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On the Theory of the Bubble Chamber

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An attempt is made to analyze the factors which determine the operation of the bubble chamber. It is concluded that the majority of bubbles in conventional chambers are nucleated by moderately energetic free electrons produced by the incident particles in Coulomb encounters. Nuclei are displaced too infrequently by Coulomb encounters to account for the observed densities of bubbles. The electrons deposit their kinetic energy in highly localized regions which then are the source of explosions which produce bubbles of greater than critical size in a time of the order of 10^{-10} or 10^{-11} sec. The bubbles grow subsequently by evaporation of the fluid. The temperature of the fluid should be sufficiently close to the critical temperature that the energy required to produce the bubble of critical size can be provided by an electron with a range comparable to or less than the diameter of the bubble of critical size. Otherwise the electron will be unable to localize its energy in a sufficiently small volume in any but highly improbable cases. It is also concluded that the viscosity of the liquid plays a very important role in determining the threshold energy for forming a bubble of critical size when it has a value near 1 centipoise or larger.

1. INTRODUCTION

GLASER's bubble chamber, which offers the possibility of accomplishing for the physics of high-energy particles what the Wilson cloud chamber did for classical nuclear physics, is based on the observation that ionizing radiations can nucleate bubbles in a superheated liquid. The bubbles form along the trajectory of the incident particle and, under favorable circumstances, define the path in a relatively unambiguous manner. Most liquids appear to function as suitable media for bubble chambers under the right conditions. Thus the effect appears to be a general one. Organic liquids, such as propane, have been employed; simpler liquids such as liquid hydrogen and liquid rare gases can be as effective in appropriate temperature ranges.

Glaser[1] suggested originally that the formation of bubbles might be associated with the development of electrostatic forces produced by the charges induced by ionization, but eventually (private communication) abandoned this view in favor of a picture which is closely related to that which will be developed in this paper. He and his colleagues have also observed that the effect is related to the production of free electrons (δ rays) since the number of bubbles per unit length of path is correlated with the corresponding number of δ rays in propane. In the following work we shall attempt to substantiate the opinion that the bubbles are related to the production of free electrons, however the analysis will indicate that the mechanism probably depends upon the production of highly localized hot regions, or "temperature spikes," within the liquid which literally explode into bubbles of larger than critical size which can grow through evaporation of the superheated liquid. The point of view adopted in this presentation is closely related to that used in discussing the effects produced by ionizing radiations in solid materials and gathered together under the designation "radiation damage." The principal difference between the behavior of solids and liquids would appear to be a consequence of the great difference in viscosity of the two media. This presentation must be regarded as speculative, not merely because the theory is an approximate one but because the opportunities for comparison between theory and experiment are meager at the present time. Most of the investigators who construct bubble chambers are, quite naturally, much more interested in using them for high energy research than as devices for the study of bubble production. Fortunately, Glaser, Rahm, and Dodd[1] have published some information on the quantitative aspects of the operation of propane chambers which we shall find to be of crucial interest if we assume it is typical of bubble chambers in general. Similar results for hydrogen have been presented by Nagle, Hildebrand, and Plano.[2]

2. THRESHOLD ENERGY FOR A STATIC BUBBLE

There are two critical steps involved in the formation of a bubble of visible size. The first is related to the formation of the smallest bubble, or nucleus, which can grow spontaneously as a result of evap-

[1] D. A. Glaser, Nuovo cimento 11, 361 (1954). Glaser has prepared a general review of the field of bubble chambers for a forthcoming volume of the *Handbuch der Physik*; D. A. Glaser and D. C. Rahm, Phys. Rev. 97, 474 (1955); Glaser, Rahm, and Dodd, *ibid.* 102, 1654 (1956).

[2] Nagle, Hildebrand, and Plano, Rev. Sci. Instr. 27, 203 (1956).

oration of the fluid; the second is related to the growth of this nucleus to a macroscopic bubble. Most of the present article will be devoted to the first step since it involves the influence of the ionizing radiation in a critical way. The second step has been subject to a careful analysis by Plesset and Zwick,[3] who have resolved the factors which determine the ultimate rate of growth under fairly general assumptions. We shall employ their results in Sec. 11.

Pless and Plano[4] have discussed the contributions to the energy of formation of a static bubble when it is generated in a reversible manner. We shall repeat the main points of their discussion since it forms part of the basis for the presentation given here.

A spherical bubble possesses an effective pressure which tends to cause it to collapse as a consequence of the surface tension. The effective pressure is

$$p_c = 2\sigma/r \tag{1}$$

in which r is the radius of the bubble and σ is the surface tension. In a statically stable bubble, this pressure is balanced by the difference between the pressure of the fluid and that of the gas in the bubble, that is

$$p_c = p_i - p_f \tag{2}$$

in which p_i is the pressure of the gas and p_f that of the fluid. In a typical bubble chamber p_f is reduced from the equilibrium value, of the order of tens of kg per cm^2, to a value near one atmosphere (1 kg per cm^2) to sensitize the system. Assuming it is filled with gas at the equilibrium pressure p_e, a stable bubble satisfies the condition

$$2\sigma/R_c = p_e - p_f \cong p_e \tag{3}$$

immediately after reduction of the pressure in the fluid. Here R_c is the critical or equilibrium radius. Thus the equilibrium radius is

$$R_c = 2\sigma/p_e. \tag{4}$$

A bubble having a larger radius and containing the vapor at equilibrium pressure will grow as the liquid evaporated into it, provided p_f remains small compared to p_e.

The primary problem of the mechanism of the bubble chamber is that of determining the physical circumstances which lead to the formation of bubbles of radius equal to or larger than (4) and containing the equilibrium pressure of vapor. The minimum energy required for the process is the sum of the surface energy and the heat of vaporization of the vapor in the bubble, if we assume the initial bubble is formed so quickly that the energy of vaporization cannot be furnished from the ambient energy. Thus

$$E_m = 4\pi\sigma R_c^2 + \frac{4\pi}{3} R_c^3 n_v H_s \tag{5}$$

in which n_v is the number of moles per unit volume in the vapor at pressure p_e, and H_s is the heat of sublimation per mole. This may be written in the form

$$E_m = 16\pi \frac{\sigma^3}{p_e^2}\left(1 + \frac{2}{3}\frac{n_v}{p_e} H_s\right) \tag{6}$$

with the use of (4). The quantity in parenthesis may be expressed as

$$(1 + 2H_s/3Z_eR_gT) \tag{7}$$

in which Z_e is the compressibility factor p_eV/R_gT, with V the molar volume, and R_g is the gas constant. It may be noted that (6) is relatively sensitive both to p_e and to σ, decreasing with increasing values of the first and decreasing values of the second. In general p_e increases with increasing temperature whereas the surface tension decreases as one approaches the critical temperature and pressure. Thus E_m decreases as the temperature of operation of the chamber rises.

If the pressure p_f of the fluid is not entirely negligible, a third term representing the work done in expanding the bubble against this pressure must be added to the foregoing. We shall not carry this term along with the other two in the following since it will not play a decisive role in the discussion when p_f is small compared with p_e.

Table I contains[5] values of the quantities appearing in the preceding equations for liquid hydrogen and propane in typical cases of interest. It may be seen from the table that the surface energy of the bubble, namely $16\pi\sigma^3/p_e^2$, is about 1.12 ev for hydrogen and 27.1 ev for propane. The energy of evaporation of the gas contained in the bubbles is

[3] M. S. Plesset and S. A. Zwick, J. Appl. Phys. **23**, 95 (1952); **25**, 493 (1954).

[4] I. A. Pless and R. J. Plano, Rev. Sci. Instr. **27**, 935 (1956).

[5] Most of the empirical data appearing in the tables were obtained from standard tables, such as *The American Institute of Physics Handbook* (McGraw-Hill Book Company, Inc., New York, 1957); Lange's *Handbook of Chemistry* (Handbook Publishers, Inc., Sandusky, Ohio); J. D'Ans and E. Lax, *Taschenbuch für Chemiker und Physiker* (Springer-Verlag, Göttingen, 1949); Landoldt-Bornstein, *Physikalischen Zahlenwerte und Funktionen* (Springer-Verlag, Berlin). The value of the surface tension of hydrogen was obtained from the *Cryogenic Data Book* by D. B. Chelton and Douglas R. Mann of The National Bureau of Standards. The value for propane was obtained from the paper by D. L. Katz and W. Saltman, Ind. Eng. Chem. **31**, 91 (1939).

TABLE I. Values of the quantities[5] appearing in Eqs. (4), (6), and (7) for liquid hydrogen and propane. (Z_e is assumed to be unity.)

	H_2	C_3H_8
$p_e - p_f$ (atmos)	5	10
T(°K)	27	328
σ(dynes/cm²)	0.97	4.46
R_c(cm)	3.83×10^{-7}	8.81×10^{-7}
$V_c = 4\pi R_c^3/3$(cm³)	2.35×10^{-19}	2.86×10^{-18}
H_s(kg cal/mole)	0.215	3.9
$16\pi\sigma^3/p_e^2$(ev)	1.12	27.1
$(1 + 2H_s/3RT)$	3.65	4.81
E_m(ev)	4.08	131.

larger than this by a factor 2.65 and 3.81 in the two cases.

Quantities related to the equation of state of hydrogen and propane are given in Table II.

TABLE II. Quantities[5] related to the equation of state hydrogen and propane.

	H_2	C_3H_8
Boiling temperature T_b(°K)	20.3	231(−42.2°C)
Density at boiling point ρ_b (g/cc)	0.0709	0.585
Critical temperature T_c(°K)	33.3	370(96.8°C)
Critical pressure p_f (atmos.)	12.8	42.0
Critical density ρ_c	0.031	0.22

We conclude from the values of E_m that the minimum energy required to form a bubble in hydrogen is of the order 4 ev, whereas that in propane and similar organic liquids is of the order of 131 ev. We must investigate the mechanism of formation more carefully before we may conclude that point thermal spikes having these energies actually could generate bubbles.

3. THE CROSS SECTION FOR ENERGY TRANSFER

Glaser, Rahm, and Dodd[1] have counted the number of bubbles along the tracks of positive pions and protons having a momentum of 915 Mev/c in a propane chamber operated at 55.5°C. The numbers were about 16 and 27 per cm, respectively. Similarly, Nagle, Hildebrand, and Plano[2] have found that the density of bubbles along the path of bubbles along the path of a fast electron in a hydrogen chamber operated at 27°K is about 15 per cm. We shall assume that each bubble is associated with the transfer of a threshold energy E_t to either an electron or a hydrogen nucleus by the incident particle as a result of Rutherford scattering. Table III contains the empirical cross sections per molecule calculated from the bubble densities in the two cases. In the table, n is the density of bubbles along the track (only one of the cases for propane is included in the table), M is the molecular weight, ρ is the density of the liquids estimated at the operating temperatures of the counters, σ_e is the empirical cross section per molecule. E_t is the threshold energy associated with the cross section, described below.

Glaser informs the writer (private communication) that closely spaced bubbles may have fused in typical tracks, so that the bubble densities given for propane may be lower limits to the actual values. This would imply, in turn, that the values of σ_e and E_t given in Table III for propane represent lower and upper limits to more accurate values. Actually, the measured distribution of bubbles corresponded closely to a Poisson distribution function for mean densities close to those given previously except for small spacings. Thus, it is doubtful if the corrections are highly significant in a quantitative sense.

TABLE III. Empirical cross section per molecule for formation of a bubble, obtained using bubble densities observed by Nagle, Hildebrand, and Plano in H_2 and by Glaser and Rahm in propane.

	H_2	C_3H_8
n(cm⁻¹)	15	16
M(g)	2.0	44.1
ρ(g/cm³)	0.050	0.330
σ_e(cm²)	0.996×10^{-21}	3.55×10^{-21}
E_t(kev)	0.513	1.890

An expression for the cross section for transfer of an energy greater than E_t to an electron by Rutherford scattering adequate for our purposes is

$$\sigma_e = 8\pi a_h^2 R_h^2/m_e v^2 E_e. \tag{8}$$

Here a_h is the Bohr radius, R_h the Rydberg energy, m_e the electronic mass, and v the velocity of the incident particle. Relativistic effects are neglected since we are concerned with the magnitude of σ_e. The cross section per molecule is found to be

$$\sigma_{e,m} = 18.74 \times 10^{-21} Z_e R_h/E_t\beta^2 \text{ (cm}^2\text{)} \tag{9}$$

in which Z_e is the number of electrons per molecule, namely, 2 for hydrogen and 26 for propane and $\beta = v/c$. The latter is 0.676 for the protons and 0.988 for the pions in the case of the propane chamber.

$$E_t = 4.832 \times 10^{-19} R_h/\sigma_{e,m} \text{ (cm}^2\text{)} \beta^2. \tag{10}$$

Thus we find that the empirical threshold energies are 513 ev and 1890 ev for the two cases listed in Table III. It may be added that the threshold value

found for the protons in propane is the same as that for the pions within the error of measurement.

We note that the values of E_t derived in this way, assuming that the agents responsible for the bubbles consist of knocked-on electrons, are substantially larger, by a factor of about 100 for hydrogen and about 10 for propane, than the values of E_m given in Table I. Evidently dynamical factors must be included if we are to conclude that delta rays actually are the source.

The calculated value of E_t which would be required if the agents responsible for the bubbles were protons ejected from the molecules by the incident particles rather than electrons is at least three orders of magnitude smaller than that found above. Hence it is unreasonably small. The cross section σ_p for ejection of a proton also varies as $1/E_t$ in the Rutherford range. For given E_t, the ratio of σ_p to σ_e is

$$\sigma_p/\sigma_e = n_p m_e/Z_e M_p \tag{11}$$

in which n_p is the number of hydrogen atoms per molecule and M_p is the mass of the proton. This ratio is of the order of 10^{-3} for hydrogen and 10^{-4} for propane. Thus E_t would have to be as small as 1 ev or less if the bubbles observed in hydrogen or in propane were to originate in ejected protons. The corresponding energy would be inadequate to produce a stable bubble, in a typical case, even in a reversible manner.

To summarize, we are lead to the conclusion that the bubbles are related primarily to energy transferred from the incident particle to the electronic system and not to the nuclei. Thus bubble formation in the liquids used resembles in a broad way the radiation damage[6] found in organic solids and the alkali halides and stands in contrast to the damage found in metals and semiconductors, which is produced primarily by the displacement of atoms through close Rutherford encounters.

Glaser, Rahm, and Dodd reached a similar conclusion as a result of the observation that the density of bubbles correlate with the density of delta rays along the tracks. Actually, the density of displaced protons, and indeed of atoms in general, would also correlate with the density of delta rays so their argument is not sufficiently restrictive. When combined with the foregoing analysis, however, it seems safe to conclude that delta rays having a threshold near 1 kev are responsible for the bubbles in hydrogen and propane.

[6] See, for example, the survey by F. Seitz and J. S. Koehler in *Solid State Physics* (Academic Press, Inc., New York, 1956), Vol. 2.

4. FATE OF ENERGY TRANSFERRED TO ELECTRONIC SYSTEMS

The energy transferred to the electronic system of a substance can reappear in several forms. The simplest form is associated with atomic or nuclear vibrations, that is with heat in the normal sense; however some may be emitted in the form of fluorescent radiation, or may be stored in the form of metastable dissociation products. Even the most efficient cathodoluminescent crystals do not have an energy efficiency[7] larger than fifteen or twenty percent for conversion of incident energy into light, most of the remaining energy reappearing as heat. Similarly, careful studies[8] of the coloration of alkali halides indicate that only a small part of the energy of incident x-rays, which produce electrons by photoionization, is stored. It seems safe to conclude that most of the energy transmitted to the electronic systems of liquid hydrogen and propane reappears in the form of heat distributed among the excited molecule and its neighbors.

Glaser has pointed out to the writer that there is evidence to show that this conclusion is not the case in very pure rare gases, particularly xenon with which he and his colleagues have carried out extensive experiments.[9] They have found that very pure xenon is not radiation sensitive, but it becomes so when very small amounts of ethylene are added. Parallel experiments, carried out at Los Alamos and at Argonne National Laboratory, on the ability of xenon to serve as a fluorescent counter, showed that the luminescent efficiency of the liquid is very high until traces of hydrocarbons are added. Apparently a large fraction of the energy transmitted to the pure xenon is able to diffuse out of the region excited initially, perhaps in the form of excitons, without contributing to the thermal spike. A large part of this energy would appear to be emitted, in turn, as radiation. The hydrocarbons presumably serve as catalysts for the conversion of the electronic energy into energy of atomic motion.

5. BEHAVIOR OF ELECTRONS HAVING ENERGIES NEAR OR BELOW 10 KEV

Electrons having energies equal to or less than 10 kev are completely in the classical region of energy and possess a range which varies[5] approximately as the square of the energy. Actually the

[7] See, for example, W. J. Van Sciver and L. Bogart, Bull. Am. Phys. Soc. Ser. II, 2, 142 (1957).

[8] H. U. Harten, Z. Physik **126**, 619 (1949); E. Feldtkeller, thesis, Technical University, Stuttgart, 1957.

[9] Brown, Glaser, and Perl, Phys. Rev. **102**, 586 (1956).

straggling in energy and in direction are very marked, increasing rapidly with decreasing energy, so that the range has a pragmatic rather than precise meaning. Since a 10-kev electron loses between 25 and 50 ev in an average collision, the straggling in range along the actual trajectory is of the order of five % and the straggling in direction, principally as a result of elastic scattering, is of the order of ten radians. Thus the motion is almost diffusive. The practical range of 10-kev electrons which undergo the least angular straggling is about 0.12 mg/cm^2 in aluminum. This may be generalized to the form

$$R\ (\text{cm}) = 0.58E^2\ (\text{kev})/\rho(Z/A) \qquad (12)$$

for other materials. Here ρ is the density and Z/A is the ratio of the number of electrons to the molecular weight in units in which the mass of the oxygen atom is 16. Z/A is 1.0 for H_2 and 0.59 for propane. In view of the diffusive motion, the average range of the electrons is substantially smaller than the practical value, which corresponds to a straight-line trajectory. For example it is about half the practical value for 10-kev electrons and relatively less for those of lower energy.

Table IV contains the values of E for which the practical range, given by (12), is equal to $2R_c$ when R_c has the values given in Table I. It is interesting to note that these values of E are of the same magnitude as the values of E_t in Table III, being about half as large.

TABLE IV. Values of the electron energy for which the practical range is twice the radius of the bubble of critical size (Table I).

	H_2	C_3H_8
R_c(cm)	3.83×10^{-7}	8.81×10^{-7}
E(kev)	0.257	0.769

We may conclude from the form of (12) that if the energy required to form the bubbles of critical size were substantially larger than the threshold values E_t, given in Table III, it would be difficult for a typical electron to concentrate its energy within the sphere of critical radius. Only a relatively exceptional group of electrons which undergo relatively large angular scattering and are, in consequence, confined to a small volume would be effective. That is, only those electrons which have energies close to the threshold value and for which the actual range is much smaller than the practical range would produce bubbles.

Since R_c is equal to $2\sigma/p_e$ [Eq. (4)], the energy for which the practical range is equal to $2R_c$ satisfies the equation

$$E_c^2 = 1.75\rho \times 10^6(Z/A)(4\sigma/p_e), \qquad (13)$$

in which σ/p_e is to be expressed in cm. We see from Eq. (6) that E_m is much more sensitive to the parameters σ and p_e, varying as σ^3/p_e^2. Since the ratio E_m/E_e presumably should be appreciably less than unity, it follows that the chamber should operate under conditions relatively close to the critical point, where σ is relatively low and p_e relatively large. This appears to be one of the conditions placed upon the operation of a bubble chamber if the bubbles are to be produced by the electrons excited along the path of the incident particle with a density characteristic of the experiments summarized in Table III.

Knock-on protons which have energies in excess of M_hE_e/m_e may lose energy by exciting the electronic system. Here M_h is the mass of the proton and m_e that of the electron, whereas E_e is the first electronic excitation energy. Since such protons possess a velocity much smaller than electrons of comparable energy, they dissipate energy at a much greater rate per unit length of path. Thus typical knock-on protons having energies above the threshold energy for bubble formation could generate bubbles in cases in which the threshold is high and the range of a typical electron having the threshold energy is too long. The density of bubbles formed in this way would, of course, be much smaller than that observed when the threshold energy is sufficiently small that bubbles may be formed by electrons having energies near the threshold value. Presumably useful bubble chambers normally will operate in such a way that the number of bubbles formed by displaced atoms is negligible.

6. TIME FOR DISSIPATION OF HEAT

The analysis of the preceding sections show that there is a substantial difference between the threshold energy E_t required to form bubbles of an interesting size under the conditions in which bubble chambers are usually operated (Table III) and the minimum energy E_m required to form bubbles of critical size "statically" (Table I). It seems reasonable to suppose that the difference stems from the fact that the actual bubbles must be formed relatively quickly under conditions which are far from those associated with equilibrium. One of the most obvious limitations upon the time permitted for

bubble formation is imposed by the finite thermal conductivity of the medium. The thermal spike produced by the electron will dissipate in a time determined by the thermal conductivity. Unless the void essential for the formation of a bubble is produced before the thermal energy is dissipated, the bubble either will not form, or will be produced at a later time as a result of more diffusely distributed thermal effects which would not necessarily locate it immediately on the trajectory of the incident particle. For example vacancies formed over a relatively large volume might coagulate into a bubble.

We shall attempt to establish a criterion related to the energy deposited by the excited electron which will guarantee that a bubble be formed before the thermal energy is dissipated. The problem is a complex one and the criterion established here will be somewhat approximate in nature.

For simplicity, we shall assume that the electron transmits its energy to the liquid in such a way that the volume within a sphere of radius R_0 is uniformly heated, R_0 being somewhat smaller than R_c. This is equivalent to assuming that the trajectory of the electron is coiled into a more or less spherical region having a radius R_0.

For definiteness in the ensuing discussion we shall assume that the volume V_0 which is heated initially is equal to that of a cylinder having the length $2R_c$ and a cross section πa^2, where $4\pi a^3/3$ is the mean molecular volume. One readily finds that the ratio V_0/V_c, in which $V_c = 4\pi R_c^3/3$, is

$$V_0/V_c = 3(1/n_m V_c)^{\frac{2}{3}}/2. \tag{14}$$

Here n_m is the number of molecules per unit of volume of the liquid. Table V contains values of the ratio for hydrogen and propane.

TABLE V. Adopted values of the ratio V_0/V_c for use in estimates.

	H_2	C_3H_8
$n_m V_c$	3543	12 900
V_0/V_c	6.45×10^{-3}	2.73×10^{-3}
R_0/R_c	0.186	0.140

The relaxation time τ_h for dissipation of heat in a spherical thermal spike of radius r is

$$\tau_h = r^2/4D \tag{15}$$

in which D is the diffusion coefficient for heat, related to the coefficient of thermal conductivity λ, the density ρ, and the specific heat c by the equation

$$D = \lambda/\rho c. \tag{16}$$

Values of these quantities appropriate for our present interest are given in Table VI, along with the value τ_c of τ_h obtained when $r = R_c$. Fortunately none of the quantities appearing in D are highly sensitive to pressure in the range of interest to us so that the values are probably accurate to a factor of 2.

TABLE VI. The diffusion coefficient of heat and values of τ_h [Eq. (15)] for the values of R_c appearing in Table I.

	H_2	C_3H_8
λ(cal/cm-deg)	2.8×10^{-4}	4×10^{-4}
ρ(g/cc)	0.050	0.330
c(cal/g)	1.74	0.60
D(cm²/sec)	3.22×10^{-3}	2.02×10^{-3}
τ_c(sec)	1.139×10^{-11}	9.60×10^{-11}
$v_c = R_c/\tau_c$(cm/sec)	3.36×10^4	0.918×10^4

The quantity

$$v_c = R_c/\tau_c = 4D/R_c \tag{17}$$

is also given in the table. It is the average minimum velocity with which the walls of the bubble must move if the bubble is to achieve stability before the heat responsible for its formation has dissipated.

It is interesting to compare v_c with the adiabatic acoustical velocity in the medium. The latter is given by the relation

$$c_a = (1/\rho\beta_a)^{\frac{1}{2}} \tag{18}$$

in which ρ is the density and β_a is the adiabatic compressibility. Values of c_a are given in Table VII. The values of β_a are estimated from the values of the compressibility in the literature.[5]

TABLE VII.

	H_2	C_3H_8
β_a(cm²/dyne) $\times 10^{12}$	500	100
c_a	1.7×10^5	1.9×10^5

We may note that the values of c_a are a factor of ten greater than the values of v_c given in Table VI. It follows that the average velocity with which the bubble must expand in order to be formed in a time near τ_c is moderately subsonic.

7. ASSUMPTIONS CONCERNING EXPANSION

We shall now attempt to estimate the energy which must be furnished in order to provide the pressure needed to form the bubble before the heat

is dissipated. The following assumptions will be made in an initial attempt to resolve the problem:

(1) As mentioned previously, it will be assumed that the thermal spike is localized in a spherical volume of radius R_0 within which the temperature and pressure are uniform. The values of R_0 given in Table V will be employed when explicit values are needed.

(2) If the fluid were not set in motion by the pressure gradient, we would assume that the radius of the heated zone increases in accordance with the relation

$$R = 2(Dt)^{\frac{1}{2}} \tag{19}$$

in which D is the thermal diffusion coefficient and t is the time. Actually, we shall assume that the motion arising from dynamical factors is significant and brings about the displacement of the initial heated boundary from the value R_0 to R_c in time τ_c.

(3) It will be assumed that the pressure is proportional to the thermal energy per unit volume and varies in the manner

$$p = p_0(R_0/R)^{3\gamma}. \tag{20}$$

Here p_0 and R_0 are the initial pressure and radius, respectively, and γ is a parameter which takes account of the fact that the expansion of the heated zone may be accompanied by the expenditure of thermal energy in doing work. This work reappears in shock waves which radiate from the heated region. If the energy lost were small, γ would be effectively unity; if the energy were large, γ would be close to the ratio of specific heats at constant pressure and temperature. We shall see that the second choice is more nearly correct, for the dynamic motion of the boundary is much more rapid than that arising from the flow of heat, as described by (19), until the radius of the boundary reaches the value R_c. Table VIII contains values of γ appropriate for adiabatic expansion under various conditions. The first value, γ(M), is that to be used for the ideal gases when all the degrees of freedom are excited in the classical manner. The molar heat of H_2 at constant volume is then $7R_g/2$ and that of propane is $30R_g$. The second set of values, namely γ(BP), are estimates for the boiling points under the assumption that $C_p = C_v + R_g$. The third set of values, γ, are those which will be adopted in treating the adiabatic expansion of the superheated liquid under adiabatic conditions. They are chosen somewhat arbitrarily as intermediate between the other two values. Reasons for preferring the adiabatic values will emerge later. It may be noted that γ is near unity for propane in any case because of the many internal degrees of freedom of the molecules.

TABLE VIII. Values of the parameter γ appearing in Eq. (20). See text for meaning of the various values.

	H_2	C_3H_8
γ(M)	9/7 = 1.286	31/30 = 1.0333
γ(BP)	1.58	1.076
γ	1.40	1.05
$\gamma - 1$	0.40	0.05
$(V_c/V_0)^{\gamma-1}$	7.518	1.343

It should perhaps be emphasized at this point that the density of the material within the expanding sphere of radius R can be expected to vary with time both because new regions of the liquid become heated and are incorporated within it, and because it expands by pushing away the surrounding material. In any case, it will be assumed that the heated zone is homogeneous and exerts a pressure determined by its energy content, in the manner of a perfect gas.

8. EQUATIONS FOR EXPANSION

One of the convenient features of the theory of displacement of a fluid in three dimensions under circumstances in which the expansion is directed radially from the center and in which the flow is not supersonic is that a fairly good approximation can be obtained either by using the theory of sound or by treating the medium as if incompressible. Such approximate solutions are useful provided the viscosity is not large. We shall assume initially that the viscosity can be neglected and shall investigate its influence later. It will turn out that it probably is very important in fluids such as propane in which the viscosity coefficient is of the order of 10^{-2} poise, but probably is not critically important in liquid hydrogen if the viscosity coefficient is as small as 10^{-4} poise, the value[10] for the normal fluid.

In the case of a uniformly expanding sphere within a gas or liquid, G. I. Taylor[11] has shown that the solution obtained by superimposing sound waves is very closely the same as that derived by introducing a shock wave, provided the velocity of the sphere is moderately subsonic. The more exact theory gives a somewhat lower value of the pressure near the sphere than the sound wave theory.

Michael Rogers of the Astronomy Department of the University of Illinois has demonstrated to

[10] The viscosity of liquid hydrogen is taken from the *Cryogenic Data Book* (reference 5).

[11] G. I. Taylor, Proc. Roy. Soc. (London) **186**, 273 (1946).

the writer that the hydrodynamical solution given by the theory of the incompressible fluid is a good approximation to that obtained by introducing a shock wave and is probably adequate for our discussion under circumstances in which the viscosity can be neglected, which, as mentioned above, is questionable.

Consider an expanding sphere of radius $R(t)$. The equations of motion of the incompressible fluid[12] are

$$\partial u/\partial t + u\,\partial u/\partial r = -(\partial p/\partial r)(1/\rho) \tag{21}$$

and

$$\partial(r^2u)/\partial r = 0. \tag{22}$$

Here u is the velocity of a fluid element at a radial position r and p is the pressure, which depends upon both time and position. The first is Newton's equation of motion and the second that of continuity of the incompressible fluid. It is readily found that an appropriate solution of the second equation is

$$u = \dot{R}R^2/r^2 \tag{23}$$

in which the dot signifies the first derivative with respect to time in the usual way.

If this is substituted into (21) and the equation is integrated with respect to r under the boundary condition that p approach the value p' at infinity, we find

$$(p - p')/\rho = (R^2\ddot{R} + 2R\dot{R}^2)/r - R^4\dot{R}^2/2r^4. \tag{24}$$

In the immediate vicinity of the sphere, the pressure $P(t)$ satisfies the relation

$$(P(t) - p')/\rho = R\ddot{R} + (3\dot{R}^2/2). \tag{25}$$

If the pressure is constant and if $R = R_0$, $\dot{R} = 0$ at $t = 0$, the first integral of (24) yields[13]

$$\dot{R}^2 = 2c^2[1 - (R_0/R)^3]/3. \tag{26}$$

If $R_0 = 0$, the second integral is

$$R = c(2/3)^{\frac{1}{2}}t. \tag{27}$$

Here $c^2 = (p_c - p')/\rho$ in which p' is the constant value of P. This solution, coupled with the preceding relation, is the incompressible fluid analog of Taylor's solution for a uniformly expanding sphere.

The case in which P satisfies the relation (20) can be integrated when p' is taken to be zero. This situation evidently is of great interest in our problem. The first integral of (24) is

$$\dot{R}^2 = 2c_0^2[(R_0/R)^3 - (R_0/R)^{3\gamma}]/3(\gamma - 1). \tag{28}$$

Here $c_0^2 = p_0/\rho$ in which p_0 is the initial pressure. This may be integrated further when $\gamma = \frac{4}{3}$, a value close to that assumed in Table VIII for hydrogen. The result is

$$c_0t/R_0 = (2z)^{\frac{1}{2}}(1 + 2z/3 + z^2/5) \tag{29}$$

in which $z = R/R_0 - 1$.

We may note that the term in $\ddot{R}$ in (25) plays a significant role in the foregoing solution, for (28) would take the form

$$\dot{R}^2 = 2c_0^2(R_0/R)^{3\gamma}/3 \tag{30}$$

if the term in $\ddot{R}$ were dropped.

9. EQUATIONS FOR DETERMINING p_0V_0

We shall now establish relationships for determining the quantity $p_0V_0 = 4\pi p_0R_0^3/3$ by requiring that the boundary R expand from the value R_0 to to R_c in the time τ_c. It is clear that a time substantially shorter than this would require a larger value of P_0V_0 and be unnecessarily short to form the bubble of minimum stable size, whereas a time substantially longer would allow the heat to diffuse away.

To begin with, we shall employ the relation

$$\tau_c = \int_{R_0}^{R_c} dR/\dot{R} \tag{31}$$

in conjunction with the simplified relation (30) obtained from (25) by assuming the term in $\ddot{R}$ is negligible. The lower limit of integration in (31) may be replaced by zero in evaluating the integral.

The resulting equation may be placed in the form

$$p_0V_0 = (3\rho/2)[2/(3\gamma + 2)]^2 \cdot (R_c/\tau_c)^2(R_c/R_0)^{3(\gamma-1)}V_c. \tag{32}$$

Values of this are given in Table IX.

TABLE IX. Values of p_0V_0 derived from Eq. (32) and values of the corresponding E_p.

	H_2	C_3H_8
p_0V_0(ev)	9.72	15.1
ν	2.5	15
E_p(ev)	24.3	226

The quantity p_0V_0, which is effectively $\frac{2}{3}$ of the translational energy of the molecules in the initial

[12] H. Lamb, *Hydrodynamics* (Dover Publications, New York, 1945); M. S. Plesset, J. Appl. Mech. **16**, 277 (1949).

[13] The solution represented by Eqs. (26) and (27) was obtained by M. Rogers. The solutions (28) and (29) apparently were first found by H. Lamb, Phil. Mag. **45**, 257 (1923). The author is indebted to Dr. Rogers and to Professor G. C. McVittie for valuable discussions.

thermal spike, must be multiplied by a factor ν larger than unity to obtain the total energy of the spike because of the internal degrees of freedom of the molecules. This factor, ν, would be 3.5 for hydrogen and 30 for propane if all the degrees of freedom were excited classically during the expansion of the thermal spike. We note, however, that for the values of $n_m V_c$ in Table V and of $4\pi R_0^3 p_0/3$ in Table IX, the temperature rise for all of the molecules within the sphere of volume V_c would be only of the order of 100 degrees. Thus values of ν somewhat lower than the maximum would be appropriate. We shall, somewhat arbitrarily, take the values 2.5 and 15, given in the next to the last row of Table IX. The energy of the thermal spike, namely

$$E_p = 4\pi\nu R_0^3 p_0/3, \tag{33}$$

is given in the last row of the table.

An alternative value for hydrogen may be obtained from Eq. (29) by setting $t = \tau_c$, $R = R_c$, and solving for $p_0 V_0$. The value $\gamma = 1.33$ used in deriving (29) is sufficiently close to the value of 1.4 adopted for hydrogen to make the comparison valuable. The new value for hydrogen and the corresponding value of E_p are given in Table X.

TABLE X. Values of $p_0 V_0$ and the corresponding values of E_p derived from Eq. (29) (for H_2) and from Eq. (35) (for propane).

	H_2	C_3H_8
$p_0 V_0$(ev)	0.97	2.02
ν	2.5	15
E_p(ev)	2.4	30.3

A corresponding solution for propane may be obtained in the following way. When γ is close to unity, as is the case for this liquid, we may approximate the relation (28) in the manner

$$\dot{R}^2 = 2c_0^2(R_0/R)^3 \log(R/R_0). \tag{34}$$

If we insert this relation in the integral (31) and treat the logarithmic term as if constant, we obtain

$$p_0 V_0 = (2/25)[1/\log(R_c/R_0)]\rho(R_c/\tau_c)^2 V_c. \tag{35}$$

Corresponding values are given in Table X.

We note that the values given in Table X are of the order of ten times smaller than those in Table IX, in spite of the fact that the value of $\dot{R}$ used in obtaining Eq. (32) was not zero when $R = R_0$. Evidently the high velocity of flow attained during the initial period of high pressure persists in the more exact solution derived by retaining the term in $\ddot{R}$. As we shall see in the next section the viscosity is sufficiently important that it seems unlikely that the flow persists so easily in actual fluids. Hence the writer is inclined to regard the values in Table IX as more realistic than those in Table X.

The work done when a gas expands adiabatically from volume V_0 to volume V_c is

$$W = P_0 V_0(1 - (V_0/V_c)^{\gamma-1})/(\gamma - 1) \tag{36}$$

in which γ is an appropriate value of C_p/C_v. For small values of $(\gamma - 1)\log(V_0/V_c)$ this becomes $P_0 V_0 \log(V_c/V_0)$, the value for an isothermal expansion.

We note that $P_0 V_0/(\gamma - 1)$ is essentially E_p. Thus whenever $\gamma - 1$ is not small compared to unity, that is in a case such as that of hydrogen (Table VIII), almost all the energy E_p would be expended in the process of adiabatic expansion by the time the bubble reaches the critical size. Somewhat less is expended in a case such as that of propane in which $\gamma - 1$ is small compared to unity. However the magnitude of the ratio V_c/V_0 guarantees that a large part of the energy E_p is expended in the adiabatic part of the expansion in any event.

The fact that a substantial part of the outward radial flow of the fluid occurs adiabatically in the case of the ideal fluid justifies the use of the values of γ given in the third row of Table VIII in the preceding treatment when considering the relationship between pressure and the volume of the hot zone.

It is perhaps worth noting in conjunction with this discussion that the work done per unit increase in volume dW/dV during any expansion, is equal to p. Thus the void is produced more efficiently when the pressure has diminished than when it is near its peak value p_0. Viscous behavior near time τ_0 could impede the expansion and preserve the energy in the spike for expansion at a later time when the pressure is lower. The larger energy expended when the pressure is higher appears in the form of shock waves. We shall see in the next section that the viscous behavior tends to be more marked in the later rather than the earlier phases of expansion.

10. RESISTANCE TO FLOW ARISING FROM VISCOSITY

We shall attempt to estimate the resistance to flow arising from the viscosity of the fluid under the conditions of interest to us. Unfortunately the analysis will be limited by the lack of accurate information concerning the values of the viscosity in the heated and compressed zone in which the

flow occurs. It is well known that the viscosity is sensitive to temperature, generally decreasing with increasing temperature; moreover Bridgman[14] has shown that the viscosity usually increases with increasing pressure. The portion of the fluid in which we are most interested is that around the initially heated zone in which the pressure is greatest and which is subject to the action of a shock wave which raises its temperature as well as its pressure. For lack of better information we shall assume that the two effects compensate under the circumstances of interest to us and that the appropriate value of the viscosity of hydrogen is in the vicinity of 0.01 centipoise, whereas the appropriate value for propane and similar fluids is near 1 centipoise.

The analysis will suggest that hydrogen behaves nearly like an ideal fluid during the flow, whereas propane probably behaves in a relatively viscous manner during the period in which the velocity of thermal flow would be small compared with that of the displacement flow if the fluid were ideal. Thus the picture developed in the preceding section, is born out fairly closely for hydrogen. On the other hand the picture is modified for propane. Viscous forces inhibit the rapid flow.

The radial velocity described by Eq. (23) has associated with it the radial compressive strain rate

$$\dot{\epsilon} = 2\dot{R}R^2/r^3. \tag{37}$$

The radial strain rate is counterbalanced by extensive strain rates of half the value in each of the two angular direction. If the medium possesses a viscosity η, the compressive strain rate is associated with a radial compressive stress

$$P_r = 2\eta\dot{\epsilon} = -4\eta R^2\dot{R}/r^3 \tag{38}$$

which will resist acceleration of the fluid and hence subtract from the applied pressure. The value of (38) at the moving interface is

$$P_R = -4\eta\dot{R}/R. \tag{39}$$

We may note from (24) that the first term on the right-hand side drops off more slowly with r than (38) for sufficiently large distances. Thus the effect of viscosity definitely is negligible at large distances if it is not important near $r = R$. On the other hand the solution derived using the theory of the incompressible fluid without taking account of viscosity is in error near the moving interface if the value of P_R derived using it is comparable to the applied pressure obtained from (20) for the values of p_0V_0 obtained previously. Whenever P_R is comparable to or greater than P, the medium behaves more nearly like a viscous solid for a region about the moving interface and the expressions for the pressure and velocity are different from those derived in Sec. 8.

We shall consider the value of R for which P_R is equal to the applied pressure, treating first the simple case in which $\dot{R}^2$ satisfies the relation (30). We find

$$P_R^2 = 16\eta^2\dot{R}^2/R^2 = 32\eta^2P/3\rho R^2 = P^2. \tag{40}$$

This leads in turn to the relation

$$(R/R_c)^{3\gamma-2} = (9/128\pi)(\rho/\eta^2) \cdot (p_0V_0/R_c)(R_0/R_c)^{3(\gamma-1)}. \tag{41}$$

The quantity on the right is about 0.61 for hydrogen when η is chosen to be 10^{-4} poise. Since $3\gamma - 2$ is 2.2 in this case, it follows that R/R_c is 0.80. It is readily seen that the viscous pressure at the moving boundary decreases relative to the applied pressure when R/R_c has a value smaller than this. Thus the initial flow occurs under circumstances in which the viscosity is relatively small in hydrogen, the viscosity being important only when R/R_c is close to unity.

The situation is very different for propane if the appropriate value of η is near 10^{-2} poise. The left-hand side of (41) then is 1.5×10^{-3} so that $R/R_c = 3.5 \times 10^{-3}$. In other words the viscosity is very important throughout the expansion. It follows that the ideal behavior of hydrogen suggested by the foregoing estimates is valid only if the very small value of the viscosity is appropriate.

If we employ the more accurate values for $\dot{R}^2$ obtained from (28) and discussed in the preceding section, we obtain

$$(R/R_c)^{6\gamma-5} = (9/128\pi)((\gamma - 1)\rho/\eta^2) \cdot (p_0V_0/R_c)(R_0/R_c)^{6(\gamma-1)} \tag{42}$$

in the case of hydrogen and

$$(R/R_c)^{6\gamma-5} = (3/128\pi)(1/\log(R/R_0))(\rho/\eta^2) \cdot (p_0V_0/R_c)(R_0/R_c)^{6(\gamma-1)} \tag{43}$$

for propane. The term in $\log(R/R_0)$ has been treated as a constant in deriving the second relation. It is clear that the conclusions drawn from the simpler solution are not radically altered. The first equation leads to a value $R/R_c = 0.36$. The ratio remains of the order of 10^{-3} in the second.

We may readily find a solution of the equations

[14] P. W. Bridgman, Proc. Am. Ac. Arts Sci. 61, 57 (1926).

for incompressible flow if we assume that the viscosity provides the dominant term resisting flow, outweighing inertial effects. Relation (23) remains valid, however the pressure when $r = R$ is the applied pressure (20). Thus we have Eq. (39) with P_R replaced by (20). The time τ required for the sphere to expand from R_0 to R_c is readily found to be

$$\tau = (4/3\gamma)(\eta/p_0V_0)(R_c/R_0)^{3(\gamma-1)}V_c. \quad (44)$$

If we set $\tau = \tau_c$, we obtain an equation for p_0V_0. The corresponding values are given in Table XI. It will be seen that the value for hydrogen is nearly the same as that given in Table IX; however, the value for propane is considerably larger. As previously, we may conclude that the influence of viscosity is marginal, although not negligible, for hydrogen if the value of η as small as 10^{-4} poise is valid, whereas viscosity is probably very important for propane and similar liquids which have viscosities near 10^{-2} poise.

TABLE XI. Values of p_0V_0 derived from Eq. (43) under the assumption that the appropriate viscosity for liquid hydrogen is 10^{-4} poise and that for propane is 10^{-2} poise.

	H_2	C_3H_8
p_0V_0(ev)	9.23	316
ν	2.5	15
E_p(ev)	23.1	4750

There is an indication in the preceding analysis that the energy required to form a bubble in liquid hydrogen in a practical case (Table III) is substantially larger than the theoretical values (Tables IX, X, XI), in contrast with the situation for propane (Table XI). Actually the estimates of the viscosity we have employed are sufficiently crude that the differences between experiment and theory for the two cases probably cannot be taken exceedingly seriously.

11. EXPANSION OF A RIGID SOLID

During an initial period of time after application of stress any liquid behaves like an elastic solid. The duration of this period depends upon the elastic constants and the viscosity. It is very short for fluids having viscosities in the range of interest to us. For example, if the stress involves only the shear modulus μ, the relaxation time is of the order,

$$\tau_r = 2\eta/\mu, \quad (45)$$

which we shall estimate for propane.

The shear modulus of an isotropic crystal is related to the compressibility β by the equation,

$$\mu = 3(1 - 2q)/2\beta(1 + q), \quad (46)$$

in which q is Poisson's ratio. If we assume that q is near 0.3 in propane and take β to be 1.0×10^{-10} cm^2/dyne (Table VII), we find that

$$\mu \cong 4.6 \times 10^9 \text{ dyne/cm}^2. \quad (47)$$

Thus $\tau_r \sim 5 \times 10^{-12}$ sec, if η is assumed to be near 10^{-2} poise. The value of τ_r would be even shorter for hydrogen if the assumption concerning η is correct.

It is interesting to estimate the expansion which an ideal elastic solid having the shear modulus estimated above for propane would undergo as a result of the presence of the initial heated zone. One readily finds from the theory of elasticity[6] that the amount by which an extended medium containing a spherical cavity of radius R_0 can expand when the material in the cavity exerts a pressure p_0 on its walls is

$$\Delta V = 3p_0V_0/4\mu. \quad (48)$$

If we give p_0V_0 the value 1 kev and μ the value (47), we readily find

$$\Delta V = 2.6 \times 10^{-19} \text{ cm}^3. \quad (49)$$

Thus,

$$\Delta V/V_c = 0.09.$$

12. GROWTH OF A MACROSCOPIC BUBBLE

As mentioned in Sec. 2, Plesset and Zwick[3] have treated the growth of a bubble in a superheated fluid after the nucleus of critical size has been attained. They have shown that both viscosity and inertial effects become unimportant soon after the bubble exceeds the critical radius. The ultimate rate of growth is determined primarily by the rate at which heat flows from the surrounding fluid to the surface of the bubble to provide the heat of vaporization of the liquid. In the steady state, the vapor inside the bubble is close to the boiling temperature T_b of the fluid at the pressure p_f [Eq. (2)] attained after the expansion of the chamber, whereas the main body of the liquid is at the superheated temperature T. The temperature drop $T - T_b$ takes place in a zone having an approximate thickness $2(Dt)^{\frac{1}{2}}$ corresponding to the distance over which heat diffuses in time t after the creation of the nucleating bubble of critical size.

The approximate asymptotic relation between

radius R and time t derived by Plesset and Zwick is

$$R = 2(3/\pi)^{\frac{1}{2}}\lambda(T - T_b)V_m t^{\frac{1}{2}}/H_s D^{\frac{1}{2}}, \qquad (50)$$

in which λ is the coefficient of thermal conductivity (Table III), V_m is the molar volume of the vapor, H_s is the heat of sublimation (Table I), and D is the thermal diffusion coefficient (Table III). For simplicity and without appreciable loss of generality we shall assume that p_f is one atmosphere and that T has the values given in Table I. The coefficient of $t^{\frac{1}{2}}$ in (50) then is 6.74 for hydrogen and 9.69 for propane when t is expressed in units of sec. It follows that a time of the order of a microsecond is required to produce a bubble of macroscopic size from the nucleating bubble which is formed in a time between 10^{-10} and 10^{-11} sec.

It should be added that in deriving (50) it is assumed that the number of molecules which pass from the liquid to the vapor phase is limited only by the flow of heat and that the rate coefficient for evaporation is not in itself a limiting factor. More specifically, it is assumed that the pressure inside the bubble is automatically the equilibrium value associated with the temperature of the inner wall. It is easy to show that this is the case if the activation energy for evaporation is not substantially larger than the energy of vaporization. Presumably the two energies are almost equal in molecular liquids of the type used in bubble chambers.

We have seen that the operating temperature of the bubble chamber must be fairly close to the critical temperature if the free electrons produced by the incident particle are to be able to produce a bubble of critical size in all but highly improbable cases. Thus T in (50) is fairly narrowly defined for a given fluid. Of the other quantities appearing in this relation, only T_b and V_m are adjustable. We note that the product $(T - T_b)V_m$ increases with decreasing p_f. V_m is particularly sensitive, varying inversely with p_f. Thus it would appear that one could increase the rate of forming macroscopic bubbles indefinitely by lowering p_f sufficiently. It is easy to show that the rate of vaporization would not be a limiting factor in this process if the activation energy for evaporation and the energy of vaporization are equal. Instead, it seems likely that freezing of the liquid about the bubble as a result of evaporation cooling would provide a limit to the advantage that may be expected. Presumably such cooling would occur through homogeneous nucleation when the temperature at the inner wall of the bubble drops appreciably below the freezing temperature.[15]

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[15] See for example, D. Turnbull in *Solid State Physics* (Academic Press, Inc., New York, 1956), Vol. 3, p. 225.