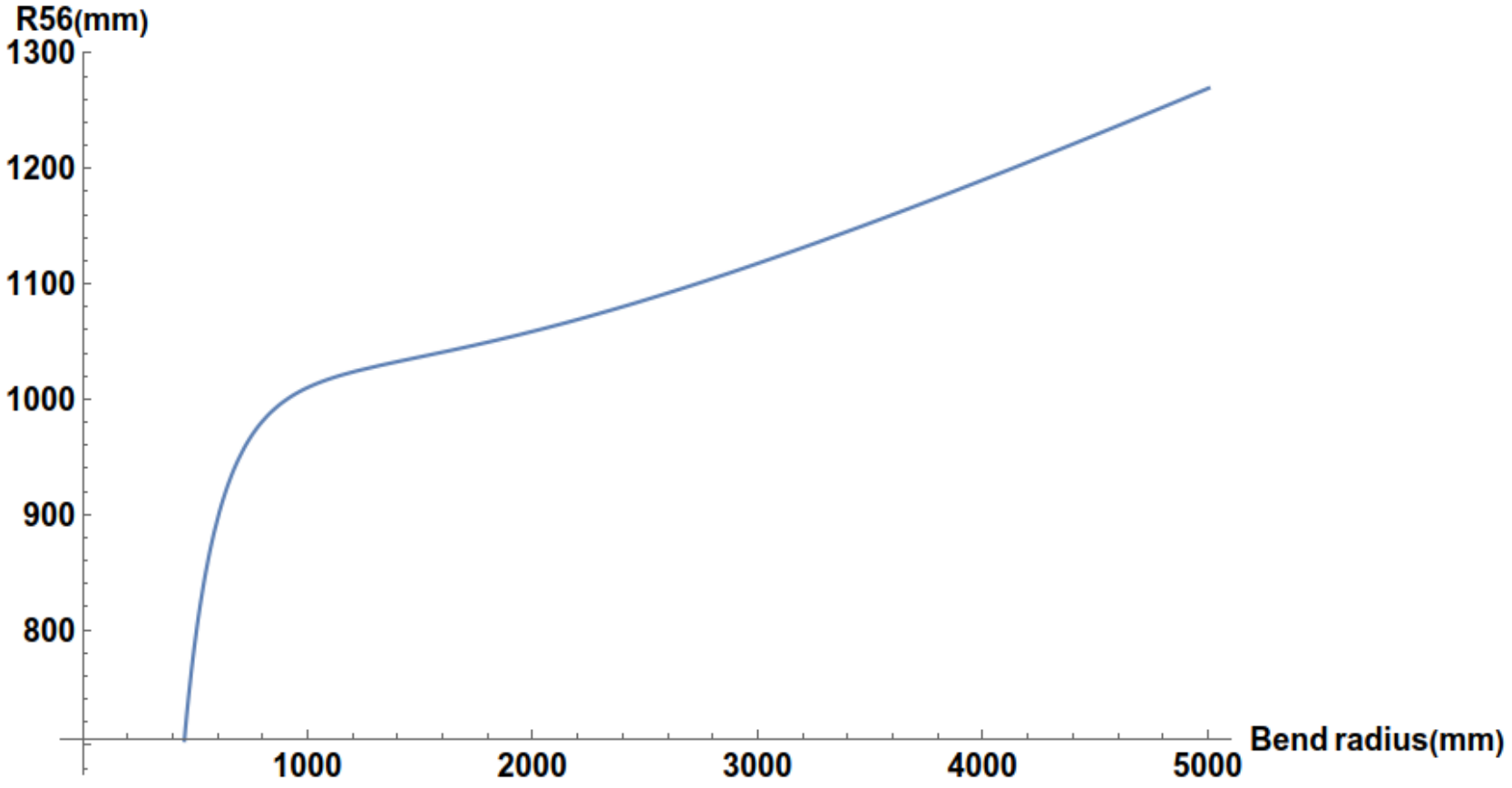
$$\begin{aligned} z_{exit} &= z_0 + R_{56}\delta_0 \\ \delta_{exit} &= \delta_0 \end{aligned}$$

- Then we can get R_{56}:

$$\begin{aligned} \Delta z &= z_{exit} - z_0 \\ R_{56} &= \frac{\Delta z}{\delta_0} \end{aligned}$$

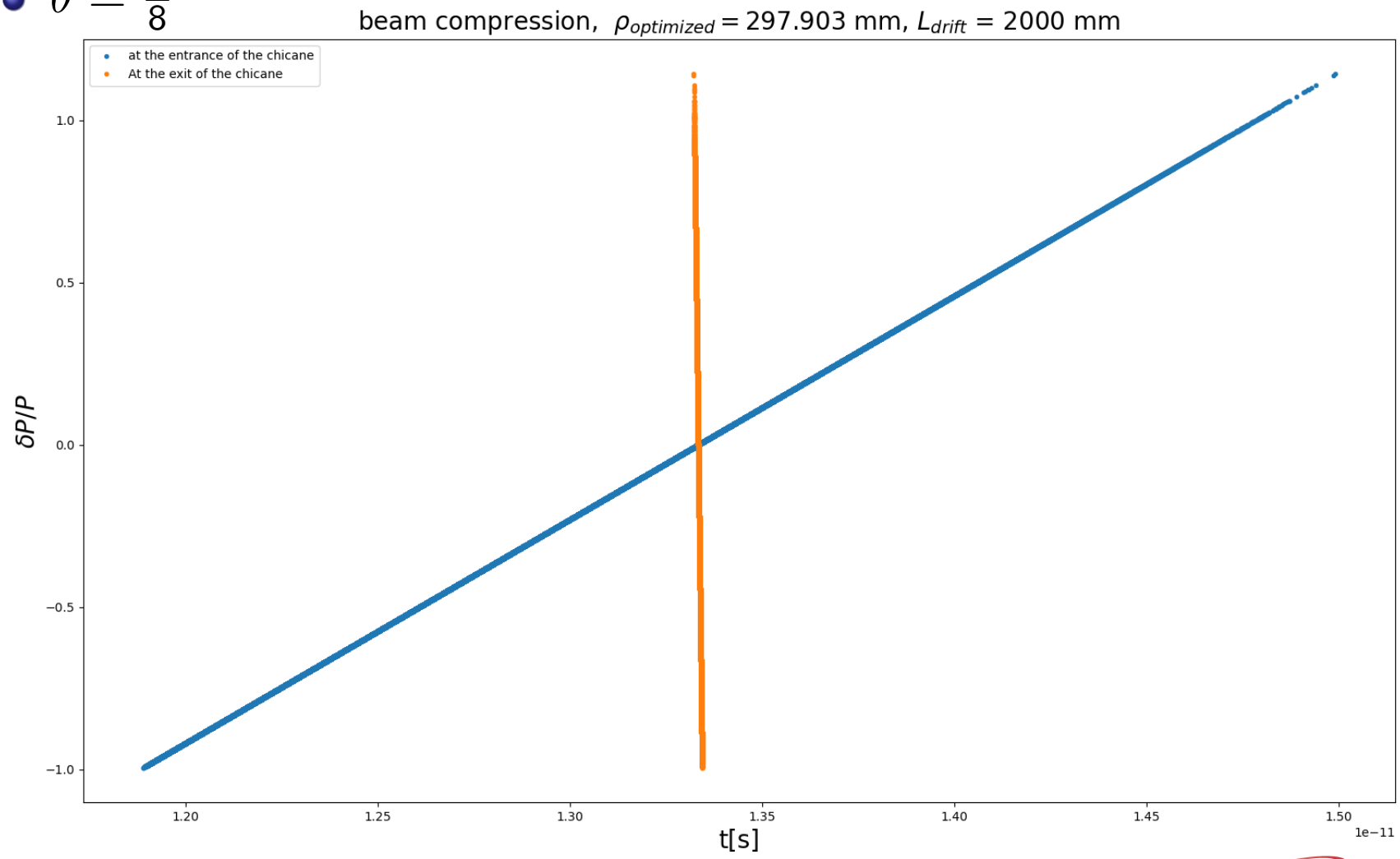
Case 1 : Uncoupled calculations

- From $M_{chicane}$, we plot the $R_{56}(\rho)$:



Case 1 : Beam compression

- $R_{56} = -4.30$ mm
- $\theta = \frac{\pi}{8}$



Plan

1. Chicane definitions
2. Case 1 : Without Dispersion
3. Case 2 : Optics insertions
4. Conclusion & Questions

Case 2 : Optics insertions

$M_{chicane} = M_{dipole}(-\rho, -\theta) \times M_{drift} \times M_{dipole}(\rho, \theta) \times M_{drift} \times M_{dipole}(\rho, \theta) \times M_{drift} \times M_{dipole}(-\rho, -\theta)$

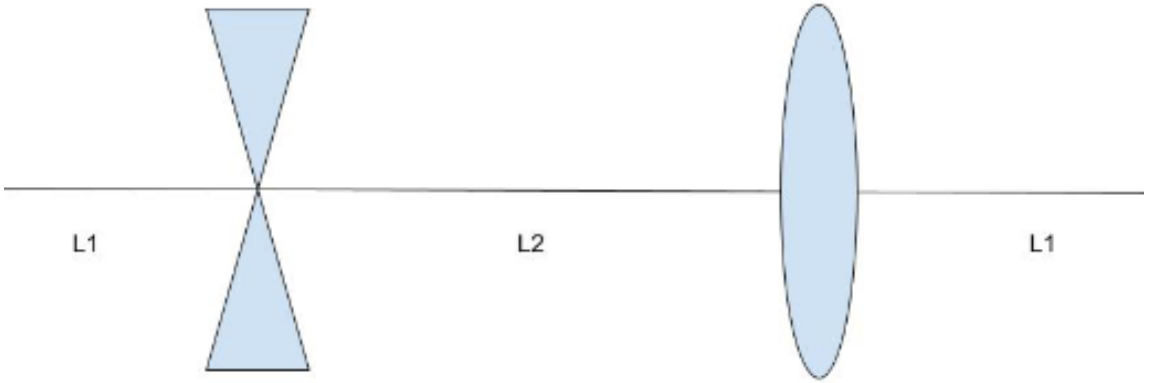
$$M_{chicane} = \begin{bmatrix} M_{11} & M_{12} & M_{15} & M_{16} \\ M_{21} & M_{22} & M_{25} & M_{26} \\ M_{51} & M_{52} & 1 & M_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} \Delta z &= M_{51}x_0 + M_{52}x_0' + M_{56}\delta_0 \\ M_{56} &= \frac{\Delta z + [coupled - term]}{\delta_0} \end{aligned}$$

Where:

$$coupled - term = M_{51}x_0 + M_{52}x_0'$$

$\frac{\pi}{2}$ insertion



$$M_{\frac{\pi}{2}} = \begin{bmatrix} 1 & L_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & L_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & L_1 \\ 0 & 1 \end{bmatrix}$$

$$M_{\frac{\pi}{2}} = \begin{bmatrix} 1 + \frac{L_2}{f} - \frac{L_1 L_2}{f^2} & 2L_1 + L_2 - \frac{L_1^2 L_2}{f^2} \\ -\frac{L_2}{f^2} & 1 - \frac{L_2}{f} - \frac{L_1 L_2}{f^2} \end{bmatrix}$$

Case 2: periodic boundary conditions

$$M_{\frac{\pi}{2}} = \begin{bmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{bmatrix} \qquad \begin{array}{l} \beta\sin\mu = 2L_1 + L_2 - \frac{L_1^2 L_2}{f^2} \\ \gamma\sin\mu = \frac{L_2}{f^2} \end{array}$$

- From twiss properties: $\gamma = \frac{1-\alpha^2}{\beta}$
- Then, we get: $L_2 = f^2 \sin\mu \times \left[\frac{1+\alpha^2}{\beta}\right]$
- To get small beam size : $\beta \rightarrow$ min $\beta \rightarrow$ L_2 Maximum
- Thus we need: $L_{2Max} \rightarrow \sin\mu = 1 \rightarrow \mu = \frac{\pi}{2}$
- Finally we have: $M_{\frac{\pi}{2}} = \begin{bmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{bmatrix} = J$
- $J^2 = -I \rightarrow J^4 = I$

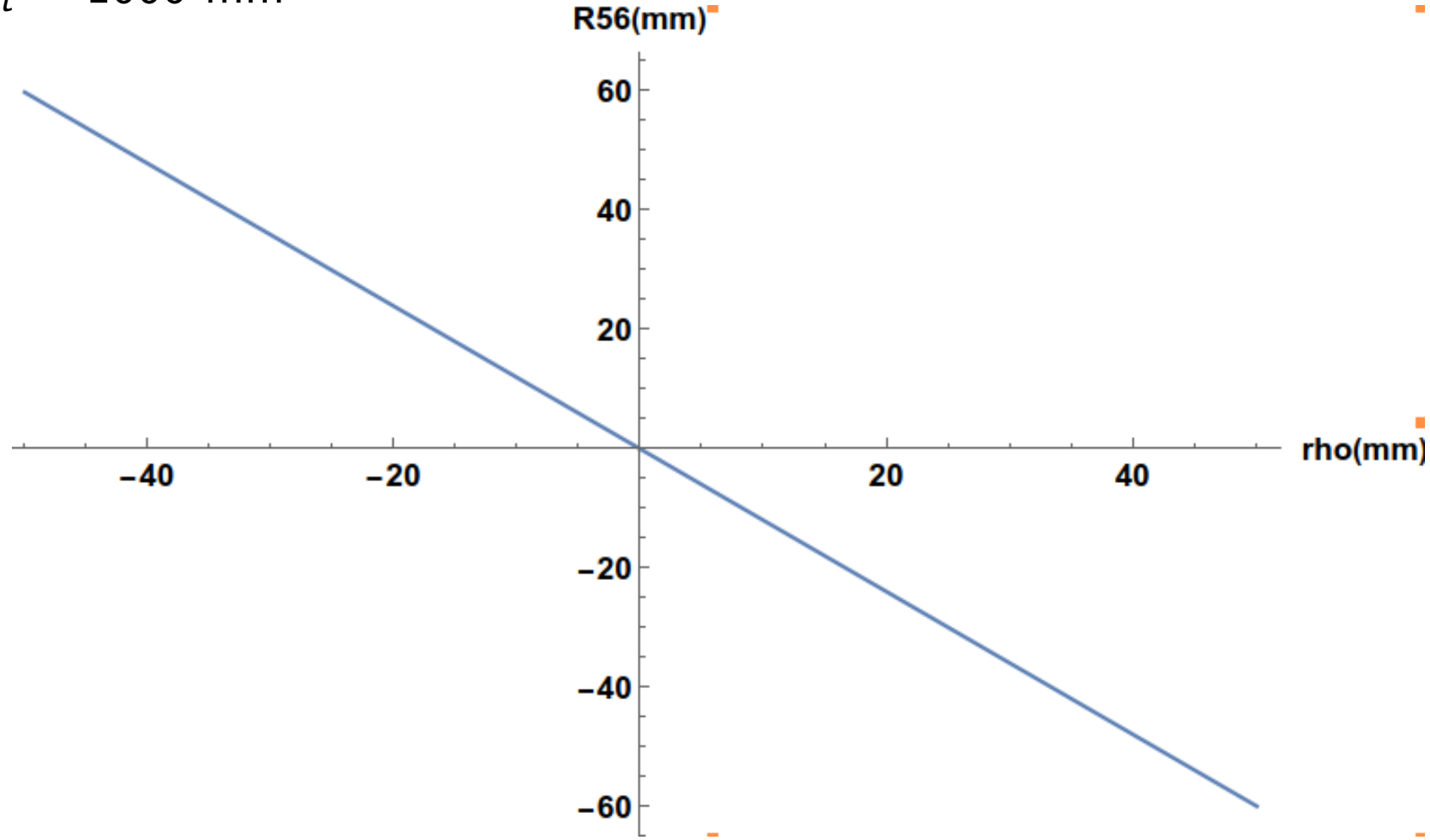
Case 2: New chicane line

$$M_{chicane^*} = M_{dipole}(-\rho, -\theta) \times J^4 \times M_{dipole}(\rho, \theta) \times M_{drift} \times M_{dipole}(\rho, \theta) \times J^4 \times M_{dipole}(-\rho, -\theta)$$

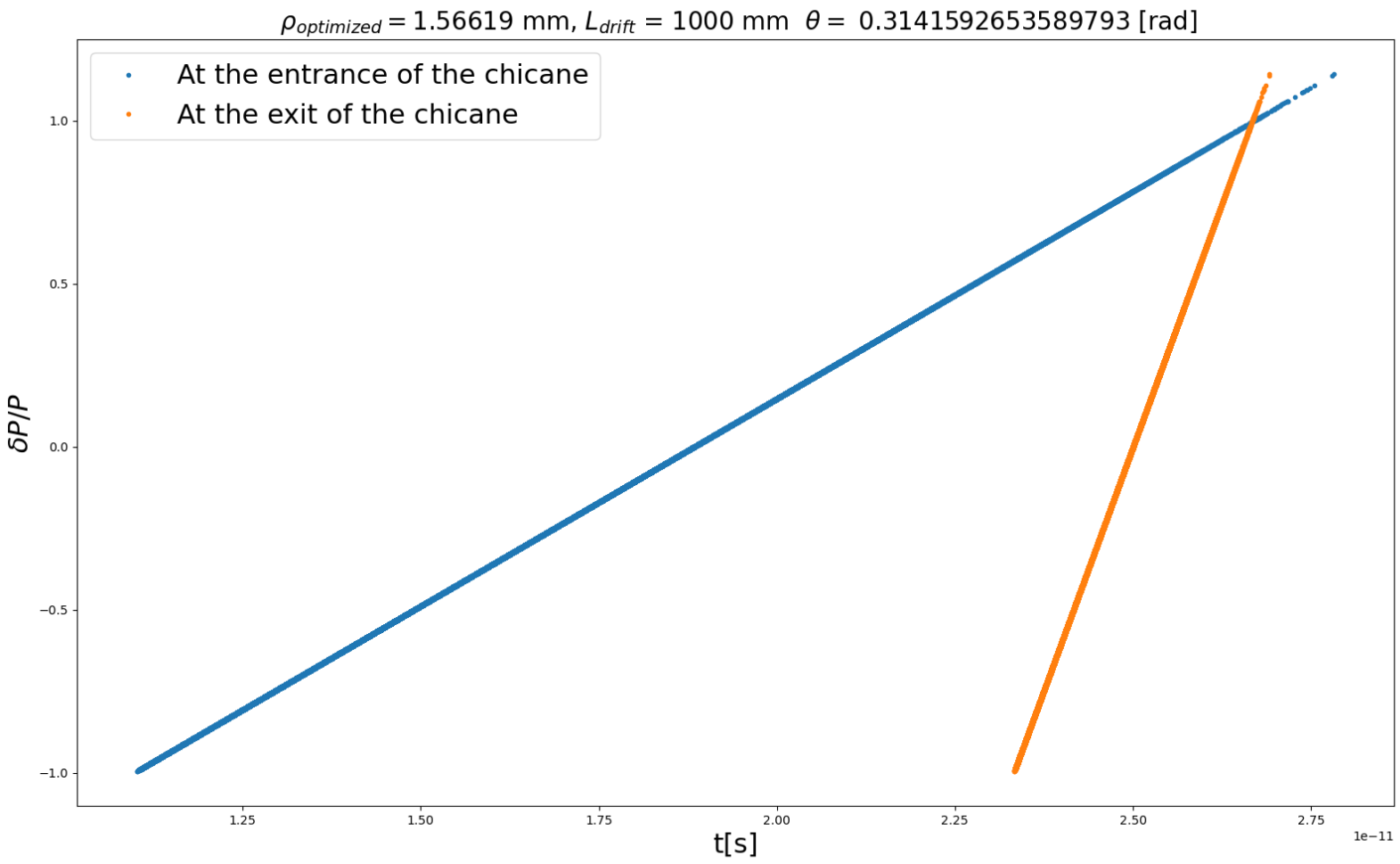
$$M_{chicane^*} = \begin{bmatrix} \cos 2\theta - \frac{L_{drift} \sin 2\theta}{2\rho} & \cos 2\theta \left(L_{drift} \cos 2\theta + 2\rho \sin 2\theta\right) & 0 & 0 \\ \frac{L_{drift} \sin \theta^2 - \rho \sin 2\theta}{\rho^2} & \cos 2\theta - \frac{L_{drift} \sin 2\theta}{2\rho} & 0 & 0 \\ 0 & 0 & 1 & \frac{L_{drift}}{\gamma^2 \beta^2} - 4\rho \left[\theta + (-1 + \cos\theta) \sin\theta\right] \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Case 2 : R56

- $\theta = \pi/10$
- $L_{drift} = 1000$ mm



Case 2 : Compression



Plan

1. Chicane definitions
2. Case 1 : Without Dispersion
3. Case 2 : Optics insertions
4. Conclusion & Questions

Conclusion

- Dipole dispersion affect the chicane compression.
- Achromatics optics using specials combinations of quadrupoles.
- More useful disperssion supressors?
- GPT chicane compression.