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Talman SPIN2016 07Oct2016 comments

There is a Wiki page at JLAB with a title 'Resonant Polarimeter' `https://wiki.jlab.org/ciswiki/index.php/Resonant_Polarimeter`.

- My comments on Talman's original SPIN 2016 talk are posted there (unbeknownst to myself, until today = 17 Oct 2016) `https://wiki.jlab.org/ciswiki/images/4/44/Talman_SPIN2016_Sateesh_comments.pdf`.
- There is also a post of Talman's revised paper for the SPIN 2016 proceedings, dated 7 Oct 2016 `https://wiki.jlab.org/ciswiki/images/5/57/Talman_Spin2016-paper_07Oct2016.pdf`
- A link to Talman's original presentation for SPIN 2016 (slides) can be found here `https://indico.cern.ch/event/570680/contributions/2310168/attachments/1341808/2026008/Talman_Spin2016-talk.pdf`.

All of my statements in this note pertain to Talman's revised paper dated 7 Oct 2016. Talman states that the transverse (horizontal) momentum deflection induced by the Stern-Gerlach force, upon passage through a quadrupole (employing an impulse or kick approximation), is (eq. 3, Talman)

$$(\Delta p_x^{SG})_{\text{Talman}} = \frac{\mu_x^*}{v}\, L_q\, \frac{\partial B_x}{\partial x}\,. \tag{1}$$

This expression is for passage through a skew quadrupole. In my previous comments, I treated a normal quadrupole, but it is simple to treat a skew quadrupole. We simply write

$$\boldsymbol{B} = \frac{\partial B_x}{\partial x}\, x\, \hat{\boldsymbol{x}} + \frac{\partial B_y}{\partial y}\, y\, \hat{\boldsymbol{y}}\,. \tag{2}$$

The spin-orbit term in the Hamiltonian is, to this level of approximation,

$$\begin{aligned} H_{\text{spin-orbit}} = \boldsymbol{\Omega}\cdot\boldsymbol{s} &\simeq -\left(a + \frac{1}{\gamma}\right)\frac{e}{mc}\boldsymbol{B}\cdot\boldsymbol{s} \\ &= -\frac{\gamma a + 1}{\gamma}\frac{e}{mc}\left(\frac{\partial B_x}{\partial x}\, x s_x + \frac{\partial B_y}{\partial y}\, y s_y\right). \end{aligned} \tag{3}$$

The equation of motion for p_x is

$$\frac{dp_x}{dt} = -\frac{\partial(\boldsymbol{\Omega}\cdot\boldsymbol{s})}{\partial x} \simeq \frac{\gamma a + 1}{\gamma}\frac{e s_x}{mc}\frac{\partial B_x}{\partial x}\,. \tag{4}$$

In an impulse approximation, the integrated kick Δp_x^{SG} is given by multiplying by the time of flight $\Delta t = L_q/v$. Hence I obtain

$$(\Delta p_x^{SG})_{\rm Mane} = \frac{\gamma a + 1}{\boldsymbol{\gamma}} \frac{e s_x}{mc} \frac{L_q}{v} \frac{\partial B_x}{\partial x} \,. \tag{5}$$

Let us express this in Talman's notation. First, for 6 MeV electrons, Talman correctly notes that the value of γa is negligible. Next $\mu_x^* = e s_x/(mc)$. Hence my expression for the transverse horizontal momentum deflection is

$$(\Delta p_x^{SG})_{\rm Mane} = \frac{\mathbf{1}}{\boldsymbol{\gamma}} \frac{\mu_x^*}{v} L_q \frac{\partial B_x}{\partial x} = \frac{\mathbf{1}}{\boldsymbol{\gamma}} (\Delta p_x^{SG})_{\rm Talman} \,. \tag{6}$$

This is the claimed factor of $1/\gamma$.

- When performing a Lorentz boost, *the magnetic dipole moment also undergoes a Lorentz transformation*, **an important fact which is entirely absent from Talman's analysis.**

- It is best to express the magnetic dipole moment in terms of the spin, and thence a covariant spin four-vector S_μ. Then the entire calculation is formulated in terms of manifestly Lorentz covariant quantities. This is precisely the approach taken by Bargmann, Michel and Telegdi in the derivation of their eponymous equation. The covariant spin four-vector S_μ can be expressed in terms of the rest-frame spin vector $\boldsymbol{s}$ (e.g. see the textbook by Jackson), which leads to the more widely employed form of the BMT equation.

- Given that the spin precession is determined by a spin precession vector $\boldsymbol{\Omega}$ in the lab frame, it should be immediately obvious that the Stern-Gerlach force in the lab frame is proportional to the gradients $\partial \boldsymbol{\Omega}/\partial p_{x,y}$. Someone like Derbenev will confirm this fact.

Turning to other matters, Talman's revised paper contains a curious 'footnote 1' which states

> For anomalous electron angular momentum $G = 0.00117$ the spin precession angle occurring during angular deflection $\Delta\theta$ of approximately $G_e\gamma\Delta\theta/(2\pi)$ is negligible. Also, the effect of non-zero electric field in the electron rest frame (obtained by Lorentz transformation of the quadrupole magnet at rest in the laboratory), can be included in the rest frame orbit equation and shown to be negligible.

- Anomalous or not, the electron (intrinsic) angular *momentum* is always $\frac{1}{2}\hbar$.
- The best value for the electron *magnetic moment* (not angular momentum) anomaly is $a_e = 0.0011\mathbf{59}\ldots$ Much effort has gone into determining this number. The currently accepted value for the *muon* magnetic moment anomaly is $a_\mu = 0.0011\mathbf{6592}\ldots$ which rounds to 0.00117.
- '...the effect of non-zero electric field in the electron rest frame ... can be included in the rest frame orbit equation and shown to be negligible.'
  - I am at a loss to understand what this means.
  - If the reference to the 'orbit equation' means the orbital quadrupole focusing, that is not part of the Stern-Gerlach force. In the *rest frame*, the orbital quadrupole focusing must arise *entirely* from the rest frame electric field, since $\boldsymbol{v}_{\text{rest}} \times \boldsymbol{B}_{\text{rest}} = 0$ because by definition $\boldsymbol{v}_{\text{rest}} = 0$.
  - If the reference to the 'orbit equation' means the Stern-Gerlach force, then $\boldsymbol{v}_{\text{rest}} \times \boldsymbol{E}_{\text{rest}} = 0$ because $\boldsymbol{v}_{\text{rest}} = 0$. The rest-frame electric field has no effect on a magnetic dipole moment. There is nothing that needs to be 'shown to be negligible.'
  - I *did* point out in my previous comments that the electric field is nonzero and $\boldsymbol{v} \times \boldsymbol{E} \neq 0$ in the frame K', an effect which continues to be absent in Talman's analysis. However K' is the frame where $p_s = 0$, and is *not* the rest frame, a fact which I pointed out in my previous comments. Talman's claim that the Stern-Gerlach force is proportional to (the gradient of) a magnetic dipole interaction $\boldsymbol{\mu} \cdot \boldsymbol{B}$ in the frame K' is false.
- Perhaps most signifcantly, my name does not appear in Talman's footnote 1, nor anywhere else in Talman's revised paper. One should be grateful for small mercies.